

# Three-parametric Weibully Deteriorated EOQ Model with Price Dependent Demand and Shortages under Fully Backlogged Condition

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**ABSTRACT:** Inventory is an important part of manufacturing, distribution and retail infrastructure where demand and deterioration play an important role in choosing the best inventory policy. In this paper, an inventory model with price dependent demand and three-parameter Weibull deterioration rate is considered. Shortages are allowed and are completely backlogged. Numerical example is provided to illustrate the theoretical results. Sensitivity analysis and graphical presentation of the major parameters with respect to the optimal solution is also carried out.

**KEYWORDS:** EOQ model, Three-parameter Weibull deterioration, Price dependent demand, Completely backlogged shortages.

## I. INTRODUCTION

One of the most important concerns of the management is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system should be minimum. This is somewhat more important when the inventory undergo deterioration. The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage such that the item cannot be used for its original purpose. Most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables, foodstuffs, etc. suffer from depletion by direct spoilage while kept in store. Highly volatile liquids such as gasoline, alcohol, turpentine, etc. undergo physical depletion over time through the process of evaporation. In the development of economic production lot size models, many researchers have considered the deterioration rate, demand rate, unit cost, etc. as fixed, but all of them probably will have some fluctuations for each cycle in real life situations. So, in practical situations, if these quantities are treated as fuzzy variables then it will be more realistic.

In classical inventory models, the demand rate is assumed to be constant. But in reality demand for many goods may be time dependent, stock dependent and price dependent. Burwell, Dave, Fitzpatrick and Roy [1] developed economic lot size model for price-dependent demand under quantity and freight discounts. An inventory system of ameliorating items for price dependent demand rate was considered by Mondal, Bhunia and Maiti [12]. You [16] developed an inventory model with price and time dependent demand.

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 12, December 2015

Gothi and Chatterji [10] developed an EPQ model for imperfect quality items under constant demand rate and varying holding cost. Parmar and Gothi [14] developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution and time-dependent quadratic demand. Gothi and Parmar [11] extended the above deterministic inventory model for deteriorating items following two parameter Weibull distribution under quadratic demand with time dependent IHC and partially backlogged shortages. Bhojak and Gothi [2] developed an EOQ model with time-dependent demand and Weibully distributed deterioration. Parmar, Aggarwal and Gothi [13] developed an order level inventory model for deteriorating items under varying demand condition. Chatterji and Gothi [4] developed an optimal EPQ model with Weibully distributed deterioration rate and time varying IHC. Further, Chatterji and Gothi [5] developed an EOQ model for deteriorating items under two and three parameter Weibull distribution and constant IHC with partially backlogged shortages. Recently, Chatterji and Gothi [6] have developed an inventory model for two-parameter Weibully deteriorated items with exponential demand rate and completely backlogged shortages. Also, Bhojak and Gothi [3] have developed an EPQ model with time dependent holding cost and Weibully distributed deterioration under shortages.

Saha and Chakraborty [15] proposed a fuzzy EOQ model for time dependent deteriorating items and time dependent demand with shortages. Dutta and Pavan Kumar [8] presented a fuzzy inventory model without shortages using trapezoidal fuzzy number with sensitivity analysis. Further, Dutta and Pavan Kumar [9] developed a fuzzy inventory model for deteriorating items with shortages under fully backlogged condition. Recently, Duari and Chakraborty [7] have developed an order level EOQ model for deteriorating items in a single warehouse system with price dependent demand and shortages. In this paper, an EOQ model is developed with three parameter Weibully deteriorated items along with price dependent demand and fully backlogged shortages. An analytical solution of the model is discussed and illustrated with numerical example. Sensitivity and graphical analysis of the optimal solution with respect to changes in different parametric values is also examined.

