

# A Control Lyapunov Function Approach for Nonlinear Web Winding System

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**ABSTRACT:** The aim of this paper is to design a law control of a nonlinear web winding system using the Lyapunov stability theory. The Lyapunov function is a powerful tool for the stability analysis of nonlinear dynamic systems. This system knows several constraints such as the thermal effects caused by the frictions, and the mechanical effects provoked by metal elongation, that generates dysfunctions due to the influence of the process conditions. In the following, we give the concept of the Lyapunov's Direct Method, and we discuss its application to design a control that reduces the coupling influence between the drive speed and traction forces on cold rolling system for a precise thickness, with the best possible regularity by using the control Lyapunov function. The resulting control system is simulated and the numerical results are quoted and discussed.

**KEYWORDS:** Web Winding System (WWS), Web Tension, Nonlinear Control, Control Lyapunov Functions (CLF), Cold Rolling Mill (CRM).

## I. INTRODUCTION

The web winding system dynamics (WWS) is described by its strongly nonlinear behavior. In all cases of winding or unwinding a web material, there is the flatness problem. Given the complexity of the system due to the nonlinearity and strong coupling between the speed and web tension, the researchers directed themselves increasingly to the techniques of nonlinear control. Our goal is to design a control law using Lyapunov theory. The Lyapunov function is a powerful tool for analyzing the stability of dynamic systems. Artstein [1] and Sontag [2] respectively have considered the stabilization and control systems have extended the concept of the Lyapunov function to that of the control Lyapunov function (CLF). They have shown that a nonlinear system can be stabilized by a state feedback, if and only if it ensures a CLF. Besides, Sontag [3] dealt stabilization systems refines and presented a universal feedback scheme using the CLF. These achievements have greatly motivated the search on the CLF which has been widely adopted in various design issues and control.

Our work is to present the concept of the CLF applied to WWS in connection to get a better understanding of regularity and industrial problems during rolling.

The paper is organized as follows. In Section 2 the Lyapunov direct method used in winding control design is introduced. The web winding model is described in Section 3, with a brief description of the system. A detailed description of winding control design methodology is in Section 4. In Section 5 the control system performances are evaluated using simulation of the web winding model. The last section concludes the paper.

## II. THEORETICAL BACKGROUND

In the following we brief introduce the concept of Lyapunov's direct method [4, 5] and then discuss how to apply it for designing stable control systems.

Let us consider the system:

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$$(\Sigma): \begin{cases} \dot{x} = f(x) + \sum_{i=1}^m g_i(x) \cdot u_i(x) & x \in \mathcal{R}^n, u \in \mathcal{R}^m \\ y = h(x) & x \in \mathcal{R}^n \end{cases}$$

**Definition 1.**  $(x^c, u^c)$  is the equilibrium point of the dynamic system described by a set of nonlinear equations:  $\dot{x} = f(x) + g(x) \cdot u$  if once  $x(t)$  is equal to  $x^c$ , it remains  $x^c$  for all future time. Mathematically this means that  $x^c$  satisfies:  $f(x^c) + g(x^c) \cdot u^c = 0$

In many practical problems we are not concerned with stability around an equilibrium point, but rather whether a system trajectory  $x(t)$  will remain close to its original trajectory  $x^c(t)$  if slightly perturbed away from it. This kind of stability problem can be transformed into an equivalent problem, where we study the stability of the perturbation dynamics with respect to the equilibrium point. The Lyapunov theorem for global stability allows concluding the stability of a system by examining the variations of a single scalar function.

**Theorem 1.** [6] Assume that there exists a scalar function  $V$  of the state  $x$ , with continuous first order derivatives such that:

$$i) \begin{cases} V(x) \geq 0 & \forall x \in \mathcal{R}^n \\ V(x) = 0 & \text{if and only if } x = x^c \\ dV(x) = 0 & \text{if and only if } x = x^c \\ \frac{\partial V}{\partial t} = 0 & \forall x \in \mathcal{R}^n \end{cases}$$

ii)  $Q(x) = \{f(x), ad_f^k g(x), k = 1, \dots, j, \dots\}$  of rank  $n \forall x \neq x^c$

Then:  $\eta(x) = -R(x) \frac{\partial V}{\partial t} g(x)$  return the system  $(\Sigma)$  globally asymptotically stable (GAS) in  $x = x^c$ .  $R(x)$  is the strictly positive definite function. This theorem is based on the La Salle's theorem [7]. Then the equilibrium point at  $x = x^c$  is globally asymptotically stable.

