

A Note on Uniform Vertex and Uniform Edge Fuzzy Graphs

Ch.Chaitanya¹, Dr.T.V.Pradeep Kumar²

Assistant Professor, Department of Mathematics, Lakireddy Bali Reddy College of Engineering (Autonomous),

Mylavaram , A.P, India¹

Assistant Professor, Department of Mathematics, Acharya Nagarjuna University, Guntur, A.P, India²

ABSTRACT: Fuzzy graph was introduced by Kaufmann [6] in 1973. In this paper, we introduced the concepts of uniform edge fuzzy graphs and Uniform vertex fuzzy graphs. Also the conditions under which the complement graph of a regular graph becomes regular as well as totally regular are discussed. Also we obtained some more results related to regular and totally regular graphs and complete fuzzy graphs.

KEYWORDS: compliment of a fuzzy graph, complete fuzzy graph, regular fuzzy graph, totally regular fuzzy graph, uniform edge fuzzy graph, uniform vertex fuzzy graph.

I. INTRODUCTION

The concept of a Graph is introduced for the first time by Leonhard Euler [3] in the year 1736. It is quite well known that graphs are simply models of relations.

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'Fuzzy Graph Model'. Application of fuzzy relations are widespread and important in the field of clustering analysis, neural networks, computer networks, pattern recognition, decision making and expert systems. In each of these, the basic mathematical structure is that of a fuzzy graph. We know that a graph is a symmetric binary relation on a nonempty set V . Similarly, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The first definition of a fuzzy graph was by Kaufmann[6] in 1973, based on Zadeh's fuzzy relations [12]. But it was Azriel Rosenfeld [8] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. Later on so many Mathematicians produced various results on Fuzzy Graphs in various aspects.

II. PRELIMINARIES

In this section we have given the definitions and examples related to the concepts that are discussed in the introduction.

2.1 Definition [4]: A graph G is a triple consisting of a vertex set $V(G)$, an edge set $E(G)$, and a relation that associates with each edge, two vertices called its endpoints (not necessarily distinct). Graphically, we represent a graph by drawing a point for each vertex and representing each edge by a curve joining its endpoints.

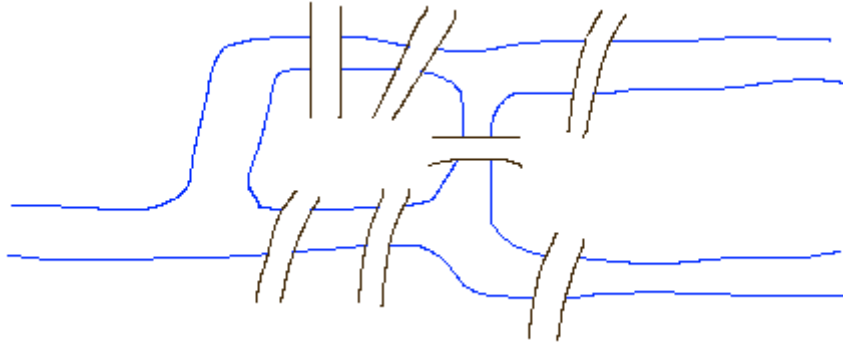
2.2 Example [3]: Konigsberg Bridge's problem

Konigsberg was a city in Prussia situated on the Pregel River in the 16th century. (Today, the city is named Kaliningrad, western Russia.) The river Pregel flowed through the town, creating an island, as in the following picture. Seven bridges spanned the various branches of the river, as shown. A famous problem concerning Konigsberg was whether it was possible to take a walk through the town in such a way as to cross over every bridge once, and only once

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015



Euler's approach was to regard the spots of land (there are 4 of them) as vertices, and the bridges as edges between those points. The mathematical essentials of the map of Königsberg can then be reduced to the following diagram, which is an example of what is called a *graph*:

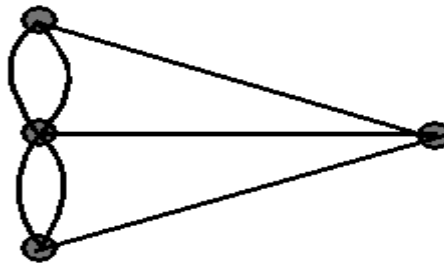


Fig 2.2.1

2.3 Definition[[8],[12]]: Let S be a non - empty set. Let $\mu: S \times S \rightarrow [0,1]$ and $\sigma: S \rightarrow [0,1]$ be two fuzzy sets. Then μ is said to be a fuzzy relation on σ if $\mu(x, y) \leq \sigma(x) \cap \sigma(y) \forall x, y \in S$.