## II. NOTATIONS

The following notations are used to develop the model:

1.  $Q(t)$  : Instantaneous rate of the inventory level at any time  $t$  ( $0 \leq t \leq T$ ).
2.  $p$  : Price of the item.
3.  $R(t)$  : Demand rate varying with price.
4.  $\theta(t)$  : Deterioration rate.
5.  $A$  : Ordering cost per order during the cycle period.
6.  $C_d$  : Deterioration cost per unit per unit time.
7.  $C_h$  : Inventory holding cost per unit per unit time.
8.  $C_s$  : Shortage cost per unit per unittime.
9.  $p_c$  : Purchase cost per unit per unit time.
10.  $S$  : Initial inventory level at time  $t = 0$ .
11.  $S_1$  : Inventory level at time  $t = \mu$ .
12.  $S_2$  : Maximum inventory level during the shortage period at time  $t = T$ .
13.  $t_2$  : Time at which shortages start,  $\mu \leq t_2 \leq T$ .

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 12, December 2015

- 14.  $T$  : Length of the replenishment cycle.
- 15. TC: The average total cost for the time period  $[0, T]$ .

### III. ASSUMPTIONS

The following assumptions are considered to develop the model:

1. A single item is considered over the prescribed period of time.
2. Replenishment rate is infinite and lead time is zero.
3. The demand rate of the product is of the form  $R(t) = ap^{-b}$ ,  $a, b > 0$ ,  $p$  is price of the item.
4. Deterioration of the units starts only after the time  $t = \mu$ .
5. Deterioration rate is  $\theta(t) = \alpha\beta(t-\mu)^{\beta-1}$  where  $t \geq \mu$ ,  $\alpha$  is scale parameter ( $0 < \alpha \ll 1$ ),  $\beta$  is shape parameter ( $\beta > 0$ ) and  $\mu$  is the location parameter ( $\mu > 0$ ).
6. Shortages are allowed and are fully backlogged.
7. No repair or replenishment of the deteriorated items takes place during a given cycle.
8. Total inventory cost is a real and continuous function which is convex to the origin.

### IV. MATHEMATICAL MODEL AND ANALYSIS

In the mathematical model, initially the stock level is  $S$ . Then the inventory level decreases to meet up the demand with demand rate  $ap^{-b}$  and the stock level becomes  $S_1$  at time  $t = \mu$ . The deterioration starts at time  $t = \mu$  and during the time interval  $[\mu, t_2]$ , mainly due to demand and partly due to deterioration with rate  $\alpha\beta(t-\mu)^{\beta-1}$ , the inventory level diminishes and ultimately falls to zero at time  $t = t_2$ . Thereafter, shortages occur during the time interval  $[t_2, T]$  and there becomes a complete backlog of  $S_2$  units till time  $t = T$ .

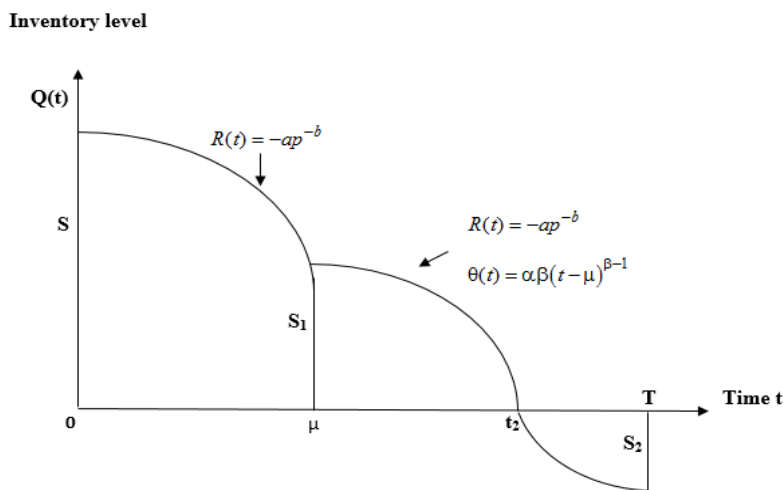


Figure 1: Graphical presentation of the inventory system

## International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 12, December 2015

The differential equations which governs the instantaneous state of  $Q(t)$  over the time intervals  $[0, \mu]$ ,  $[\mu, t_2]$  and  $[t_2, T]$  are given by

