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On Nonlinear Equations for ϕ-Contractor Couple in Fuzzy Normed Spaces

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ABSTRACT: In this chapter, I introduce the concept of ϕ-contractor couple in fuzzy normed spaces, which extends the concept of ϕ-contractor in fuzzy normed spaces given by Fang and Song [7].
By using the concept of ϕ-contractor couple we investigate the existence of solutions for set-valued nonlinear operator equations. As an application of our main theorem, a new fixed point theorem in fuzzy normed spaces is obtained. Our results improve and extend the results of Fang and Song [7].

KEY WORDS: Fuzzy normed spaces, ϕ-contractor couple.

I. INTRODUCTION

Based on the fact that in many situations the distance between two points is inexact rather than a single real number, Kaleva and Seikkala [13] initiated the concept of fuzzy metric space by describing the distance of points as a fuzzy real number. Since each usual metric space and each Menger probabilistic metric space can be considered as a special case of fuzzy metric space, the study for the fuzzy metric space has attracted many authors and several results for nonlinear mappings have been given in some literatures [5], [10], [11], [12]. The book written by Schweizer and Sklar [16] provides a number of examples of probabilistic metric spaces, all of which are from probabilistic origin. Of course, all of which can be regarded as examples of the fuzzy metric spaces.

Inspired by the work of Kaleva and Seikkala [13], Felbin [8], [9] introduced and studied the fuzzy normed linear space. It is as important as the concept of Menger probabilistic normed linear space introduced by Serstnev [17] and more over, each usual normed linear space and each Menger probabilistic normed linear space can still be considered as it's a special case. Xiao and Zhu [18] studied the linear topological structure of the fuzzy normed linear space and obtained some basic properties. Many authors proved results in fuzzy normed linear spaces including Bag and Samanta [1], Xiao and Zhu [19], [20], Fang [5], Fang and Song [7].

Altman established the contractive theory in Banach spaces, which offers a strong tool to study the existence and uniqueness of solutions for nonlinear operator equations. Inspired by Altman's work, Lee and Padgett [14], [15] studied the random contractive theory and showed the existence and uniqueness of solution for random operator equations with a random contractor. Chang [2] and Chang et al. [3], [4], Fang [6], Zeng [21] studied the probabilistic contractive theory in Menger probabilistic normed spaces and discussed the existence and uniqueness of solutions for operator equations with the probabilistic contractors.

II. PRELIMINARIES

Throughout this paper, let R = $(-\infty, +\infty)$, R+ = $[0, +\infty)$ and N be the set of all natural numbers. A fuzzy set on R i. e. a mapping x: $R\to [0, 1]$ is called a fuzzy number. A fuzzy number x is called convex if its α-level sets $[x]_\alpha$ is a
Convex set in R for every $\alpha \in (0, 1]$, where $[x]_\alpha = \{t: x(t) \geq \alpha\}$. A fuzzy number x is called normal if there exists a $t_0 \in R$ such that $x(t_0) = 1$. A fuzzy number x is nonnegative if x (t) = 0 for all t < 0. The fuzzy number $\bar{0}$ is defined by $\bar{0}(t) = 1$ for t = 0 and $\bar{0}(t) = 0$ for $t \neq 0$.

We denote the set of all nonnegative upper semi continuous normal convex fuzzy numbers by G. Obviously, if $x \in G$ then each of its α level set $[x]_\alpha$ is a closed interval $[a^\alpha, b^\alpha]$, where $a^\alpha \in R^+$ and $b^\alpha = +\infty$ is admissible. When $b^\alpha = +\infty$, then $[a^\alpha, b^\alpha]$ means the interval $[a^\alpha, +\infty)$.

Imitating the definition of fuzzy metric spaces in [13], Fang and Song [7] introduced the concept of fuzzy normed spaces.

Definition 1. Let X be a real vector space, ||. || a mapping from X into G and the mappings L, R: [0, 1] → [0, 1] be symmetric, nondecreasing in both arguments and satisfy L(0, 0) = 0, R(1, 1) = 1. Denote

$[||x||]_\alpha = [|||x|||_1^\alpha , |||x|||_2^\alpha]$ for $x \in X, \alpha \in (0, 1]$.