Note: $\sigma(x) \cap \sigma(y) = \min \{ \sigma(x), \sigma(y) \}$.

2.4 Definitions[[8],[12]]: (i) A fuzzy relation is called Symmetric if $\mu(x, y) = \mu(y, x), \forall x, y \in S$.

(ii) A fuzzy relation is called Reflexive if $\mu(x, x) = \sigma(x), \forall x \in S$

2.5 Definition-[[6],[8]]: A fuzzy graph G is a pair of functions $G: (\sigma, \mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ . The underlying crisp graph of $G: (\sigma, \mu)$ is denoted by $G^*(V, E)$ where $E \subseteq V \times V$.

2.6 Example: The following figure gives us an example for a fuzzy graph. Here ' σ ' values are written in the brackets at vertices and ' μ ' values are written in the middle of the edges.

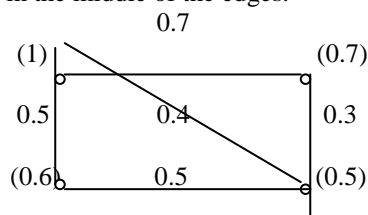


Figure 2.6.1. Here we observe that $\mu(x, y) \leq \sigma(x) \cap \sigma(y)$ at each edge.

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2.7 Note: $\sigma^* = \{x \in V / \sigma(x) > 0\}$ and $\mu^* = \{(x, y) \in V \times V / \mu(x, y) > 0\}$

2.8 Note: In any fuzzy graph $G: (\sigma, \mu)$ on $G^*(V, E)$ we assume that V is finite and μ is reflexive. Also we need not consider the loops and parallel edges.

2.9 Definition- [9]: A fuzzy graph $G: (\sigma, \mu)$ is said to be a Strong Fuzzy Graph if $\mu(u, v) = \sigma(u) \cap \sigma(v) \forall (u, v) \in \mu^*$.

2.10 Definition- [9]: A fuzzy graph $G: (\sigma, \mu)$ is said to be a Complete Fuzzy Graph if $\mu(u, v) = \sigma(u) \cap \sigma(v) \forall u, v \in \sigma^*$.

2.11 Definition [11]: Let $G: (\sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is defined as $d_G(u) = \sum_{u \neq v} \mu(u, v)$.

2.12 Definition[[5],[10]]: The complement of a fuzzy graph $G: (\sigma, \mu)$ is a fuzzy graph $\bar{G}: (\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} = \sigma$ and $\bar{\mu}(u, v) = \sigma(u) \cap \sigma(v) - \mu(u, v) \forall u, v \in V$.

2.13 Definition [7]: Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If $d_G(u) = k \forall u \in V$ (i.e.) if each vertex has same degree then it is called a Regular Graph of degree k or k -Regular Graph.

2.14 Definition [7]: Let $G: (\sigma, \mu)$ be a fuzzy graph. The Total degree of a vertex u is defined as $td_G(u) = \sum_{u \neq v} \mu(x, y) + \sigma(u)$. i.e. $td_G(u) = d_G(u) + \sigma(u)$.

2.15 Definition [7]: Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If $td_G(u) = k \forall u \in V$. i.e., if each vertex has same total degree then it is called a Totally Regular Graph of degree k or k -Totally Regular Graph.

III. RESULTS

In this section we introduced the concepts of Uniform edge fuzzy graphs and Uniform vertex fuzzy graphs and obtained some interesting results basing on these concepts.

3.1 Definition: Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Let $\sigma^* = \{x \in V / \sigma(x) > 0\}$. Then G is said to be an Uniform Vertex Fuzzy Graph if $\sigma(x) = k \forall x \in \sigma^*$ where k is some positive real such that $0 < k \leq 1$

3.2 Example: Here are the examples of an Uniform Vertex Fuzzy Graph and a non-uniform vertex fuzzy graph respectively. In both the figures 'σ' values are written in the brackets at vertices and 'μ' values are written in the middle of the edges. Fig.3.2.1 represents an uniform vertex fuzzy graph and fig. 3.2.2 represents a non-uniform vertex fuzzy graph.

