

Inclusion Relations between Euler and Taylor Methods of Summability in Non-Archimedean Fields

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ABSTRACT: The aim of this paper is to study the inclusion relations between Euler and Taylor methods of summability in complete, non-trivially valued, Non-Archimedean Fields.

KEYWORDS: Non-Archimedean, Euler method, Taylormethod[13].

I. INTRODUCTION

Throughout this paper,[7] K denotes complete, non-trivially valued, non-archimedean fields. Infinite matrices and sequences considered in the sequel have entries in K [12]. A general reference for the study of summability methods in classical one is [7]. For analysis, in non-archimedean fields, see [6].

If $A = (a_{n,k})$, $a_{n,k} \in K$, $n, k = 0, 1, 2, \dots$ is an infinite matrix, then the A -transform of a sequence $x = \{x_k\}$, $x_k \in K$, $k = 0, 1, 2, \dots$, is the sequence, where

$$(Ax)_n = \sum_{k=0}^n a_{nk} x_k, \quad n = 0, 1, 2, \dots, \text{ it being assumed that the series on the right converge..}$$

In the sequel, the following definitions and theorems are needed[8].

Definition 1. Let $\{x_k\}$ be a sequence in K and $x \in K$. We say that $\lim_{k \rightarrow \infty} x_k = x$ if for each $\epsilon > 0$, the set $\{|x - x_k| \geq \epsilon\}$ is finite. In such a case, we say that x is the limit of [9] $\{x_k\}$.

Definition 2. Let $A = (a_{n,k})$, $a_{n,k} \in K$, $n, k = 0, 1, 2, \dots$, if $\lim_{n \rightarrow \infty} (Ax)_n$ exists whenever $\lim_{n \rightarrow \infty} x_k$ exists, then

A is said to be convergence preserving. Further, if $\lim_{n \rightarrow \infty} (Ax)_n = s$

whenever $\lim_{n \rightarrow \infty} x_k = s$, we say that A is regular.

Theorem 1. $A = (a_{n,k})$ is a regular if and only if [6]

(i) $\sup_{n,k} |a_{n,k}| < \infty$

(ii) $\lim_{n \rightarrow \infty} a_{n,k} = 0, \quad k = 0, 1, \dots,$

(iii) $\lim_{n \rightarrow \infty} \sum_{k=0}^{\infty} a_{n,k} = 1$

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Fredrick Hartmann [4] derived the necessary and sufficient conditions for inclusion of sonnenchein matrix methods with complex entries in classical analysis. He also discussed the necessary and sufficient condition for the regularity of Taylor,[11] Laurent and Euler-Knopp method and proved some properties such as invertibility, inclusion of such methods.

In classical analysis, Schoonmaker[8] studied inclusion results among Euler,Taylor and Abel-Type matrix and he used $0 < q < 1$ for Abel-type matrix.[10] we prove some inclusion relations between Euler and Taylor methods of summability in K [5].

II. THEOREMS ON EULER AND TAYLOR METHODS OF SUMMABILITY

Definition 3. Let $p \in K$ be such that $|1 - p| < 1$. The Euler method $[E, p]$ is given by the matrix $(e_{n,k}^{(p)})$ as follows;

If $p \neq 1$

$$e_{n,k}^{(p)} = \begin{cases} nC_k p^k (1-p)^{n-k}, & k \leq n; \\ 0, & k > n, \end{cases}$$

If $p = 1$

$$e_{n,k}^{(1)} = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases}$$

$e_{n,k}^{(p)}$ is called the $[E, p]$ matrix[4].

Definition 4. Let $r \in K$ be such that $|r| < 1$. The Taylor method $[T, r]$ is given by the matrix $(t_{n,k}^{(r)})$ as follows;

$$t_{n,k}^{(r)} = \begin{cases} kC_n r^{k-n} (1-r)^{n+1}, & k \geq n; \\ 0, & k < n, \end{cases}$$

In classical case, Agnew [1] and Laush [5] proved inclusion theorems on Euler and Taylor summability methods respectively[14].

We proved some properties of Euler[2] and Taylor[3] methods of summability in Non-Archimedean Fields. In the sequel, the following results are needed [2,3]

Theorem 2 The $[E, p]$ method is regular if $|1 - p| < 1$

Corollary 1. The $[E, p]$ matrix is invertible and its inverse is the $\left[E, \frac{1}{p}\right]$ matrix .

Theorem 3. Let $\pi = \sup\{x/x \in K, |x| < 1\}$. Let $|r| < \pi^{\frac{1}{\pi-1}}$. Then $[T, r]$ method is regular.

Corollary 2. The $[T, r]$ matrix is invertible and its inverse is $\left[T, -\frac{r}{1-r}\right]$ matrix

Theorem 4. If $p = \frac{r}{r-1}$ and $|p| < 1$, then $[E, p] \left[T, \frac{r}{r-1}\right] = [S, p]$

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$$\begin{aligned}
 \text{Proof. Let } [E, p] \left[T, \frac{r}{r-1} \right] &= \left(\sum_{n=m}^k m C_n p^n (1-p)^{m-n} \right) \left(k C_n \left(\frac{r}{r-1} \right)^{k-n} \left(1 - \frac{r}{r-1} \right)^{n+1} \right) \\
 &= \sum_{n=m}^k m C_n p^n (1-p)^{m-n} k C_n p^{k-n} (1-p)^{n+1}, \text{ since } p = \frac{r}{r-1} \\
 &= p^k (1-p)^{m+1} \sum_{n=m}^k m C_n k C_n \quad (\text{By binomial Co-efficient relation}) \\
 &= (m+k) C_k p^k (1-p)^{m+1} \\
 &= [S, p]
 \end{aligned}$$

Theorem 5. If $p = \left| \frac{r}{r-1} \right|$, $|r| < 1$ and $|p| < 1$ then $[E, p] \supset [T, r]_{[1]}$

Proof: Suppose that $\{s_n\}$ is $[T, r]$ summable to σ . Let $\{\sigma_n\}$ denote the $[T, r]$ transform of $\{s_n\}$. Since

$\left[T, \frac{r}{r-1} \right] = [T, p]$, the Euler Transform $[E, p]$ of Taylor transform $\left[T, \frac{r}{r-1} \right]$ of $\{\sigma_n\}$ is given by

$$\begin{aligned}
 \tau_k &= \left(\sum_{n=0}^k k C_n p^n (1-p)^{k-n} \right) \left(m C_n \left(1 - \frac{r}{r-1} \right)^{n+1} \left(\frac{r}{r-1} \right)^{m-n} \sigma_m \right) \\
 &= \left(\sum_{n=0}^k k C_n p^n (1-p)^{k-n} \right) \left(\sum_{m=n}^{\infty} m C_n p^{m-n} (1-p)^{n+1} \sigma_m \right) \left(\because p = \frac{r}{r-1} \right)
 \end{aligned}$$

It can be verified that

$$\sum_{k=0}^n k C_n m C_n = (m+k) C_k = \sum_{m=0}^{\infty} \left(\sum_{n=0}^k k C_n m C_n \right) p^m (1-p)^{k+1} \sigma_m$$

(by interchanging the order of summation)

Therefore, $\tau_m = \sum_{m=0}^{\infty} (m+k) C_k p^m (1-p)^{k+1}$

$$\sigma_m = [S, p] \sigma_m$$

Since $[S, p]$ is regular and $\{\sigma_m\}$ converges to σ , we find that $\{\tau_m\}$ converges to σ . i.e., $\{s_n\}$ is $[E, p]$ summable to σ which implies that $[E, p] \supset [T, r]$.

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