The quadruple (X. ||.||, L, R) is called a fuzzy normed space (briefly, an FN-space) and ||.|| a fuzzy norm, if the following conditions are satisfied:

(FN-1) $||x|| = \bar{0}$ if and only if $x =\theta$;

(FN-2) $||kx|| = |k| \, ||x||$ for all $x \in X, k \in R$

(FN-3) for all $x, y \in X$,

(1) $||x + y||(s + t) \ge L (||x||(s),||y||(t))$ whenever $s \le |||x|||_1^1$ and $t \le |||y|||_1^1$ and $s+ t \le |||x + y|||_1^1$

(2) $||x + y||(s + t) \le R(||x||(s), ||y||(t))$ whenever $s \ge |||x|||_1^1$ and $t \ge |||y|||_1^1$ and $s+ t \ge |||x + y|||_1^1$.

Remark 1 [7]. There is a little difference between the above definition and the definition of the fuzzy normed space given in [8]. In [9], the definition requires that for all $x \in X$, $x \ne \theta$, there exists $\alpha_0 \in (0, 1]$ independent of x such that

$|||x|||_2^\alpha < \infty$ for all $\alpha \le \alpha_0$ and $\inf_{\alpha \le \alpha_0} |||x|||_1^\alpha > 0$.

Remark 2 [7] **By** induction, we can generalize the triangle inequality (FN-3) as the following:

(FN-3)' for any $n \in N$ and $x_1, \ldots x_n \in X$,

(1) $||\sum_{i=1}^n x_i||(t) \ge L((||x_1||(t_1), L(||x_2||(t_2),\ldots,L(||x_{n-1}||(t_{n-1}), ||x_n||(t_n))\ldots)),$

whenever $t_i \le |||x_i|||_1^1$ $i = 1,\ldots,n$ and $t = \sum_{i=1}^n t_i \le ||| \sum_{i=1}^n x_i|||_1^1$;

(2) $||\sum_{i=n}^n x_i||(s)\le R((||x_1||(s_1), R(||x_2||(s_2),\ldots,R(||x_{n-1}||(s_{n-1}), ||x_n||(s_n))\ldots)),$

whenever $s_i \ge |||x_i|||_1^1$ $i = 1,\ldots,n$ and $s = \sum_{i=1}^n s_i \ge ||| \sum_{i=1}^n x_i|||_1^1$.

Theorem A [7]. Let (X, ||.||, L, R) be an FN-space. If $\{R^n(t)\}$ is an equi-continuous sequence of mappings at t = 0, where

$R^1(t) = t$, $R^n(t) = R(t, R^{n-1}(t))$, $i = 2, 3\ldots$,

Then for each $\alpha \in (0, 1]$ there exists $\beta \in (0, \alpha]$ such that

(1) $|||\sum_{i=1}^n x_i|||_2^\alpha \le \sum_{i=1}^n |||x_i|||_2^\beta$ for all $x_i \in X$,

$i = 1, 2, \ldots,n$.

Theorem B [7]. Let (X, ||.||, L, R) be an FN-space. If R satisfies the condition $\lim_{a \to 0+}R(a, a) = 0$, then there exists a topology $\tau_{||.||}$ on X such that $(X, \tau_{||.||})$ is a real metrizable Hausdorff topological vector space having $ß = \{B(\varepsilon, \alpha): \varepsilon > 0, \alpha \in(0, 1]\}$ as a neighbourhood base of θ, where

$B(\varepsilon, \alpha) = \{x \in X : |||x|||_2^\alpha < \varepsilon\}$.

By Theorem 2, many topological concepts can be introduced in fuzzy normed spaces.

Definition 2 [8,9]. Let (X, ||.||, L, R) be an FN-space. A sequence $\{x_n\}$ in X is said to converges to $x \in X$ (often write $\lim_{n\to\infty} x_n = x$ or $x_n\to x$) if

$\lim_{n\to\infty}||x_n - x|| = \bar{0}$ i. e.

$\lim_{n\to\infty}|||x_n - x|||_2^\alpha = 0$, for all $\alpha \in (0, 1]$.

Definition 3 [8,9]. Let (X, ||.||, L, R) be an FN-space. A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if

$\lim_{n\to\infty, m \to\infty}||x_m - x_n|| = \bar{0}$ i. e.

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$\lim_{n\to\infty, m\to\infty} |||x_m - x_n|||_2^\alpha = 0$, for all $\alpha \in (0, 1]$.
(X, ||.||, L, R) is called complete if every Cauchy sequence in X converges in X.
Definition 4 [7]. A nonempty subset A in (X, ||.||, L, R) is said to be bounded if there exists an M > 0 such that $\mathrm{Sup}_{x\in A}|||x|||_2^\alpha \le M$, for all $\alpha \in (0, 1]$.