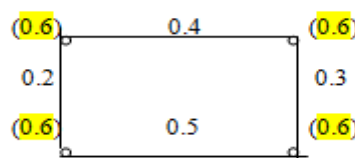


Figure-3.2.1 Here $\sigma(x) = \sigma(y) = \sigma(z) = \sigma(w) = 0.6$ where x, y, z, w are the vertices.

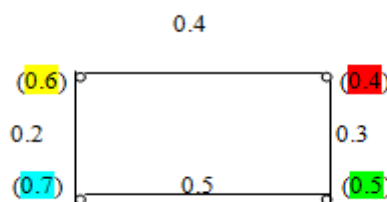


Figure-3.2.2. Here $\sigma(x) = 0.6 \neq 0.4 = \sigma(y)$ where x, y, z, w are the vertices.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

3.3 Definition:- Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Let $\sigma^* = \{x \in V / \sigma(x) > 0\}$. Then G is said to be a Uniform edge fuzzy graph if $\mu(x, y) = k \forall (x, y) \in \sigma^* \times \sigma^*$ with $x \neq y$, where k is some positive real such that $0 < k \leq 1$.

3.4 Example: Here are the examples of an Uniform Edge Fuzzy Graph and a non-uniform edge fuzzy graphs respectively. In both the figures 'σ' values are written in the brackets at vertices and 'μ' values are written in the middle of the edges. Fig.3.4.1 represents an uniform edge fuzzy graph and fig. 3.4.2 represents a non-uniform edge fuzzy graph.

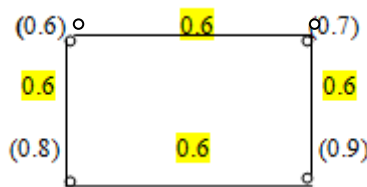


Figure- 3.4.1 Here $\mu(x, y) = \mu(y, z) = \mu(z, w) = \mu(w, x) = 0.6$

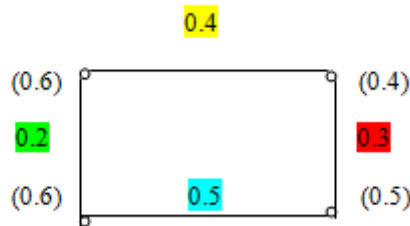


Figure-3.4.2 Here $\mu(x, y) = 0.4 \neq \mu(y, z) = 0.3$. Where x, y, z, w are the vertices.

3.5 Theorem: If $G: (\sigma, \mu)$ is a regular fuzzy graph and uniform vertex fuzzy graph on $G^*(V, E)$ then the complement fuzzy graph $\bar{G}: (\bar{\sigma}, \bar{\mu})$ is also Regular.

Proof: Let V be the finite set of vertices. Since V is finite, we have $\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite. Let $|\sigma^*| = n$.

Given G is a regular fuzzy graph.

Then $d_G(u) = \sum_{u \neq v} \mu(u, v) = m, \forall u \in \sigma^*$ for some positive real m (1).

Given that G is an Uniform vertex fuzzy graph.

Then $\sigma(x) = k \forall x \in \sigma^*$ for some real k where $0 < k \leq 1$ (2).

Let $x \in \sigma^*$. Now $d_{\bar{G}}(x) = \sum_{x \neq y} \bar{\mu}(x, y) = \sum_{x \neq y} [\sigma(x) \cap \sigma(y) - \mu(x, y)]$
 $= \sum_{x \neq y} (\sigma(x) \cap \sigma(y)) - \sum_{x \neq y} \mu(x, y) = \sum_{x \neq y} k - d_G(x)$ (by using (2))
 $= (n-1)k - m$ (by using (1))

$\therefore d_{\bar{G}}(x) = (n-1)k - m \forall x \in \sigma^*$.

Therefore $\bar{G}: (\bar{\sigma}, \bar{\mu})$ is also Regular Fuzzy graph.

This completes the proof.

3.6 Theorem: If $G: (\sigma, \mu)$ is a Totally Regular fuzzy graph and Uniform vertex fuzzy graph on $G^*(V, E)$ then the complement fuzzy graph $\bar{G}: (\bar{\sigma}, \bar{\mu})$ is also Totally Regular.