The time derivative  $\dot{V}(x)$  along the system trajectory  $x(t)$  can be calculated as follows:

$$\dot{V}(x) = \frac{dV(x)}{dt} = \frac{\partial V}{\partial x_1} \frac{\partial x_1}{\partial t} + \frac{\partial V}{\partial x_2} \frac{\partial x_2}{\partial t} + \dots + \frac{\partial V}{\partial x_n} \frac{\partial x_n}{\partial t}$$

The Lyapunov's direct method can be used in control design problems, where our task is to find an appropriate control law for a given plant. The control design procedure requires hypothesizing a Lyapunov function candidate and then finding the control law to make this candidate a real Lyapunov function, namely:

1. For given dynamic system  $\dot{x} = f(x) + g(x) \cdot u$  find an appropriate LFC  $V(x)$  that is an explicit function of the control variables.
2. Select the control law that maximizes the negative value of  $\dot{V}(x)$  at all points along the system trajectory  $x(t)$ .
3. Select locally available signals to execute the chosen control law.

### III. MATHEMATICAL WWS MODEL

#### 1. WWS description and Mathematical model

The web system is very important in a rolling mill, because its parameters determine the strip quality. Among its parameters, we quote:

1. The entry and exit of the traction forces.

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2. The entry and exit velocities ensured by the winders motors, and the work rolls velocity (Fig.1).
3. The pressure force or the variations between the work rolls and their parallelism.

The variation of the exit strip flatness evolves because of the thermal dilation of the cylinders [8, 9], but also due to the elasticity forces [10]. To avoid this phenomenon, the traction forces are applied to limit the elasticity of the rolled material.

The thickness control is ensured by programmable automats, which are called AGC (Automatic Gauge Control system) [11]. Their goal is to maintain the strip thickness uniform in spite of the acting factors to change it. Considering the complexity of the Cold Rolling Mill System (CRM), the modeling and the control of the WWS should be studied to minimize the flatness defaults. With this intention, we start with the development of a mathematical model describing the dynamic behavior of the system.

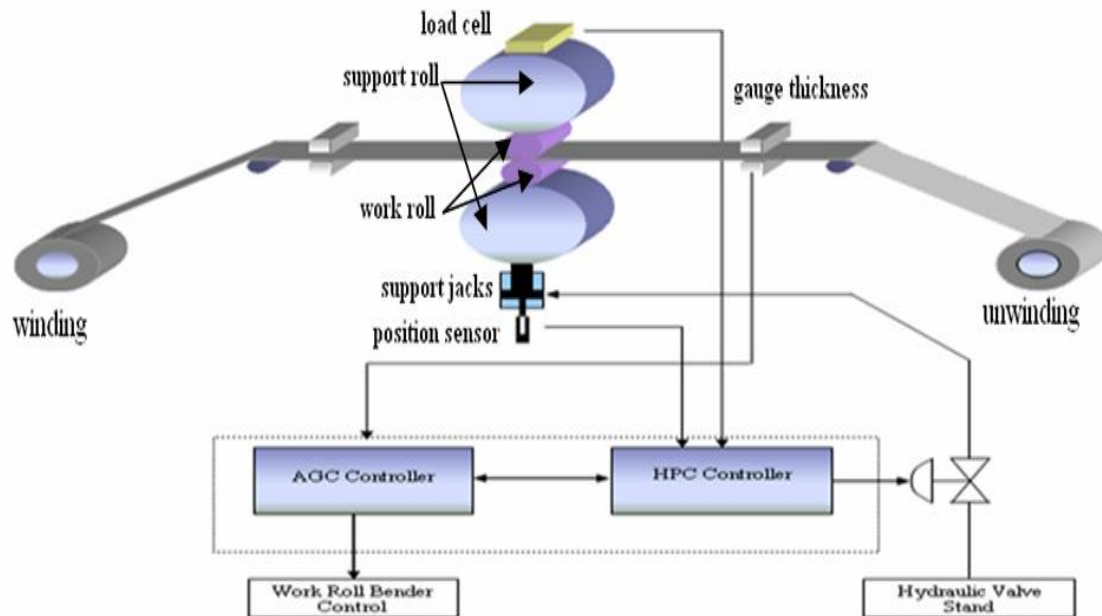


Fig.1 Interactions between the components of the cold rolling system.