$$\frac{dQ(t)}{dt} = -ap^{-b} \quad (0 \leq t \leq \mu) \quad (1)$$

$$\frac{dQ(t)}{dt} + \alpha\beta(t-\mu)^{\beta-1} Q(t) = -ap^{-b} \quad (\mu \leq t \leq t_2) \quad (2)$$

$$\frac{dQ(t)}{dt} = -ap^{-b} \quad (t_2 \leq t \leq T) \quad (3)$$

Using boundary conditions

$$Q(0) = S, \quad Q(\mu) = S_1, \quad Q(t_2) = 0 \text{ and } Q(T) = -S_2 \quad (4)$$

The solutions of equations (1), (2) and (3) are given by

$$Q(t) = S - ap^{-b}t \quad (0 \leq t \leq \mu) \quad (5)$$

$$Q(t) = ap^{-b} \left\{ (t_2 - t) \left[ 1 - \alpha(t - \mu)^\beta \right] + \frac{\alpha}{\beta + 1} \left[ (t_2 - \mu)^{\beta+1} - (t - \mu)^{\beta+1} \right] \right\} \quad (\mu \leq t \leq t_2) \quad (6)$$

$$Q(t) = ap^{-b}(t_2 - t) \quad (t_2 \leq t \leq T) \quad (7)$$

Substituting  $Q(\mu) = S_1$  in equation (5), we get

$$S_1 = S - ap^{-b}\mu \quad (8)$$

Substituting  $Q(t_1) = S_1$  in equation (6), we get

$$S_1 = ap^{-b} \left\{ (t_2 - \mu) + \frac{\alpha}{\beta + 1} (t_2 - \mu)^{\beta+1} \right\} \quad (9)$$

Comparing equations (8) and (9) we get

$$S = ap^{-b} \left\{ t_2 + \frac{\alpha}{\beta + 1} (t_2 - \mu)^{\beta+1} \right\} \quad (10)$$

By placing  $Q(T) = -S_2$  in equation (7), we get

$$S_2 = ap^{-b}(T - t_2) \quad (11)$$

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 12, December 2015

$$\Rightarrow T = t_2 + \frac{S_2}{a} p^b \tag{12}$$

**Cost Components:**

The total cost per replenishment cycle consists of the following cost components:

**1. Operating Cost**

The operating cost  $OC$  over the period  $[0, T]$  is

$$OC = A \tag{13}$$

**2. Deterioration Cost**

The deterioration cost  $DC$  over the period  $[\mu, t_2]$  is

$$DC = \int_{\mu}^{t_2} \theta Q(t) dt$$

$$\therefore DC = C_d \left\{ \alpha a p^{-b} \frac{(t_2 - \mu)^{\beta+1}}{(\beta+1)(\beta+2)} \right\} \tag{14}$$

**3. Inventory Holding Cost**

The holding cost for carrying inventory over the period  $[0, t_2]$  is

$$IHC = C_h \left[ \int_0^{\mu} Q(t) dt + \int_{\mu}^{t_2} Q(t) dt \right]$$

$$\therefore IHC = C_h \left\{ \left( S\mu - \frac{ap^{-b}}{2} \mu^2 \right) + ap^{-b} \left[ \frac{1}{2} (t_2 - \mu)^2 + \frac{\alpha\beta (t_2 - \mu)^{\beta+2}}{(\beta+1)(\beta+2)} \right] \right\} \tag{15}$$