We denote the collection of all bounded subset in X by Ω_X.

Definition 5 [7]. Let (X, ||.||, L, R) be an FN-space and A, B $\in \Omega_X$. For $\alpha \in (0, 1]$, we define $|||A|||_2^\alpha$ and $\rho_\alpha(A, B)$ as follows:

(2) $|||A|||_2^\alpha = \inf_{a\in A} |||a|||_2^\alpha$ and

(3) $\rho_\alpha(A, B) = \max\{\sup_{a\in A}\inf_{b\in B} |||a - b|||_2^\alpha, \sup_{b\in B}\inf_{a\in A} |||a - b|||_2^\alpha\}$,
respectively.

By using the definitions of $|||A|||_2^\alpha$ and $\rho_\alpha(A, B)$, Fang and Song [7] showed that the following theorem is true.
Theorem C. Let (X, ||.||, L, R) be an FN-space which satisfies the following conditions:
(4) $\lim_{t\to 0+}R(t, t) = 0$ and $\lim_{t\to 0+}||x||(t) = 0$, for all $x \in X$.
If A, B $\in \Omega_X$, then:

$|||A|||_2^\alpha = 0$ for all $\alpha \in (0, 1]$ if and only if $\theta \in A$;

$|||kA|||_2^\alpha = |k|\, |||A|||_2^\alpha$, $k \in R$;

$\rho_\alpha(x + A, x + B) = \rho_\alpha(x - A, x - B) = \rho_\alpha(A, B)$ for all $x \in X$.
Definition 6 [6]. A function $\varphi: R^+ \to R^+$ is said to satisfy the condition
(Φ), if it is nondecreasing and $\sum_{n=1}^{\infty} \varphi^n(t) < +\infty$ for all $t > 0$, where $\varphi^n(t)$ denotes the n^{th} iteration of $\varphi(t)$.

Remark 3. [6]. Let $\varphi(t)$ satisfying the condition (Φ), then we have
$\lim_{n\to\infty} \varphi^n(t) = 0$, $\varphi(t) < t$, for all $t > 0$, and $\varphi(0) = 0$.
In the sequel, we always suppose that an FN-space (X, ||.||, L, R) satisfies the following conditions:
(a) $\lim_{t\to\infty}||x||(t) = 0$, for all $x \in X$.
(b) $R(0, a) = a$ and $\{R^n(t)\}$ is equi-continuous at $t = 0$.
Definition 7 [7]. Let (X, ||.||, L, R) and (Y, ||.||, L, R) be two fuzzy normed spaces. A mapping T: Y $\to$ X is said to be odd, if T (-y) = -Ty for all y $\in$Y. S(Y, X) denotes the set of all odd mappings from Y to X.
Definition 8 [7]. A set valued mapping P: D(P) $\subset$ X $\to \Omega_Y$ (respectively single-valued mapping P: D(P) $\subset$ X $\to$Y) is said to be closed, if for any $x_n \in D(P)$ and $y_n \in P(x_n)$ (respectively $y_n = P(x_n)$) we have $x \in D(P)$ and $y \in P(x)$ (respectively y = P(x)) whenever $x_n \to x$ and $y_n \to y$.
Definition 9. Let (X, ||.||, L, R) and (Y, ||.||, L, R) be two fuzzy normed spaces, P: D(P) $\subset$ X $\to \Omega_Y$ and Q: D(Q) $\subset$ X $\to \Omega_Y$ be two set-valued mappings. Let Γ_1, Γ_2: X $\to$ S(Y, X) be two mappings. (Γ_1, Γ_2) is called a Φ- contractor couple of P and Q with respect to u if there exists a function $\varphi: R^+ \to R^+$ satisfying condition (Φ) such that
(5) $\rho_\alpha(P(x + \Gamma_1(x) y), Q(X) + y))$

$$\le \varphi\,(\max\{|||y|||_2^\alpha, |||P(x + \Gamma_1(x)y) - u|||_2^\alpha, |||Q(x)- u + y|||_2^\alpha\})$$

for all $\alpha \in (0, 1]$, for all $x \in D(Q)$, $y \in \{y \in Y : x + \Gamma_1(x)y \in D(P)\}$.
(6) $\rho_\alpha(Q(x + \Gamma_2(x)y), P(X) + y))$

$$\le \varphi\,(\max\{|||y|||_2^\alpha, |||Q(x + \Gamma_2(x)y) - u|||_2^\alpha, |||P(x)- u + y|||_2^\alpha\})$$

for all $\alpha \in (0, 1]$, for all $x \in D(P)$, $y \in \{y \in Y : x + \Gamma_2(x)y \in D(Q)\}$.

