

Density of States of an Electron Gas in Lower Dimensions

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ABSTRACT : We discuss the detailed theoretical aspects of the density of states of an electron gas in lower dimensions. This is one of the important parameter for calculating various electronic properties. From this study it is clear that the density of states of an electron gas in one and three dimensions is proportional to Energy and for two dimensions it remains constant.

KEY WORDS: Density of states , Energy.

I. INTRODUCTION

The density of states plays an important role in the kinetic theory of solids [1,4,5,6]. The product of the density of states and the probability distribution function is the number of occupied states per unit volume at a given energy for a system in thermal equilibrium. This value is widely used to investigate various physical properties of matter [2,3,7]. The following are examples, using two common distribution functions, of how applying a distribution function to the density of states can give rise to physical properties.

Physics of low dimensional system has been a topic of interest from the theoretical point of view for a long time. This is mainly due to the fact that simple models are amenable for an exact solution from which more complicated results may be interpreted.

Some of the well known 1D models in condensed matter physics are

- * Kronig penny model for a band structure of a crystalline solid.

- * Lattice vibrations and Phonon Dispersions

- * Linear array of interacting spins

Real electrons are 3D but can be made to behave as though they are free to move in fewer dimensions. This can be achieved by trapping them in a narrow potential well that restricts their motion in one dimension to discrete energy levels. If the separation between these energy levels is large enough the electron will appear to be frozen into the ground state and no motion will be possible in this dimension. The result is a 2D electron gas. The same effect can be achieved with a 2D potential well which leaves the electron free to move in 1D only as a quantum wire. [7,8]

II. DENSITY OF STATES

2.1 Density of states of an electron gas in 3D

Density of states is an important parameter which plays a major role in calculating various properties. It is defined as the number of states or orbital per unit energy range.

If N is the total number of orbital with energy $\leq E$ then the Density of states is ,

$$D_3(E) = \frac{dn}{dE} \quad \text{where } n \text{ is the number of states or orbital per unit volume.}$$

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Energy is given by $E = \hbar^2 K^2 / 2m$

If K_f is the Fermi wave vector then the volume is , $V = \frac{4}{3}\pi K_f^3$

In a volume $(\frac{2\pi}{L})^3$ there is one allowed state. Therefore in unit volume there will be $(\frac{L}{2\pi})^3$ states.

$$N = (\frac{L}{2\pi})^3 K_f^3$$

$$K_f = (3\pi^2 n)^{\frac{1}{3}}$$

$$E_f = \hbar^2 K_f^2 / 2m$$

Substituting the value of k ,

$$E_f = \hbar^2 / 2m * (3\pi^2 n)^{\frac{2}{3}}$$

$$E_f = C n^{\frac{2}{3}}$$

$$dE_f / dn = C * \frac{2}{3} (n^{-\frac{1}{3}}) = \alpha n^{\frac{1}{3}}$$

Simplifying we get ,

Density of states at E_f is ,

$$D_3(E_f) = 1/\alpha * C^{1/2} E_f^{1/2}$$

$$D_3(E) = \frac{\sqrt{2Em^2}}{\pi^2 \hbar^3} \dots\dots\dots (1)$$

This is the equation for Density of states of an electron gas in 3D.

2.2 Density of states of an electron gas in 2d

The expression for the Density of states of an electron gas can be divided in a similar way as the 3D provided the volume is replaced by area.[9,10]

$$\text{Area of Fermi surface} = 4\pi K_f^2$$

$$\text{Radius of Fermi surface} = \frac{2\pi}{L}$$

In an area $(\frac{2\pi}{L})^2$ there is only one allowed state. Therefore in unit area the number of states will be $(\frac{L}{2\pi})^2$.

$$\text{Number of states} = (\frac{L}{2\pi})^2 4\pi K_f^2$$

$$K_f = (n\pi)^{\frac{1}{2}}$$

Energy is given by $E = \hbar^2 K^2 / 2m$

$$E = \hbar^2 / 2m * n\pi$$

$$dE/dn = \hbar^2 / 2m * \pi$$

Density of states of an electron gas in 2D is,

$$D_2(E) = 2m/\pi \hbar^2 \dots\dots\dots (2)$$

This is the equation for Density of states of an electron gas in 2D.

2.3 Density of states of an electron gas in 1D

In one dimension, the area of the Fermi surface in 2D is replaced by the radius of the Fermi sphere[11,12]

$$\text{Radius of Fermi surface} = \frac{2\pi}{L}$$

In a length $\frac{2\pi}{L}$ there is an allowed state.

$$\text{Number of states per unit length} = 2n\pi$$

Energy is given by $E = \hbar^2 K^2 / 2m$

$$E = 2 \hbar^2 \pi^2 n^2 / m$$

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$$dE/dn = 2 \hbar^2 \pi^2 n^2 / m$$

Density of states of an electron gas in 1D is,

$$D_1(E) = 1/C E^{-1/2} \dots\dots\dots (3)$$

This is the equation for Density of states of an electron gas in 1D.

III. RESULTS AND DISCUSSIONS

Density of states is one of the important parameter for calculating various electronic properties. Hence the expression for density of states of an electron gas in one, two and three dimensions have been studied. The main results are

$$D_1(E) \propto E^{-1/2},$$

$$D_2(E) = \text{Constant and}$$

$$D_3(E) \propto E^{1/2} \text{ respectively.}$$

The density of states of an electron gas in three dimensions is derived from the energy of an electron. To find three dimensions its volume is considered and as a result it shows that it is proportional to the squareroot of energy. For two dimensions the volume is replaced by area and here the density of states remains constant. Then for one dimension its area is replaced by the radius and it is inversely proportional to the squareroot of energy.

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