**4. Shortage Cost**

The shortage cost  $SC$  over the period  $[t_2, T]$  is

$$SC = -C_s \int_{t_2}^T Q(t) dt$$

## International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 12, December 2015

$$\therefore SC = C_s \left\{ ap^{-b} \frac{(T-t_2)^2}{2} \right\} \quad (16)$$

### 5. Purchase Cost

The purchase cost  $PC$  during the period  $[0, T]$  is

$$PC = P_c \{S + S_2\} \quad (17)$$

Hence, the average total cost for the time period  $[0, T]$  is given by

$$TC = \frac{1}{T} [OC + DC + IHC + SC + PC]$$

$$\therefore TC = \frac{1}{T} \left[ \begin{aligned} & A + C_d \left\{ \alpha ap^{-b} \frac{(t_2 - \mu)^{\beta+1}}{(\beta+1)(\beta+2)} \right\} + C_h \left[ \left( s\mu - \frac{ap^{-b}}{2} \mu^2 \right) + ap^{-b} \left\{ \frac{1}{2} (t_2 - \mu)^2 + \frac{\alpha\beta(t_2 - \mu)^{\beta+2}}{(\beta+1)(\beta+2)} \right\} \right] \\ & + C_s \left\{ ap^{-b} \frac{(T-t_2)^2}{2} \right\} + P_c \{S + S_2\} \end{aligned} \right] \quad (18)$$

$t_2^*$  and  $T^*$  are the optimum values of  $t_2$  and  $T$  respectively, which minimize the cost function  $TC$

and they are the solutions of the equations  $\frac{\partial TC}{\partial t_2} = 0$  &  $\frac{\partial TC}{\partial T} = 0$  such that

$$\left. \begin{aligned} & \left[ \left( \frac{\partial^2 TC}{\partial t_2^2} \right) \left( \frac{\partial^2 TC}{\partial T^2} \right) - \left( \frac{\partial^2 TC}{\partial t_2 \partial T} \right)^2 \right]_{t_2=t_2^*, T=T^*} > 0 \\ & \left[ \frac{\partial^2 TC}{\partial t_2^2} \right]_{t_2=t_2^*, T=T^*} > 0 \end{aligned} \right\} \quad (19)$$

### V. NUMERICAL EXAMPLE

To illustrate the proposed model, an inventory system with the following hypothetical values is considered. By taking  $A = 300$ ,  $\alpha=0.0001$ ,  $\beta=2$ ,  $a=3$ ,  $b=5$ ,  $p=4$ ,  $\mu=8$ ,  $C_h=16$ ,  $C_s=17$ ,  $C_d=28$  and  $p_c=15$  (with appropriate units).

## International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 12, December 2015

The optimal values of  $t_2$  and  $T$  by using maple mathematical software are  $t_2^*=66.54266794$ ,  $T^*=145.2076423$  units and the optimal total cost per unit time  $TC=3.972148137$  units.

### VI. SENSITIVITY ANALYSIS AND GRAPHICAL ANALYSIS

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. Here, we study the sensitivity for the cycle length  $T$  and total cost per time unit  $TC$  with respect to the changes in the values of the parameters  $A$ ,  $\alpha$ ,  $\beta$ ,  $a$ ,  $b$ ,  $p$ ,  $\mu$ ,  $C_h$ ,  $C_s$ ,  $C_d$  and  $p_c$ .

The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all other remaining parameters as fixed.