III. MAIN RESULTS

Theorem 1. Let (X, ||.||, L, R) and (Y, ||.||, L, R) be two complete fuzzy normed spaces. Let P: D (P) $\subset$ X $\to \Omega_Y$ and Q: D (Q) $\subset$ X $\to \Omega_Y$ be two
Closed set-valued mappings and let Γ_1, Γ_2: X $\to$ S(Y, X). If the following conditions are satisfied:

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(i) $x + \Gamma_1(x)y \in D(P)$ for all $x \in D(Q)$ and $y \in Y$,
(ii) $x + \Gamma_2(x)\, y \in D(Q)$ for all $x \in D(P)$ and $y \in Y$.
(iii) there exists a constant $M > 0$ such that

$|||\Gamma_1(x)\, y|||_2^{\alpha} \le M\, |||y|||_2^{\alpha}$ for all $x \in D(Q)$ and $y \in Y$,

$|||\Gamma_2(x)\, y|||_2^{\alpha} \le M\, |||y|||_2^{\alpha}$ for all $x \in D(P)$ and $y \in Y$.

(iv) for given $x \in D(P)$, $y \in P(x)$, there exists $z_1 \in P(x - \Gamma_1(x)y)$ such that

$|||z_1|||_2^{\alpha} \le \rho_\alpha(P(x - \Gamma_1(x)y), Q(x) - y))$,

For given $x \in D(Q)$, $y \in Q(x)$, there exists $z_2 \in Q(x - \Gamma_2(x)y)$ such that

$|||z_2|||_2^{\alpha} \le \rho_\alpha(Q(x - \Gamma_2(x)\, y), P(x) - y))$.

(V) (Γ_1, Γ_2) is Φ- contractor couple of P and Q with respect to u.
Then the following system of nonlinear set-valued operator equations

(7) $u \in P(x),\ u \in Q(x)$

has a solution.

Proof. Without loss of generality, we can suppose that $u = \theta$. In this case (5) can be written as follows:

(8) $\rho_\alpha(P(x + \Gamma_1(x)\, y), Q(x) + y))$

$$\le \varphi\,(\max\{|||y|||_2^{\alpha}, |||P(x + \Gamma_1(x)\, y)|||_2^{\alpha}, |||Q(x) + y|||_2^{\alpha}\})$$

In fact if $u \ne \theta$, putting $T_1(x) = P(x) - u$ and $T_2(x) = Q(x) - u$ for all $x \in D(P)$, for all $x \in D(Q)$ then $u \in P(x)$, $u \in Q(x)$is equivalent to $\theta \in T_1(x)$,
$\theta \in T_2(x)$. Besides, by (iii) of Theorem 3, we know that P and Q satisfies (5) is equivalent to that T_1 and T_2 satisfies
$\rho_\alpha(T_1(x + \Gamma_1(x)\, y), T_2(x) + y))$
$\le \varphi\,(\max\{|||y|||_2^{\alpha}, |||T_1(x + \Gamma_1(x)\, y)|||_2^{\alpha}, |||T_2(x) + y|||_2^{\alpha}\})$,
For all $\alpha \in (0, 1]$ and so we can turn to discuss equation $\theta \in P(x)$, $\theta \in Q(x)$ under the condition (8).
For any $x_0 \in D(Q)$ take $y_0 \in Q(x_0)$ and let $x_1 = x_0 - \Gamma_1(x_0)y_0$. By condition (i) we have $x_1 \in D(P)$. From (2) and $\theta \in Q(x_0) - y_0$, it follows that

$|||Q(x_0)|||_2^{\alpha} \le |||y_0|||_2^{\alpha}$ and $|||Q(x_0) - y_0|||_2^{\alpha} = 0$.

Replacing x and y in (8) by x_0 and $- y_0$ respectively, we get

$\rho_\alpha(P(x_1), Q(x_0) - y_0)) \le \varphi\,(\max\{|||y_0|||_2^{\alpha}, |||P(x_1)|||_2^{\alpha}, |||Q(x_0) - y_0|||_2^{\alpha}\})$. (9)

We can prove that

$\rho_\alpha(P(x_1), Q(x_0) - y_0)) \le \varphi\,(|||y_0|||_2^{\alpha})$, for all $\alpha \in (0, 1]$.

