

Applications of D'Alemberts Solution

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ABSTRACT: The method of D'Alembert's provides a solution to the one-dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

(or) $u_{xx} = \frac{1}{c^2} u_{tt}$ i.e. $u_{tt} - c^2 u_{xx} = 0$. In this paper we derive the equation of D'Alembert's solution and also solve the problem by the use of D'Alembert's solution.

KEYWORDS: Onedimensional wave equation, Cauchy type.

I. DERIVATION OF D'ALEMBERT'S SOLUTION

Many authors for example [5] have studied the concept of D'Alemberts method to solve differential equations. The differential equations of homogeneous and non-homogenous equations[4]
Consider the initial value problem of Cauchy type described as[3] ,

$$\text{PDE: } u_{tt} - c^2 u_{xx} = 0, -\infty < x < \infty, t \geq 0 \quad (1)$$

$$\text{ICs: } u(x,0) = \eta(x), u_t(x,0) = v(x) \quad (2)$$

Where the curve on which the initial data $\eta(x)$ and $v(x)$ are prescribed as the axis[5].

$\eta(x)$ and $v(x)$ are assumed to be twice continuously differentiable. Here the string considered as an infinite extent[2].

Let $u(x, t)$ denote the displacement for any x and t .

At time $t = 0$, the displacement and velocity of the string be prescribed . We know that the general solution of the wave equation is given by[6],

$$u(x, t) = \phi(x + ct) + \psi(x - ct) \quad (3)$$

Where ϕ and ψ are arbitrary functions. Substituting the ICs (2) in (3), we obtain

$$\phi(x) + \psi(x) = \eta(x) \quad (4)$$

$$c[\phi'(x) - \psi'(x)] = v(x)$$

Integrating the second equation of (4), we get

$$\phi(x) - \psi(x) = \frac{1}{c} \int_0^x v(\xi) d\xi$$

Addition and subtraction of this equation with the first relation of Equations (4) yield

$$\phi(x) = \frac{\eta(x)}{2} + \frac{1}{2c} \int_0^x v(\xi) d\xi$$

$$\psi(x) = \frac{\eta(x)}{2} - \frac{1}{2c} \int_0^x v(\xi) d\xi \text{ respectively. Substituting these relations for } \phi(x) \text{ and } \psi(x) \text{ in Eq (2), we}$$

at once get[7]

$$u(x, t) = \frac{1}{2} [\eta(x + ct) + \eta(x - ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} v(\xi) d\xi \quad (5)$$

This is known as the D'Alembert's solution of the one dimensional wave equation.

If $v = 0$, i.e., if the string is released from the rest, the required solution is[1]

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$$u(x, t) = \frac{1}{2}[\eta(x + ct) + \eta(x - ct)] \quad (6)$$

The D'Alembert's solution has an interesting geometrical interpretation.

II. APPLICATIONS OF D'ALEMBERT'S SOLUTION

Example:1

Consider the initial value problem $u_{tt} = 4u_{xx}$, $t > 0$, $-\infty < x < \infty$ satisfying the conditions $u(x, 0) = x$, $u_t(x, 0) = 0$ [8].

Solution:

The D'Alembert's solution for the IVP defined as

$$\text{PDE: } u_{tt} = c^2 u_{xx} \quad t > 0, \quad -\infty < x < \infty$$

$$\text{ICs: } u(x, 0) = \eta(x) \quad u_t(x, 0) = v(x) \quad \text{is}$$

$$u(x, t) = \frac{1}{2}[\eta(x + ct) + \eta(x - ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} v(\xi) d\xi$$

For the problem considered, $v(x) = 0$, $c = 2$ and $\eta(x) = x$.

Therefore, the solution to the given problem is

$$u(x, t) = \frac{1}{2}[\eta(x + 2t) + \eta(x - 2t)] = \frac{1}{2}[x + 2t + x - 2t] = x$$

Hence the solution of the given IVP is $u(x, t) = x$.

Example 2: Let $u = \psi(x, t)$ be the solution to the initial value problem

$u_{tt} = u_{xx}$ for $-\infty < x < \infty$, $t > 0$ with $u(x, 0) = \sin x$, $u_t(x, 0) = \cos x$, then the value of $\psi(\frac{\pi}{2}, \frac{\pi}{6})$ is obtained through D'Alembert's solution[9].

Solution:

The D'Alembert's solution for the IVP defined as

$$\text{PDE: } u_{tt} = c^2 u_{xx} \quad t > 0, \quad -\infty < x < \infty$$

$$\text{ICs: } u(x, 0) = \eta(x) \quad u_t(x, 0) = v(x) \quad \text{is}$$

$$u(x, t) = \frac{1}{2}[\eta(x + ct) + \eta(x - ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} v(\xi) d\xi$$

For the given problem, $v(x) = \cos x$, $c = 1$ and $\eta(x) = \sin x$.

$$\text{Therefore, } u(x, t) = \frac{1}{2}[\eta(x + t) + \eta(x - t)] + \frac{1}{2} \int_{x-t}^{x+t} \cos \xi d\xi$$

$$= \frac{1}{2}[\sin(x + t) + \sin(x - t)] + \frac{1}{2}[\sin(x + t) - \sin(x - t)] = \sin 2t = \frac{\sqrt{3}}{2}$$

Hence the solution is $\frac{\sqrt{3}}{2}$

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