**Table – 1: Partial Sensitivity Analysis**

Parameter	Values	$t_2$	$T$	TC	% change in TC
A	- 40	54.94777431	115.3730777	3.053408668	- 23.13
	- 20	61.37979650	131.5587436	3.539185841	- 10.90
	+ 20	71.16740962	157.9767229	4.367456031	+ 09.95
	+ 40	75.10198610	169.2730834	4.734107385	+ 19.18
A	- 40	70.63287382	148.7283663	3.933466909	- 0.97
	- 20	68.48346981	146.9888837	3.953882917	- 0.46
	+ 20	65.08951278	144.2940313	3.988701606	+ 0.42
	+ 40	63.70351726	143.2122901	4.003854898	+ 0.80
B	- 40	80.48300011	157.0386865	3.856777352	- 2.90
	- 20	77.20052572	154.2689311	3.882313157	- 2.26
	+ 20	51.64808500	133.9567967	4.143304976	+ 4.31
	+ 40	39.33711510	125.4816334	4.334346125	+ 9.12
a	- 40	79.68975775	182.9795829	3.112957666	- 21.63
	- 20	72.19812659	160.8964298	3.569229270	- 10.14
	+ 20	62.32751868	134.0185134	4.337391482	+ 09.20
	+ 40	58.79403447	124.9475729	4.674182270	+ 17.67
b	- 40	20.05438497	39.18516757	15.94796738	+ 301.49
	- 20	38.14976194	76.68939144	7.853598066	+ 97.42
	+ 20	104.8597957	270.0061740	2.067252271	- 47.96
	+ 40	151.9633147	503.0476644	1.095599478	- 72.42
p	- 40	22.27346122	43.62410897	14.24006754	+ 258.50
	- 20	42.88511378	87.13549516	6.859807521	+ 72.70
	+ 20	90.77600393	218.6915618	2.577940015	- 35.10
	+ 40	114.1634789	308.0251507	1.803408790	- 54.60

## International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 12, December 2015

$\mu$	- 40	65.97040042	145.0587014	3.982913428	+ 0.27
	- 20	66.40197140	145.3550278	3.976177616	+ 0.10
	+ 20	67.01732489	145.7820999	3.966800318	- 0.13
	+ 40	67.37449807	146.0326832	3.961491642	- 0.27
$C_s$	- 40	59.79112295	172.6055731	3.415158375	- 14.02
	- 20	63.77302553	156.3182760	3.731295137	- 6.06
	+ 20	68.87704288	137.8014288	4.163254314	+ 04.81
	+ 40	70.63402235	131.9501381	4.319307287	+ 08.74
$p_c$	- 40	66.72584534	145.5816527	3.953756028	- 0.46
	- 20	66.69496558	145.5589505	3.962952372	- 0.23
	+ 20	66.63337000	145.5136752	3.981343324	+ 0.23
	+ 40	66.60265385	145.4911018	3.990537937	+ 0.46
$C_d$	- 40	66.77950031	145.6211056	3.970626826	- 0.04
	- 20	66.72172487	145.5786232	3.971388491	- 0.02
	+ 20	66.60674620	145.4941092	3.972905771	+ 0.02
	+ 40	66.54954087	145.4520758	3.973661405	+ 0.04
$C_h$	- 40	85.95055442	156.6008675	3.562662077	- 10.31
	- 20	74.91869605	150.2003990	3.793327005	- 4.50
	+ 20	60.15227130	141.9146246	4.116093770	+ 3.62
	+ 40	54.83757222	138.9874182	4.235002096	+ 6.62

