

Improved Ranking of Generalized Trapezoidal Fuzzy Numbers

S. Sagaya Roseline¹, E.C. Henry Amirtharaj²

Assistant Professor, Department of Mathematics, Bishop Heber College, Tiruchirappalli, Tamilnadu, India¹

Associate Professor, Department of Mathematics, Bishop Heber College, Tiruchirappalli, Tamilnadu, India²

ABSTRACT : Ranking of fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. In this paper, an improved method of ranking of generalized trapezoidal fuzzy numbers based on rank, perimeter, mode, divergence and spread is proposed. The proposed method is very simple and easy to apply in the real life problems. Numerical examples are provided to illustrate the proposed ranking method and compared with different existing approaches. And the proposed ranking method is applied in solving the generalized fuzzy transportation problem.

KEYWORDS: Fuzzy numbers, Trapezoidal Fuzzy Numbers, Generalized Trapezoidal Fuzzy Numbers, Ranking of generalized fuzzy numbers.

I. INTRODUCTION

Zadeh [1,2] introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life. Fuzzy numbers must be ranked before an action is taken by a decision maker. An efficient method for ordering the fuzzy numbers is by the use of a ranking function, which maps each fuzzy number into the real line, where a natural order exists. The method for ranking was first proposed by Jain[3]. Yager[4] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0,1]. An approach is presented for the ranking of fuzzy numbers in Kaufmann and Gupta [7]. Cheng[6] presented a method for ranking fuzzy numbers by using the distance method. Abbasbandy and Hajjari[9] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α – levels of trapezoidal fuzzy numbers. Chen and Chen[10] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Kumaretal [11] proposed RM approach for ranking the generalized trapezoidal fuzzy numbers. Amit kumar, Pushpinder Singh, Jagdeep Kaur [12,13] proposed a ranking of generalized fuzzy numbers. In this paper, a ranking method of generalized trapezoidal fuzzy numbers is proposed based on area, perimeter, mode, divergence, left spread and right spread. Numerical examples are provided to illustrate the new ranking method and compared with different existing approaches. Finally the ranking method is applied to order generalized fuzzy numbers in solving Generalized Fuzzy Transportation Problem(GFTP).

II. PRELIMINARIES

In this section some basic definitions are reviewed (Kaufmann and Gupta [7])

Definition

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicates the membership grade of the element in the set A . The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) ; x \in X \}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

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Definition

A fuzzy set \tilde{A} defined on the universal set of real numbers R , is said to be a fuzzy number if its membership function has the following characteristics:

- (i) $\mu_{\tilde{A}} : R \rightarrow [0,1]$ is continuous.
- (ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
- (iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
- (iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a \leq b \leq c \leq d$.

Definition

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

Definition

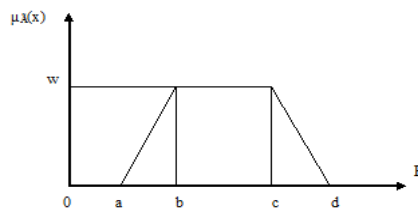
A fuzzy set \tilde{A} defined on the universal set of real numbers R , is said to be generalized fuzzy number if its membership function has the following characteristics

- (i) $\mu_{\tilde{A}} : R \rightarrow [0,1]$ is continuous.
- (ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
- (iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
- (iv) $\mu_{\tilde{A}}(x) = w$, for all $x \in [b, c]$, where $0 \leq w \leq 1$.

Definition

A generalized fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)}, & a \leq x \leq b \\ w, & b \leq x \leq c \\ \frac{w(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$



A Generalized trapezoidal fuzzy number

The arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers R , are reviewed as in [5,8]

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

- (i) $\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_1, w_2))$, where $a_1, b_1, c_1, d_1, a_2, b_2, c_2$ and d_2 are any real numbers
- (ii) $\tilde{A} \ominus \tilde{B} = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2; \min(w_1, w_2))$, where $a_1, b_1, c_1, d_1, a_2, b_2, c_2$ and d_2 are any real numbers

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- (iii) $\tilde{A} \otimes \tilde{B} = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2; \min(w_1, w_2))$, where $a_1, b_1, c_1, d_1, a_2, b_2, c_2$ and d_2 are positive real numbers
- (iv) $\tilde{A} \oslash \tilde{B} = (a_1/a_2, b_1/b_2, c_1/c_2, d_1/d_2; \min(w_1, w_2))$, where $a_1, b_1, c_1, d_1, a_2, b_2, c_2$ and d_2 are all nonzero positive real numbers