In fact if $|||P(x_1)|||_2^{\alpha} > |||y_0|||_2^{\alpha}$ then from (9) and Remark 3, it follows that

$\rho_\alpha(P(x_1), Q(x_0) - y_0)) \le \varphi\,(|||P(x_1)|||_2^{\alpha}) < |||P(x_1)|||_2^{\alpha}$,

Which is a contradiction. Therefore

$\rho_\alpha(P(x_1), Q(x_0) - y_0)) \le \varphi\,(|||y_0|||_2^{\alpha})$, for all $\alpha \in (0, 1]$.

On the other hand by condition (iv) there exists $y_1 \in P(x_1)$ such that

$|||y_1|||_2^{\alpha} \le \rho_\alpha(P(x_1), Q(x_0) - y_0))$, for all $\alpha \in (0, 1]$.

So

$|||y_1|||_2^{\alpha} \le \varphi\,(|||y_0|||_2^{\alpha})$ for all $\alpha \in (0, 1]$.

Put $x_2 = x_1 - \Gamma_2(x_1)y_1$. By (i) $x_2 \in D(Q)$.
By the method as stated above there exists $y_2 \in Q(x_2)$ such that

$|||y_2|||_2^{\alpha} \le \varphi\,(|||y_1|||_2^{\alpha}) \le \varphi^2(|||y_0|||_2^{\alpha})$, for all $\alpha \in (0, 1]$.

Inductively, we can obtain two sequences $\{x_n\}$ in X and $\{y_n\}$ in Y, satisfying the following conditions:

$$(10)\qquad \left.\begin{aligned} x_{2n+1} &= x_{2n} - \Gamma_1(x_{2n})y_{2n}, \\ x_{2n+2} &= x_{2n+1} - \Gamma_2(x_{2n+1})y_{2n+1} \end{aligned}\right\}$$

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and $y_{2n} \in Q(x_{2n})$, $y_{2n+1} \in P(x_{2n+1})$,

and

(11) $|||y_n|||_2^\alpha \le \varphi^n(|||y_0|||_2^\alpha)$, for all $\alpha \in (0, 1]$.

By (10), (11) and condition (iii), we have

$$|||x_{n+1} - x_n|||_2^\alpha = |||\Gamma_1(x_n)y_n|||_2^\alpha$$
$$\le M|||y_n|||_2^\alpha \quad (12)$$
$$\le M\varphi^n(|||y_0|||_2^\alpha), \text{ for all } \alpha \in (0, 1].$$

Since $\{R^n(t)\}$ is equi-continuous at $t = 0$, by Theorem A, for each $\alpha \in (0, 1]$ there exists a $\beta \in (0, \alpha]$ such that

$$|||x_n - x_m|||_2^\alpha \le \sum_{i=n}^{m-1} |||x_{i+1} - x_i|||_2^\beta \quad (13)$$
$$\le M \sum_{i=n}^{m-1} \varphi^i(|||y_0|||_2^\beta)$$

Since $\sum_{n=0}^{\infty} \varphi^n(|||y_0|||_2^\beta) < +\infty$ for given $\varepsilon > 0$ there exists an $n \in N$ such that (14)

$\sum_{i=n}^{m-1} \varphi^I(|||y_0|||_2^\beta) < \varepsilon/M$,

Whenever $m > n \ge N$. From (13) and (14) it follows that

$|||x_n - x_m|||_2^\alpha < \varepsilon$, for all $\alpha \in (0, 1]$ and $m > n \ge N$.

So $\{x_n\}$ is a Cauchy sequence in X. By the completeness of X, we can suppose that $x_n \to x_* \in X$. Moreover by (12) and Remark (3) we obtain

$y_n \to \theta$. Notice that $y_n \in Q(x_n)$ and the set valued mapping P and Q are closed . Hence we have $\theta \in P(x_*)$, $\theta \in Q(x_*)$. This completes the proof.