We present the constitutive model parts of the Winders/Strip during cold rolling mill (CRM) [12]. In usual practice, the metal translates to the roll gap at a  $V_1$  speed (Fig.2) lower than the linear circumferential speed of the work rolls  $V_2$ , and exit with  $V_3 > V_2$ . Close to the roll gap entry, the metal advances less quickly than the cylinder. Friction tends to involve it downstream. But close to the exit, the metal product goes more quickly. It is thus slowed down by the cylinders (resistant friction) [13]. Therefore, when a difference velocity intervenes, traction is modified and results a supplementary elongation.

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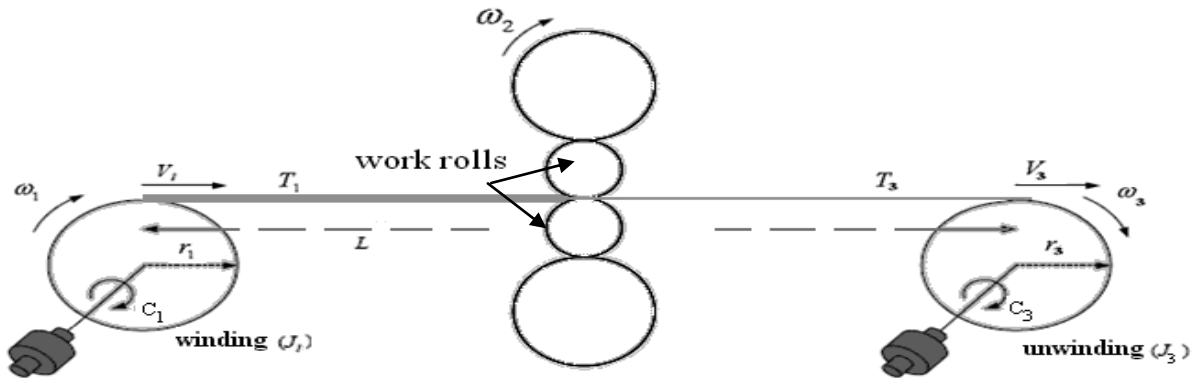


Fig.2 Synoptic model of the web winding system

## 2. Global model

The mathematical model of the WWS is given as follows [12]:

$$\begin{aligned} \frac{dT_1}{dt} &= -\frac{V_2}{L}T_1 + \frac{ES}{L}V_2 - \frac{ES}{L}V_1 \\ \frac{dT_3}{dt} &= -\frac{V_2}{L}T_3 - \frac{ES}{L}V_2 + \frac{ES}{L}V_3 \\ \frac{dV_1(t)}{dt} &= -\frac{1}{\tau_{em1}}V_1(t) + \frac{gr_1^2}{J_1}T_1 + \frac{r_1}{\tau_{em1} \cdot k_1} \cdot U_1(t) \\ \frac{dV_2(t)}{dt} &= -\frac{1}{\tau_{em2}}V_2(t) - \frac{gr_2^2}{J_2}(T_1 - T_3) + \frac{r_2}{\tau_{em2} \cdot k_2} \cdot U_2(t) \\ \frac{dV_3(t)}{dt} &= -\frac{1}{\tau_{em3}}V_3(t) - \frac{gr_3^2}{J_3}T_3 + \frac{r_3}{\tau_{em3} \cdot k_3} \cdot U_3(t) \end{aligned}$$

with:  $V_2(t) = \omega_2(t) \cdot r_2(t)$

Let us consider the definite nonlinear model by the state representation (E) which can be put in the general form of the nonlinear affine control system:

$$(E) : \begin{cases} \dot{x} = f(x) + g(x)u & x \in \mathfrak{R}^n \quad u \in \mathfrak{R}^m \\ y = h(x) & y \in \mathfrak{R}^p \end{cases}$$

with :

$$x^T = [V_1 \quad T_1 \quad V_2 \quad T_3 \quad V_3] \quad y^T = [T_1 \quad V_2 \quad T_3] \quad u^T = [U_1 \quad U_2 \quad U_3]$$