Also according to [10], if $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ are two generalized trapezoidal fuzzy numbers then

- (i) $\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \text{minimum}(w_1, w_2))$
- (ii) $\tilde{A} \ominus \tilde{B} = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2; \text{minimum}(w_1, w_2))$
- (iii) $\tilde{A} \otimes \tilde{B} = (a^1, b^1, c^1, d^1; \text{minimum}(w_1, w_2))$, where $a^1 = \text{minimum}(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$,
 $b^1 = \text{minimum}(b_1 b_2, b_1 c_2, c_1 b_2, c_1 c_2)$, $c^1 = \text{maximum}(b_1 b_2, b_1 c_2, c_1 b_2, c_1 c_2)$,
 $d^1 = \text{maximum}(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$
- (iv) $\lambda \tilde{A} = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1), \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1), \lambda < 0 \end{cases}$

III. IMPROVED RANKING METHOD OF GENERALIZED TRAPEZOIDAL FUZZY NUMBERS

In this section, an improved method of ranking of generalized trapezoidal fuzzy numbers to the ranking in [15] is proposed as follows

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ be a generalized trapezoidal fuzzy number then

- (i) Rank $\mathfrak{R}(\tilde{A}) = \frac{w_1}{2} (c_1 - b_1 + d_1 - a_1)$
- ii) Perimeter $P(\tilde{A}) = \sqrt{(a_1 - b_1)^2 + w_1^2} + \sqrt{(d_1 - c_1)^2 + w_1^2} + (c_1 - b_1) + (d_1 - a_1)$
- (iii) mode $(\tilde{A}) = \frac{w_1(b_1 + c_1)}{2}$, (iv) divergence $(\tilde{A}) = w_1(d_1 - a_1)$
- (v) Left spread $(\tilde{A}) = w_1(b_1 - a_1)$, (vi) Right spread $(\tilde{A}) = w_1(d_1 - c_1)$

Two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ can be compared using the following approach

Step 1: Find $\mathfrak{R}(\tilde{A})$ and $\mathfrak{R}(\tilde{B})$

- Case (i) If $\mathfrak{R}(\tilde{A}) > \mathfrak{R}(\tilde{B})$ then $\tilde{A} > \tilde{B}$
- Case (ii) If $\mathfrak{R}(\tilde{A}) < \mathfrak{R}(\tilde{B})$ then $\tilde{A} < \tilde{B}$
- Case (iii) If $\mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$ then go to step 2

Step 2: Find $P(\tilde{A})$ and $P(\tilde{B})$

- Case (i) If $P(\tilde{A}) > P(\tilde{B})$ then $\tilde{A} > \tilde{B}$
- Case (ii) If $P(\tilde{A}) < P(\tilde{B})$ then $\tilde{A} < \tilde{B}$
- Case (iii) If $P(\tilde{A}) = P(\tilde{B})$ then go to step 3

Step 3: Find mode (\tilde{A}) and mode (\tilde{B})

- Case (i) If mode $(\tilde{A}) > \text{mode}(\tilde{B})$ then $\tilde{A} > \tilde{B}$
- Case (ii) If mode $(\tilde{A}) < \text{mode}(\tilde{B})$ then $\tilde{A} < \tilde{B}$
- Case (iii) If mode $(\tilde{A}) = \text{mode}(\tilde{B})$ then go to step 4

Step 4: Find divergence (\tilde{A}) and divergence (\tilde{B})

- Case (i) If divergence $(\tilde{A}) > \text{divergence}(\tilde{B})$ then $\tilde{A} > \tilde{B}$
- Case (ii) If divergence $(\tilde{A}) < \text{divergence}(\tilde{B})$ then $\tilde{A} < \tilde{B}$
- Case (iii) If divergence $(\tilde{A}) = \text{divergence}(\tilde{B})$ then go to step 5

Step 5: Find Left spread (\tilde{A}) and Left spread (\tilde{B})

- Case (i) If Left spread $(\tilde{A}) > \text{Left spread}(\tilde{B})$ then $\tilde{A} > \tilde{B}$
- Case (ii) If Left spread $(\tilde{A}) < \text{Left spread}(\tilde{B})$ then $\tilde{A} < \tilde{B}$

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Case (iii) If Left spread (\tilde{A}) = Left spread (\tilde{B}) then go to step 6