Proof: Since V is finite, we have $\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite.

Let $|\sigma^*| = n$. Given that G is a totally regular fuzzy graph.

Then $td_G(u) = d_G(u) + \sigma(u) = m \forall u \in \sigma^*$ for some positive real m (1).

Given that G is a uniform vertex fuzzy graph.

Then $\sigma(x) = k \forall x \in \sigma^*$ for some real k where $0 < k \leq 1$ (2).

Let $x \in \sigma^*$. Now $td_{\bar{G}}(x) = d_{\bar{G}}(x) + \bar{\sigma}(x)$
 $= \sum_{x \neq y} \bar{\mu}(x, y) + \bar{\sigma}(x) = \sum_{x \neq y} [\sigma(x) \cap \sigma(y) - \mu(x, y)] + \bar{\sigma}(x)$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

$$\begin{aligned}
 &= \sum_{x \neq y} (\sigma(x) \cap \sigma(y)) - \sum_{x \neq y} \mu(x, y) + \sigma(x) \\
 &= \sum_{x \neq y} (\sigma(x) \cap \sigma(y)) - \sum_{x \neq y} \mu(x, y) - \sigma(x) + \sigma(x) + \sigma(x) \\
 &= \sum_{x \neq y} (\sigma(x) \cap \sigma(y)) - [\sum_{x \neq y} \mu(x, y) + \sigma(x)] + 2\sigma(x) \\
 &= \sum_{x \neq y} (k) - [d_G(x) + \sigma(x)] + 2\sigma(x) = (n-1)k - d_G(x) + 2k \\
 &= (n-1)k - m + 2k = (n+1)k - m \\
 \therefore d_G(x) &= (n+1)k - m \quad \forall x \in \sigma^* . \\
 \therefore \bar{G}: (\bar{\sigma}, \bar{\mu}) &\text{ is also Totally Regular Fuzzy graph. This completes the proof.}
 \end{aligned}$$

3.7 Theorem:- If $G: (\sigma, \mu)$ is a Uniform Edge fuzzy graph on $G^*(V, E)$ then G is a Regular graph.

Proof: Since V is finite, we have $\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite.

Let $|\sigma^*| = n$. Given G is a Uniform Edge fuzzy graph.

Then $\mu(x, y) = k \quad \forall (x, y) \in \sigma^* \times \sigma^*$ with $x \neq y$ for some real k where $0 < k \leq 1$ (1).

Let $u \in \sigma^*$. Now $d_G(u) = \sum_{u \neq v} \mu(u, v) = \sum_{u \neq v} k = (n-1)k$

$\therefore d_G(u) = (n-1)k \quad \forall u \in \sigma^*$. Therefore G is regular Fuzzy graph. This completes the proof.

3.8 Theorem:- If $G: (\sigma, \mu)$ is both Uniform Edge fuzzy graph and Uniform Vertex fuzzy graph on $G^*(V, E)$ then G is a totally regular graph.

Proof: Since V is finite, we have $\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite.

Let $|\sigma^*| = n$. Given G is a Uniform Edge fuzzy graph.

Then $\mu(x, y) = m \quad \forall (x, y) \in \sigma^* \times \sigma^*$ with $x \neq y$ for some real m where $0 < m \leq 1$ (1)

Given G is a Uniform vertex fuzzy graph.

Then $\sigma(x) = k \quad \forall x \in \sigma^*$ for some real k where $0 < k \leq 1$ (2).

Let $u \in \sigma^*$. Now $td_G(u) = \sum_{u \neq v} \mu(u, v) + \sigma(u) = \sum_{u \neq v} m + k = (n-1)m + k$
(since by (1) and (2))

$\therefore td_G(u) = (n-1)m + k \quad \forall u \in \sigma^*$. Therefore G is Totally Regular Fuzzy graph.

This completes the proof.

3.9 Theorem:- If $G: (\sigma, \mu)$ is a Complete fuzzy graph and Uniform vertex fuzzy graph on $G^*(V, E)$ then G is Regular.

Proof: Since V is finite, we have $\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite.

Let $|\sigma^*| = n$. Given G is a Uniform vertex fuzzy graph.