$$f(x) = \begin{bmatrix} \frac{gr_1^2}{J_1}x_2 - \frac{1}{\tau_{em1}}x_1 \\ -\frac{1}{L}x_2x_3 - \frac{ES}{L}x_1 + \frac{ES}{L}x_3 \\ -\frac{gr_2^2}{J_2}x_2 + \frac{gr_2^2}{J_2}x_4 - \frac{1}{\tau_{em2}}x_3 \\ -\frac{1}{L}x_3x_4 - \frac{ES}{L}x_3 + \frac{ES}{L}x_5 \\ -\frac{gr_3^2}{J_3}x_4 - \frac{1}{\tau_{em3}}x_5 \end{bmatrix} \quad h(x) = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad g(x) = [g_1, g_2, g_3] = \begin{bmatrix} \frac{r_1}{\tau_{em1} \cdot k_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{r_2}{\tau_{em2} \cdot k_2} & 0 \\ 0 & 0 & \frac{r_3}{\tau_{em3} \cdot k_3} \end{bmatrix} \quad (1)$$

These parameters characterize the system: it is multivariable, strongly coupled, nonlinear and time-varying.

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Let us note that it is a nonlinear model affine control system [14]. The whole of the equilibrium points drawn by following expression  $f(x^c) + g(x^c).u^c = 0$ , are defined by:

$$\left\{ \begin{array}{l} x_1^c = \frac{ES - x_2^c}{ES} . x_3^c; \\ x_5^c = \frac{ES + x_4^c}{ES} . x_3^c; \\ u_1^c = \frac{k_1}{r_1} . \left( -\frac{1}{ES} . x_2^c . x_3^c + x_3^c \right) - \frac{g . r_1 . k_1 . \tau_{em1}}{J_1} x_2^c; \\ u_2^c = \frac{k_2}{r_2} . x_3^c + \frac{g . r_2 . k_2 . \tau_{em2}}{J_2} x_2^c - \frac{g . r_2 . k_2 . \tau_{em2}}{J_2} x_4^c; \\ u_3^c = \frac{k_3}{r_3} . \left( \frac{1}{ES} . x_3^c . x_4^c + x_3^c \right) + \frac{g . r_3 . k_3 . \tau_{em3}}{J_3} x_4^c; \\ x_2^c, x_3^c, x_4^c \text{ are unspecified.} \end{array} \right. \quad (2)$$

### IV. CONTROL DESIGN

The web control design is based on the decentralized and centralized control principle [15, 16]. More specifically, the local stabilizing control is designed using only the isolated subsystem state variables.

Consider the web winding model in general form:

$$\dot{x} = f(x) + g(x).u \quad (3)$$

The control design consists of the following steps:

1. For the winding system described by the equation (3), the Lyapunov function candidates  $V(x)$  are chosen in as simple form as possible.
2. The winders control law is proposed so that the following condition is satisfied:  $\frac{dV(t)}{dt} \leq 0$

i.e.  $[grad V(x)]^T [f(x) + g(x).u(x)] \leq 0$  (4)

where the control  $u(x)$  is designed using the linearization principle:

$$u(x) = u^c + \tilde{u}(x) \quad (5)$$

The  $u^c$  term in equation (5) defines the operating point with designed control mentioned previously. The  $\tilde{u}(x)$  control is chosen so that the system state  $x(t)$  tends towards set point  $x^c$ .

By introducing the control expression (5), we obtain the following form:  $\dot{x} = \tilde{f}(x) + g(x).\tilde{u}(x)$ , with:  $\tilde{f}(x) = f(x) + g(x).u^c$ .

3. The higher hierarchical level control can be derived using the information from winding system, but in a considerably reduced form. The main design principle is to reduce the influence of the coupling between web velocities and the web tractions exerted on dynamic system. The best solution is to design the control in the form:

$$\tilde{u}(x) = -R(x) \frac{\partial V}{\partial x} g(x) \quad (6)$$

Then the dynamical system is globally asymptotically stable in the equilibrium point at the  $x = x^c$ . The web winding control design is based on the Lyapunov theory of stability. Let us choose the following Lyapunov function candidate:

$$V(x) = \frac{C1}{2} (x_1 - x_1^c)^2 + \frac{C2}{2} (x_3 - x_3^c)^2 + \frac{C3}{2} (x_5 - x_5^c)^2 \quad (7)$$

where C1, C2 and C3 are the positive constants.

$$V(x^c) = 0 \text{ and } V(x) > 0 \quad (8)$$

for all  $x_i \in (-\infty; +\infty)$   $i = 1, 2, \dots, 5$ .