Step 6 : Find right spread (\tilde{A}) and right spread (\tilde{B})

Case (i) If right spread (\tilde{A}) > right spread (\tilde{B}) then $\tilde{A} > \tilde{B}$

Case (ii) If right spread (\tilde{A}) < right spread (\tilde{B}) then $\tilde{A} < \tilde{B}$

Case (iii) If right spread (\tilde{A}) = right spread (\tilde{B}) then go to step 7

Step 7 : Find w_1 and w_2

Case (i) If $w_1 > w_2$ then $\tilde{A} > \tilde{B}$

Case (ii) If $w_1 < w_2$ then $\tilde{A} < \tilde{B}$

Case (iii) If $w_1 = w_2$ then $\tilde{A} \sim \tilde{B}$

IV. NUMERICAL EXAMPLES

In this section seven sets of fuzzy numbers are compared using the proposed method and existing approaches and the results are shown in the table.

Set 1

Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.7)$ be generalized trapezoidal fuzzy numbers.

By Step 1, $\Re(\tilde{A}) = 0.14$ and $\Re(\tilde{B}) = 0.14$.

Since $\Re(\tilde{A}) = \Re(\tilde{B})$, by step 2, $P(\tilde{A}) = 1.61$ and $P(\tilde{B}) = 1.81$

Since $P(\tilde{A}) < P(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$

Set 2

Let $\tilde{A} = (0.1, 0.2, 0.4, 0.5; 1)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be generalized trapezoidal fuzzy numbers.

By Step 1, $\Re(\tilde{A}) = 0.3$ and $\Re(\tilde{B}) = 0.2$

Since $\Re(\tilde{A}) > \Re(\tilde{B}) \Rightarrow \tilde{A} > \tilde{B}$

Set 3

Let $\tilde{A} = (0.1, 0.2, 0.4, 0.5; 1)$ and $\tilde{B} = (1, 1, 1, 1; 1)$ be generalized trapezoidal fuzzy numbers.

By Step 1, $\Re(\tilde{A}) = 0.3$ and $\Re(\tilde{B}) = 0$

Since $\Re(\tilde{A}) > \Re(\tilde{B}) \Rightarrow \tilde{A} > \tilde{B}$

Set 4

Let $\tilde{A} = (-0.5, -0.3, -0.3, -0.1; 1)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be generalized trapezoidal fuzzy numbers.

By Step 1, $\Re(\tilde{A}) = 0.2$ and $\Re(\tilde{B}) = 0.2$.

Since $\Re(\tilde{A}) = \Re(\tilde{B})$, by step 2, $P(\tilde{A}) = 2.4$ and $P(\tilde{B}) = 2.4$

Since $P(\tilde{A}) = P(\tilde{B})$, by step 3, mode (\tilde{A}) = -0.3 and mode (\tilde{B}) = 0.3 $\Rightarrow \tilde{A} < \tilde{B}$

Set 5

Let $\tilde{A} = (0.3, 0.5, 0.5, 1; 1)$ and $\tilde{B} = (0.1, 0.6, 0.6, 0.8; 1)$ be generalized trapezoidal fuzzy numbers.

By Step 1, $\Re(\tilde{A}) = 0.35$ and $\Re(\tilde{B}) = 0.35$.

Since $\Re(\tilde{A}) = \Re(\tilde{B})$, by step 2, $P(\tilde{A}) = 2.84$ and $P(\tilde{B}) = 2.84$

Since $P(\tilde{A}) = P(\tilde{B})$, by step 3, mode (\tilde{A}) = 1 and mode (\tilde{B}) = 0.6

Since mode (\tilde{A}) > mode (\tilde{B}) $\Rightarrow \tilde{A} > \tilde{B}$

Set 6

Let $\tilde{A} = (0, 0.4, 0.6, 0.8; 1)$, $\tilde{B} = (0.2, 0.5, 0.5, 0.9; 1)$ and $\tilde{C} = (0.1, 0.6, 0.7, 0.8; 1)$ be generalized trapezoidal fuzzy numbers.

By Step 1, $\Re(\tilde{A}) = 0.5$, $\Re(\tilde{B}) = 0.35$ and $\Re(\tilde{C}) = 0.4$

Since $\Re(\tilde{B}) < \Re(\tilde{C}) < \Re(\tilde{A})$, $\tilde{B} < \tilde{C} < \tilde{A}$

Set 7

Let $\tilde{A} = (0.1, 0.2, 0.4, 0.5; 1)$, $\tilde{B} = (-2, 0, 0, 2; 1)$ be generalized trapezoidal fuzzy numbers.