Then $\sigma(x) = k \quad \forall x \in \sigma^*$ for some real k where $0 < k \leq 1$ (1).

Given G is a Complete Fuzzy Graph.

Then $\mu(u, v) = \sigma(u) \cap \sigma(v) \quad \forall u, v \in \sigma^*$.

Let $u \in \sigma^*$. Now $d_G(u) = \sum_{u \neq v} \mu(u, v) = \sum_{u \neq v} [\sigma(u) \cap \sigma(v)]$
(Since G is complete)

$= \sum_{u \neq v} (k) = (n-1)k$

$\therefore d_G(u) = (n-1)k \quad \forall u \in \sigma^*$.

$\therefore G$ is Regular Fuzzy graph.

This completes the proof.

3.10 Theorem:- If $G: (\sigma, \mu)$ is a Complete fuzzy graph and Uniform vertex fuzzy graph on $G^*(V, E)$ then G is Totally Regular.

Proof: Given V is finite. Then $\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite.

Let $|\sigma^*| = n$.

Given G is a Uniform vertex fuzzy graph.

Then $\sigma(x) = k \quad \forall x \in \sigma^*$ for some real k where $0 < k \leq 1$ (1).

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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Given G is a Complete Fuzzy Graph.

Then $\mu(u, v) = \sigma(u) \cap \sigma(v) \quad \forall u, v \in \sigma^*$.

Let $u \in \sigma^*$.

$$\begin{aligned} td_G(u) &= \sum_{u \neq v} \mu(u, v) + \sigma(u) \\ &= \sum_{u \neq v} [\sigma(u) \cap \sigma(v)] + \sigma(u) \quad (\because G \text{ is complete}) \\ &= \sum_{u \neq v} (k) + k = (n-1)k + k = nk. \end{aligned}$$

$$\therefore td_G(u) = nk \quad \forall u \in \sigma^*.$$

$\therefore G$ is Totally Regular Fuzzy graph. This completes the proof.

Theorem-3.11: If $G: (\sigma, \mu)$ is a Regular fuzzy graph and Uniform vertex fuzzy graph on $G^*(V, E)$ then G is Totally Regular.

Proof: Since V is finite, we have $\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite.

Let $|\sigma^*| = n$. Given G is a Uniform vertex fuzzy graph.

Then $\sigma(x) = k \quad \forall x \in \sigma^*$ for some real k where $0 < k \leq 1$ (1).

Given that G is a regular fuzzy graph.

Then $d_G(u) = m \quad \forall u \in \sigma^*$ for a positive real m .

Let $u \in \sigma^*$. Now $td_G(u) = d_G(u) + \sigma(u) = m + k$.

$$\therefore td_G(u) = m + k \quad \forall u \in \sigma^*.$$

$\therefore G$ is Totally Regular Fuzzy graph. This completes the proof.

IV. CONCLUSION

In this paper we introduced the concepts of uniform vertex and uniform edge fuzzy graphs with illustrative examples. Also we produced various results related to these concepts. These are useful in studying regularity and total regularity of various special kinds of fuzzy graphs.

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BIOGRAPHY

	<p>. Ch.Chaitanya is an Assistant professor in the Department of Mathematics in Lakireddy Bali Reddy College of Engineering (Autonomous), Mylavaram – 521 230, Krishna (DT), Andhra Pradesh, India. He received his Master's degree in mathematics in the year 2006 from P. B. Siddhartha college of arts & sciences ,Vijayawada, A.P. He is been imparting mathematics into both the graduate and post graduate students for the past eight years. His main research field is fuzzy graphs and their applications.</p>
	<p>Dr. Tumurukota Venkata Pradeep Kumar is presently working as Assistant Professor in Mathematics in University Engineering College, Acharya Nagarjuna University, Nagarjuna Nagar, Guntur (dt)., A.P., India. He obtained his M. Sc and Ph.D., from Acharya Nagarjuna University under the guidance of Prof. Dr. Bhavanari Satyanarayana. He had brilliant academic career. His field of work is Algebra. He has nearly two decades of teaching experience in various capacities. He has published more than 40 research papers in various National and International Journals. He also published 10 books for U.G / P.G students. He attended more than 20 conferences/workshops / seminars and presented papers.</p>