### Graphical Presentation

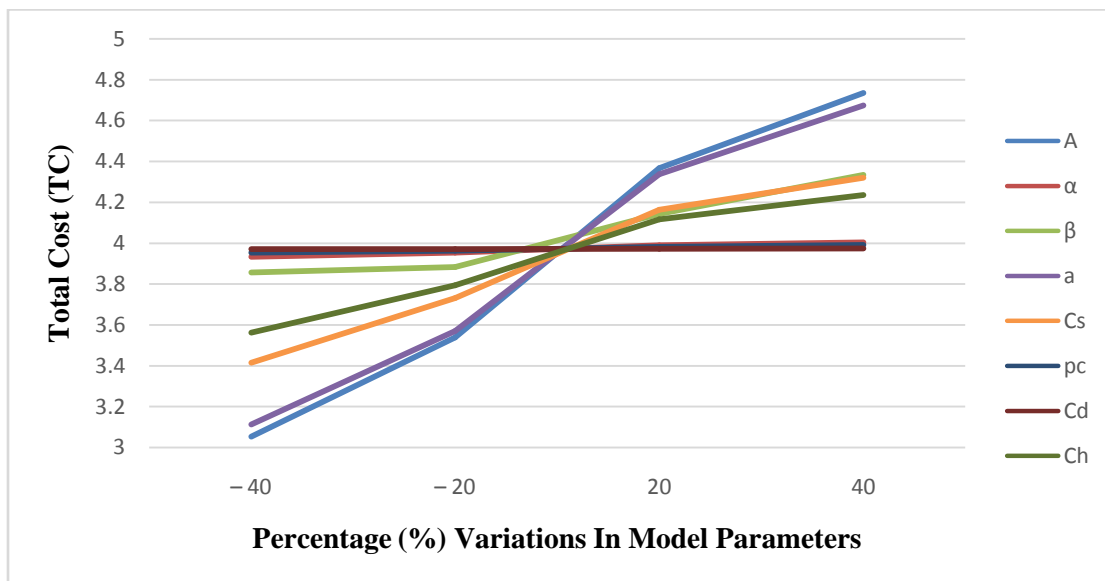


Figure – 2



# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 12, December 2015

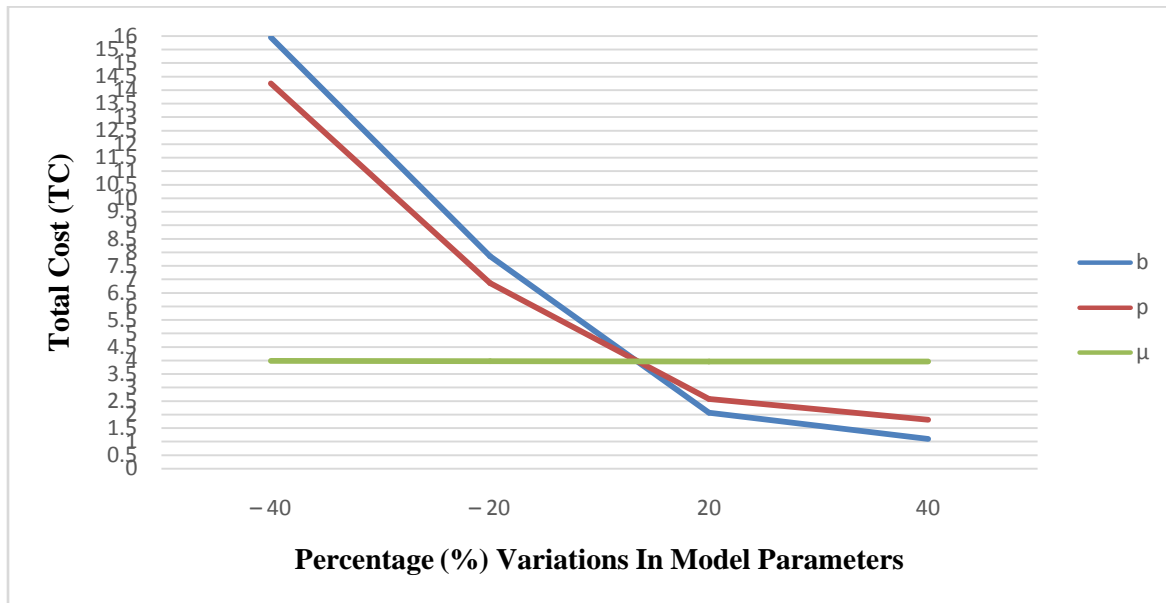


Figure – 3

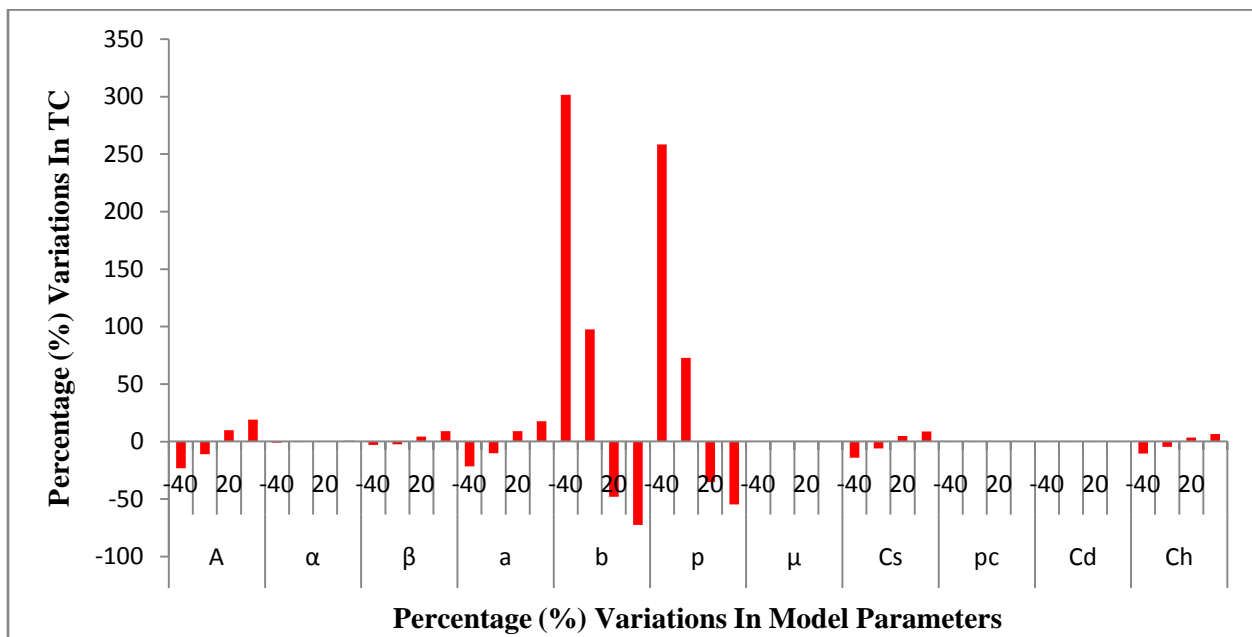


Figure – 4

# International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 4, Issue 12, December 2015

The graphical presentation of sensitivity analysis of model parameters is represented in the above figure 4 which shows the effect of percentage change in the values of parameters on the percentage change in the minimum total cost.

## VII. CONCLUSION

From partial sensitivity analysis and graphical analysis following conclusions can be made:

- From Figure-2 it is observed that as the scale parameter  $\alpha$ , shape parameter  $\beta$ , operating cost  $A$ , deterioration cost  $C_d$ , shortage cost  $C_s$ , purchase cost  $p_c$ , initial rate of demand  $a$ , shortage cost  $C_s$  and holding cost  $C_h$  increase the total cost TC increases.
- From Figure-2 it is also observed that total cost TC is highly sensitive to the change in the values of  $A$ ,  $a$  and  $C_s$ , moderately sensitive to the change in the values of  $C_h$  and  $\beta$  and slightly sensitive to the change in the values of  $\alpha$ ,  $p_c$  and  $C_d$ .
- From partial sensitivity analysis it is observed that as  $b$ ,  $p$  and location parameter  $\mu$  increase the total cost  $TC$  decreases.
- From Figure-3 it is observed that total cost TC is slightly sensitive to the change in the values of  $\mu$  and highly sensitive to the change in the values of  $b$  and  $p$ .
- Also from Figure-4 of percentage variations in model parameters, it can be easily observe that total cost TC is highly sensitive to the parameters  $b$  &  $p$ , moderately sensitive to the parameters  $A$  &  $a$ , and very slightly sensitive to the parameters  $\alpha$ ,  $\beta$ ,  $\mu$ ,  $C_s$ ,  $p_c$ ,  $C_d$  &  $C_h$ .

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