Theorem 2. Let $(X, \|.\|, L, R)$ be a complete FN-space. Let T, S: $X \to \Omega_X$ be two closed set-valued mappings satisfying the following conditions:

(i) for all $\alpha \in (0, 1]$ and $x, y \in X$

(15) $\rho_\alpha(Ty, Sx) \le \varphi(\max\{|||x - y|||_2^\alpha, |||y - Ty|||_2^\alpha, |||y - Sx|||_2^\alpha\}$

(16) $\rho_\alpha(Tx, Sy) \le \varphi(\max\{|||x - y|||_2^\alpha, |||y - Tx|||_2^\alpha, |||y - Sy|||_2^\alpha\}$

where $\varphi: R^+ \to R^+$ satisfies the condition (Φ),

(ii) for given $x \in X$ and $a \in Tx$, there exists $b_1 \in a - Ta$ such that

(17) $|||b_1|||_2^\alpha \le \rho_\alpha(Ta, Sx)$, for all $\alpha \in (0, 1]$

for given $x \in X$ and $a \in Sx$, there exists $b_2 \in a - Sa$ such that

(18) $|||b_2|||_2^\alpha \le \rho_\alpha(Sa, Tx)$, for all $\alpha \in (0, 1]$

Then T and S have a fixed point in X, i. e. there exists $x_* \in X$ such that

$x_* \in T(x_*)$ and $x_* \in S(x_*)$.

Proof. Replacing y by x + y in (15), we get

$$\rho_\alpha(Tx+y, Sx) = \rho_\alpha(x + y - Tx+y, x + y - Sx)$$
$$\le \varphi(\max\{|||y|||_2^\alpha, |||x + y - T(x + y)|||_2^\alpha, |||x + y - Sx|||_2^\alpha\},$$

for all $\alpha \in (0, 1]$.

Replacing a by x - y in (17), we get

$$|||b_1|||_2^\alpha \le \rho_\alpha(Tx - y, Sx)$$
$$= \rho_\alpha(x - y - T(x - y), (x - y) - Sx), \text{ for all } \alpha \in (0, 1].$$

Put $P(x) = x - Tx$, $Q(x) = x - Sx$, $u = \theta$ and $\Gamma_1(x) = I_X$, $x \in X$, $\Gamma_2(x) = I_X$, $x \in X$. (I_X is the identity mapping on X). It is easy to see that all the conditions of Theorem 1 are satisfied. Therefore there exists $x_* \in X$ such that $\theta \in P(x_*) = x_* - Tx_*$ and $\theta \in Q(x_*) = x_* - Sx_*$, i. e. x_* is fixed point of T and x_* is fixed point of S.

Remark 4. Notice that the usual normed space (X, ||.||) can be considered as a special case of the FN- space. Indeed, each nonnegative real number x can be considered as a fuzzy number $\tilde{x}$ defined by

$$\tilde{x}(t) = \begin{cases} I, & t = x \\ 0, & t \neq x. \end{cases}$$

If taking L(a, b) ≡ 0 and R(a, b) = max{a, b}, then (X, ||.||, L, R) is an FN-space satisfying conditions (a) and (b). Therefore, using Theorem 2, we can obtain a fixed point theorem for set valued mappings in the usual normed spaces.

Corollary 1. Let (X, ||.||) be a Banach space and T, S: $X \to \Omega_X$ be two closed set-valued mappings satisfying the following conditions:

(i) $H(Ty, Sx) \leq \varphi(\max\{\|x - y\|, \|y - Ty\|, \|y - Sx\|\})$ x, y ∈ X,

(ii) $H(Tx, Sy) \leq \varphi(\max\{\|x - y\|, \|y - Tx\|, \|y - Sy\|\})$ x, y ∈ X.

(iii) For given x ∈ X and a ∈Tx there exists $b_1 \in a - Ta$ such that
$\|b_1\| \leq H(Ta, Sx)$,
for given x ∈ X and a ∈Sx there exists $b_2 \in a - Sa$ such that
$\|b_2\| \leq H(Sa, Tx)$,

Where H(A, B) is a Hausdorff metric induced by ||.|| and $\varphi: R^+ \to R^+$ satisfies condition (Φ). Then T and S have a fixed point in X, i. e. there exists $x_* \in X$ such that $x_* \in T(x_*)$ and $x_* \in S(x_*)$

IV. CONCLUSIONS

Fang and Song [7] defined ϕ-contractor in fuzzy normed spaces. In this paper, we have extended the concept of ϕ-contractor and defined ϕ-contractor couple in fuzzy normed spaces. With the help of ϕ-contractor couple we have proved the existence theorem of solutions for set-valued nonlinear operator equations in fuzzy normed spaces. We have applied our existence theorem to prove a new fixed point theorem in fuzzy normed spaces.

Many questions are raised by this work. First one such, the examination of the conditions which enable one to easily apply the existence theorem and fixed point theorem, we have discussed, which are mostly stated as purely mathematical results. The second question of which our theorems can give constructive proofs.

Other questions can be posed and indeed all are under investigation and will be considered elsewhere.

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