The Lyapunov function gradients must be continuous functions:

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$$(\text{grad } V)^T = [C1.(x_1 - x_1^c) \ 0 \ C2.(x_3 - x_3^c) \ 0 \ C3.(x_5 - x_5^c)] \tag{9}$$

Moreover the Lyapunov function candidate (7) is required to satisfy the theorem 1:

$$\frac{dV}{dt} = (\text{grad } V)^T \frac{dx}{dt} = C1.(x_1 - x_1^c)\dot{x}_1 + C2.(x_3 - x_3^c)\dot{x}_3 + C3.(x_5 - x_5^c)\dot{x}_5 \tag{10.a}$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial x} \tilde{f}(x) + \frac{\partial V}{\partial x} g(x) \cdot \tilde{u}(x) \tag{10.b}$$

By application to the theorem 1, we quote that:  $\frac{\partial V}{\partial x} \tilde{f}(x) = 0 \ \forall x \in \mathcal{X}^n$ .

However:

$$\frac{dV}{dt} = \frac{\partial V}{\partial x} g(x) \cdot \tilde{u}(x) = -R(x) \cdot \left( \frac{\partial V}{\partial x} g(x) \right)^2 \leq 0 \tag{11}$$

Thus, we can deduce the control  $\tilde{u}(x)$  as follows:

$$\tilde{u}_1(x) = -R_1 \frac{\partial V}{\partial x} g_1(x) = -R_1 \frac{C1.(x_1 - x_1^c).r_1}{\tau_{em1}.k_1} x_1$$

$$\tilde{u}_2(x) = -R_2 \frac{\partial V}{\partial x} g_2(x) = -R_2 \frac{C2.(x_3 - x_3^c).r_2}{\tau_{em2}.k_2} x_3$$

$$\tilde{u}_3(x) = -R_3 \frac{\partial V}{\partial x} g_3(x) = -R_3 \frac{C3.(x_5 - x_5^c).r_3}{\tau_{em3}.k_3} x_5 \tag{12}$$

and the global CLF of a web winding system is given as :

$$u_i(x) = u_i^c + \tilde{u}_i(x) \quad i = 1, 2, 3. \tag{13}$$

## V. SIMULATION AND RESULTS DISCUSSION

This section presents the temporal evolution of the exit variables by using the CLF quoted previously.

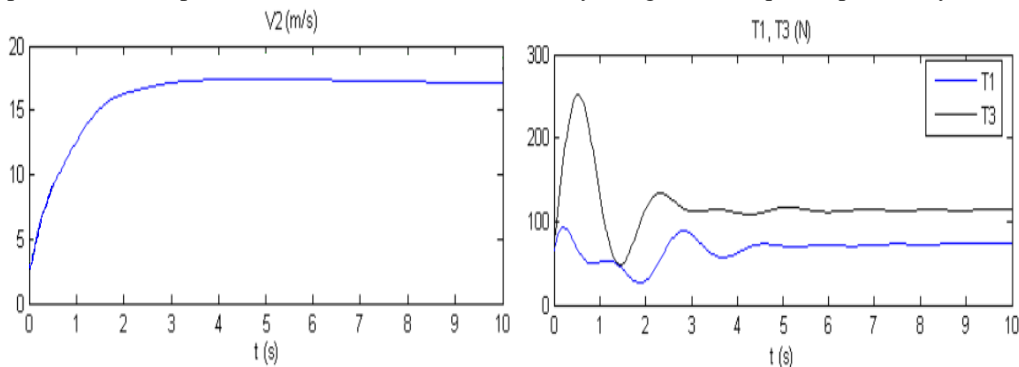


Fig. 3 for C1=C2=C3= 1, (a) Web velocities response (b) Web tensions response through the WWS. (b)

Figure 3(a) and 3(b) illustrate the tensions and winding velocity through the WWS for C1=C2=C3=1. The velocity trajectory is similar to a 1st order system, with a response time of 5 seconds. But, we note that trajectory is further from the reference imposed (20m/s). The upstream and downstream web tensions (or traction forces) manifest some undesirable oscillations (disturbances) which are cancelled at 5 seconds.