By Step 1, $\Re(\tilde{A}) = 0.3$, $\Re(\tilde{B}) = 2$

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Since $\mathfrak{R}(\tilde{A}) < \mathfrak{R}(\tilde{B})$, $\tilde{A} < \tilde{B}$

The comparison of proposed Generalized fuzzy ranking is given in the following table

A Comparison of proposed improved fuzzy ranking with existing ranking methods

Methods	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7
Cheng (1998)	$\tilde{A} < \tilde{B}$	$\tilde{A} \sim \tilde{B}$	Not Comparable	$\tilde{A} \sim \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	Not Comparable
Chu and Tsao (2002)	$\tilde{A} < \tilde{B}$	$\tilde{A} \sim \tilde{B}$	Not Comparable	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	Not Comparable
Chen and Chen (2007)	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} > \tilde{B}$
Abbasbandy and Hajjari (2009)	Not Comparable	$\tilde{A} \sim \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Chen and Chen (2009)	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Kumar et al. (2010)	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Thorani (2012)	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Proposed method	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B}$

V. APPLICATION OF IMPROVED RANKING OF GENERALIZED FUZZY NUMBER IN GFTP

In this section, proposed ranking method is applied to order generalized fuzzy number in solving GFTP.

Example 1 :

Consider a GFTP with rows representing three generalized fuzzy origins GFO_1 , GFO_2 , GFO_3 and columns representing four destinations GFD_1 , GFD_2 , GFD_3 , GFD_4 .

	GFD ₁	GFD ₂	GFD ₃	GFD ₄	GFC
GFO ₁	[2,3,4,5,0.2]	[2,4,5,7,0.5]	[1,2,3,4,0.1]	[2,4,6,8,0.6]	[1,4,6,10,0.2]
GFO ₂	[2,3,6,13,0.3]	[1,2,3,6,0.1]	[2,3,6,14,0.1]	[2,6,11,15,0.3]	[2,4,6,8,0.2]
GFO ₃	[2,6,10,17,0.2]	[2,6,8,12,0.2]	[1,2,4,6,0.1]	[1,3,7,10,0.1]	[5,10,14,20,0.2]
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

Since $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = [8,18,26,38,0.2]$, the problem is balanced GFTP and there exists a feasible solution to the GFTP.

Using generalized fuzzy Vogel's approximation method in [14], the initial generalized fuzzy basic feasible solution is obtained as follows

By step1 in , the rank of each cell \tilde{C}_{ij} using the proposed ranking method is given in the following table

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0.4	1.5	0.2	2.4
2.1	0.3	0.75	2.7
1.9	1.2	0.35	0.65

and the generalized fuzzy penalty cost for each row and column and their rank are computed and given in following table

	GFD ₁	GFD ₂	GFD ₃	GFD ₄	GFC
GFO ₁	[2,3,4,5,0.2]	[2,4,5,7,0.5]	[1,2,3,4,0.1]	[2,4,6,8,0.6]	[1,1,1,1,0.1] (0)
GFO ₂	[2,3,6,13,0.3]	[1,2,3,6,0.1]	[2,3,6,14,0.1]	[2,6,11,15,0.3]	[1,1,3,8,0.1] (0.45)
GFO ₃	[2,6,10,17,0.2]	[2,6,8,12,0.2]	[1,2,4,6,0.1]	[1,3,7,10,0.1]	[0,1,3,4,0.1] (0.3)
GFD	[0,3,6,12,0.2] (1.5)	[1,4,5,6,0.1] (0.3)	[0,0,1,2,0.1] (0.15)	[1,1,-1,-20.1] (-0.25)	

Since the generalized fuzzy maximum penalty is [0,3,6,12,0.2], first column is selected and since the first cell has the least generalized fuzzy cost, the maximum feasible amount [1,4,6,10,0.2] depending on the generalized fuzzy capacity and generalized fuzzy demands is allocated to the first cell. Then the first row which is fully exhausted is deleted. Then the column and row generalized fuzzy penalties for the reduced GFTP are recomputed and then the procedure is repeated until all the rim requirements are satisfied. Finally, the initial basic feasible solution of the given GFTP is obtained and it is given in the following table

	GFD ₁	GFD ₂	GFD ₃	GFD ₄	GFC
GFO ₁	[1,4,6,10,0.2]				[1,4,6,10,0.2]
GFO ₂	[2,3,4,5,0.2]	[2,4,5,7,0.5]	[1,2,3,4,0.1]	[2,4,6,8,0.6]	[2,4,6,8,0.2]
GFO ₃	[2,3,6,13,0.3]	[1,2,3,6,0.1]	[2,3,6,14,0.1]	[2,6,11,15,0.3]	[5,10,14,20,0.2]
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

And the initial generalized fuzzy transportation minimum cost is
 $[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}, w_z] = [11, 59, 139, 302, 0.1]$

Using generalized fuzzy modified distribution method in [14], it is found that the above generalized fuzzy basic feasible solution is optimal and it is given in the following table.

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	GFD ₁	GFD ₂	GFD ₃	GFD ₄	U _i
GFO ₁	[1,4,6,10,0.2]	-0.9	-0.6	-0.5	[-1,-3.5,-5.5,-11,0.2]
GFO ₂	[2,3,4,5,0.2]	[2,4,5,7,0.5]	[1,2,3,4,0.1]	[2,4,6,8,0.6]	
GFO ₃	-0.05	[2,4,6,8,0.2]	-0.7	-0.55	[-2,-4.5,-4.5,-5,0.1]
	[2,3,6,13,0.3]	[1,2,3,6,0.1]	[2,3,6,14,0.1]	[2,6,11,15,0.3]	
	[1,2,3,4,0.2]	[1,2,2,2,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	[-1,-0.5,0.5,1,0.5]
	[2,6,10,17,0.2]	[2,6,8,12,0.2]	[1,2,4,6,0.1]	[1,3,7,10,0.1]	
V _j	[3,6,5,9,5,16,0.2]	[3,6,5,7,5,11,0.2]	[2,2,5,3,5,5,0.1]	[2,3,5,6,5,9,0.1]	

The optimal generalized fuzzy transportation minimum cost is

$$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}, w_Z] = [11, 59, 139, 302; 0.1]$$

Example 2 :

Consider a GFTP discussed in example 1

Using generalized fuzzy Hungarian approach in [15], the initial generalized fuzzy basic feasible solution is obtained as follows

The rank of each cell \tilde{C}_{ij} using the proposed ranking method is given in the following table

0.4	1.5	0.2	2.4
2.1	0.3	0.75	2.7
1.9	1.2	0.35	0.65

Using step3 and step 4 of the Generalized Fuzzy Hungarian approach in [15] the following table is obtained

[0,0,0,0,0.1]	[1,2,2,3,0.1]	[0,0,0,0,0.1]	[1,0,0,0,0.1]
[0,0,2,6,0.1]	[0,0,0,0,0.1]	[1,1,3,8,0.1]	[0,2,5,5,0.1]
[0,3,5,10,0.1]	[1,4,4,6,0.1]	[0,0,0,0,0.1]	[-1,-1,0,0,0.1]

Applying steps 5 to 8 as in [15], the following initial generalized trapezoidal fuzzy transportation table is obtained.

	GFD ₁	GFD ₂	GFD ₃	GFD ₄	GFC
GFO ₁	[1,4,6,10,0.2]				[1,4,6,10,0.2]
	[0,0,0,0,0.1]	[0,1,3,7,0.1]	[0,3,5,10,0.1]	[1,4,5,10,0.1]	
GFO ₂		[2,4,6,8,0.2]			[2,4,6,8,0.2]
	[1,1,1,2,0.5]	[0,0,0,0,0.1]	[2,5,7,14,0.1]	[2,7,9,11,0.1]	
GFO ₃	[1,2,3,4,0.2]	[1,2,2,2,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	[5,10,14,20,0.2]
	[0,0,0,0,0.1]	[0,0,0,0,0.1]	[0,0,0,0,0.1]	[0,0,0,0,0.1]	
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

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And the initial generalized trapezoidal fuzzy transportation minimum cost is

$$[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}, w_z] = [11, 59, 139, 302; 0.1]$$

Using generalized fuzzy modified distribution method in [14], it is found that the above generalized fuzzy basic feasible solution is optimum.

VI. CONCLUSION

In this paper, an improved method of ranking of generalized trapezoidal fuzzy numbers is proposed and numerical examples are provided and compared with different existing approaches. Also the proposed ranking method is applied to order the generalized fuzzy number in solving the generalized fuzzy transportation problem and it can be applied to order the generalized fuzzy number in solving any optimization problem.

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