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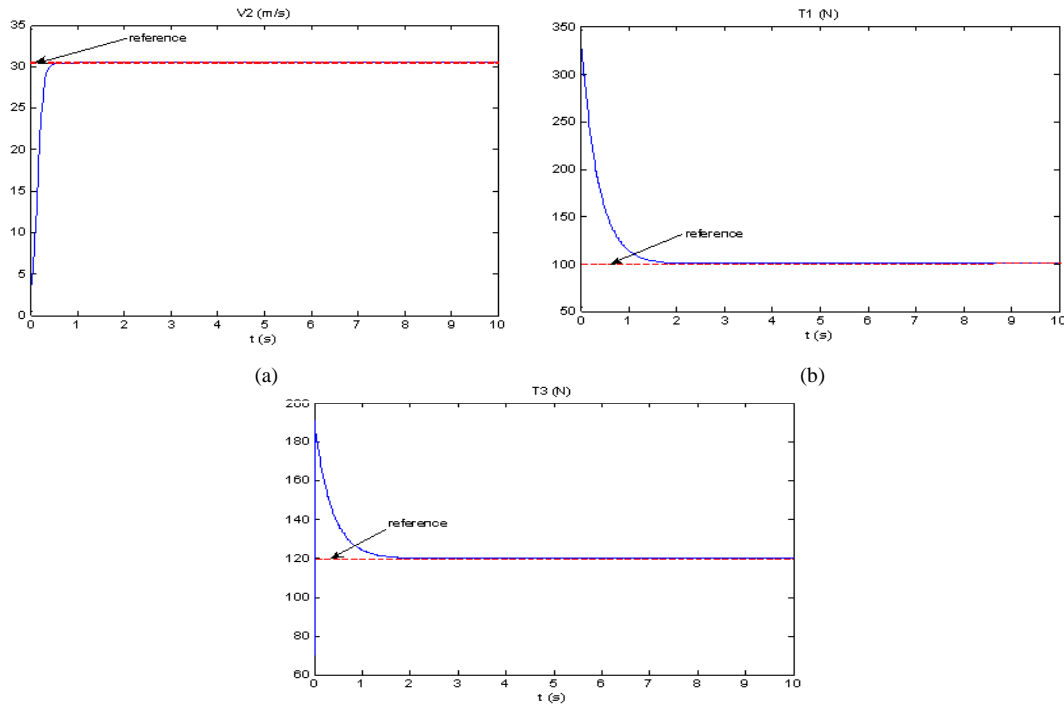


Fig. 4 for  $C_1=100$ ,  $C_2=400$ ,  $C_3=500$ , (a) Web velocities response, (b) Upstream web tension response, (c) Downstream web tension response through the WWS.

Contrary to the case of  $C_1=100$ ,  $C_2=400$  and  $C_3=500$ , we observe, in figure 4(a), the perfect follow-up of the reference velocity ( $=30$  m/s). The same, the upstream (fig.4(b)) and downstream (fig.4(c)) web tensions converge directly after 2 seconds, towards their reference, without having any oscillation (disturbance)

The outputs of the system follow perfectly their imposed reference trajectories. Obviously, more dynamics of the reference trajectory will be fast, more the control will be strong. It is important to recall that the dynamics of the reference trajectory is dependent of the design parameters.

A first observation relates to the sensitivity to the disturbances. This phenomenon is much accentuated in the case of the traction forces ( $T_1$  and  $T_3$ ). The second observation relates to the parameters ( $C_1$ ,  $C_2$ , and  $C_3$ ) which can take various values. They were introduced during development of the control law. These parameters make it possible to change the regulation dynamics. However, there are restrictions on the values which they can take. These parameters must necessarily be always positive and no null. These conditions rise directly from the Lyapunov theorems.

## VI. CONCLUSION

The paper describes an application of the approach of Lyapunov for a nonlinear MIMO system. The good choice of CLF cancels the nonlinearity and allows a very good follow-up of the references trajectories.

Finally, we have illustrated our control strategy by some numerical simulation results. Future works will consist in evaluating the robustness of our class of control laws with respect to some perturbations, such as the friction. But also, the lack of information about some the output system parameters requires the development of an observer to estimate the unmeasurable states.

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