

# On Lacunary Interpolation through g-Splines

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**ABSTRACT:** In this paper, we construct A new kind of lacunary g-splines which are the solution of  $(0, 2, 4)$ -interpolation problem and their local approximation with functions belonging to  $C^{(4)}(I)$  and  $C^{(5)}(I)$ . Our methods are of lower degree having better convergence property than the earlier investigations using piecewise polynomials with certain specific properties.

**KEYWORDS:** Spline function, lacunary interpolation, quintic splines piecewise polynomial.

## I. INTRODUCTION

In recent, years, splines functions have arise in many problems of mathematical Physics, such as for solving differential Equations in hydrodynamics viscoelasticity, electromagnetic theory, mixed boundary problems in mathematical physics biology and Engineering.

ThFawzy ([3] [4]) constructed special kinds of lacunaryquintic g-splines and proved that for functions  $f \in C^{(4)}$  the method converges faster that investigated by A.K. Verma[1] and for functions  $f \in C^{(5)}$  the order of approximation is the same as the best order of approximation using quintic g- splines. During the past twentieth boththeories of splines and experiences with their use in numerical analysis have undergone a considerable degree of development. According to Fawzy [3] theinterest in spline function is due to the fact that spline function are a good tool for the numerical approximation of functions,+

spline functions are used in many areas such as interpolation, datafitting, numerical solution of ordinary partial differential equation and also numerical solution of integral equations Lacunary interpolation by splines appears function about a function and its derivatives but without Hermite condition in which condition in which consecutive derivatives are used at each nodes, Several researchers have studied the use of spline to solve such interpolation [ 2, 5, 7 ]. In addition to the paper mentioned above dealing with best interpolation on approximation by splines there were also few papers that deal with constructive properties of space of splines interpolation. In my earlier work [6] some kinds of lacunary interpolation by g-splines have been investigated. In this paper we will continue to discuss the problem.

This paper is organized as follows- In Section 2, we construct a spline function of degree five which interpolates the lacunary data  $(0, 2, 4)$  some theorems about quintic spline functions are also studied in section 3, we construct almost quartic spline interpolant  $-(0, 2, 4)$ . Our methods are of lower degree having better convergence property than the earlier investigations made in ([8] [9] [10][11]).

## II. SPLINE INTERPOLANT $(0, 2, 4)$ FOR $F \in C^{(5)}(I)$

Let

$$\Delta: 0 = x_0 < x_1 < \dots < x_{n-1} < x_n = 1$$

be a partition of the interval  $I = [0,1]$  with  $X_{k+1} - x_k = h_k, k = 0(1)n - 1$ .

ands<sub>2,Δ</sub> be a piecewise polynomial of degree  $\leq 5$ ,which is a solution of  $(0, 2, 4)$ - interpolation for functions  $f \in C^{(5)}[x_0, x_n]$  inthe form :

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$$(2.1) \quad s_{2,\Delta}(x) \equiv s_{2,k}(x) = \sum_{j=0}^5 \frac{s_{k,j}^{(2)}}{j!} (x - x_k)^j, \quad x_k \leq x \leq x_{k+1} \quad \text{for } k = 0(1)n - 1,$$

Where  $s_{k,j}^{(2)}$ ,  $s$  are explicitly given below in terms of the prescribed data  $\{f_k^{(j)}\}$ ,  $j = 0, 2, 4$ ;  $K = 0(1)n$

In particular, for  $j = 0, 2, 4$

$$(2.2) \quad s_{k,j}^{(2)} = f_k^{(j)}, \quad k = 0(1)n - 1$$

and for  $j = 1, 3, 5$ , we have

$$(2.3) \quad s_{k,5}^{(2)} = \frac{1}{h} [f_{k+1}^{(4)} - f_k^{(4)}],$$

$$(2.4) \quad s_{k,3}^{(2)} = \frac{6}{h^3} [f_{k+1} - f_k - h s_{k,1}^{(2)} - \frac{h^2}{2!} f_k^{(2)}] - \frac{h^4}{4!} f_k^{(4)} - \frac{h^5}{5!} s_{k,5}^{(2)}$$

and

$$(2.5) \quad s_{k,1}^{(2)} = \frac{1}{h} [(f_{k+1} - f_k - \frac{h^2}{2!} f_k^{(2)} - \frac{h^4}{4!} f_k^{(4)}) - \frac{h^2}{3!} (f_{k+1}^{(2)} - f_k^{(2)} - \frac{h^2}{2!} f_k^{(4)}) + \frac{7}{3.5!} (f_{k+1}^{(4)} - f_k^{(4)})]$$

The coefficients  $s_{k,j}^{(2)}$ ,  $j = 1, 3, 5$  have been determined by the conditions :

That

$$D_L^{(p)} s_{2,k}(x_{k+1}) = D_R^{(p)} s_{2,k+1}(x_{k+1}), \quad p = 0, 2, 4; \quad k = 0(1)n - 1$$

Thus

$$s_{2,\Delta} \in C^{(0,2,4)}[I] = \{f : f^{(p)} \in C(I)\}, \quad p = 0, 2, 4\}.$$

Is a unique quintic piecewise polynomial satisfying interpolatory conditions (2.2).

If  $f \in C^{(5)}[I]$ , then owing to (2.3) - (2.5) and using Taylor's expansion, we have

$$(2.6) \quad |s_{k,j}^{(2)} - f_k^{(j)}| \leq C_{k,j}^{(2)} h^{5-j} \omega(f^{(5)}; h), \quad j = 1, 3, 5; \quad k = 0(1)n - 1$$

Where the constant  $C_{k,j}^{(2)}$  are given by :

$$C_{k,1}^{(2)} = \frac{1}{36}, \quad C_{k,3}^{(2)} = \frac{13}{60} \quad \text{and} \quad C_{k,5}^{(2)} = 1.$$

Using (2.1) - (2.6) and a little computation gives :

### Theorem 2.1

Let  $f \in C^{(5)}[I]$  and  $s_{2,\Delta} \in C^{(0,2,4)}(I)$  be the unique spline interpolant  $(0, 2, 4)$  given in (2.1) - (2.5),

then

$$(2.7) \quad \|D^{(j)}(f - S_{2,\Delta})\|_{L_\infty[X_k, x_{k+1}]} \leq c_{2,k}^j h^{5-j} \omega(f^{(5)}; h), \quad j = 0(1)5; \quad k = 0(1)n - 1$$

Where the constants  $c_{2,k}^j$ ,  $s$  are given by :

$$c_{2,k}^0 = \frac{13}{180}, \quad c_{2,k}^{(1)} = \frac{8}{45}, \quad c_{2,k}^2 = \frac{23}{60}, \quad c_{2,k}^{(3)} = \frac{43}{60}, \quad c_{2,k}^{(r)} = 1, \quad r = 4, 5.$$

### III. ALMOST QUARTIC SPLINE INTERPOLANT $(0, 2, 4)^*$ FOR $f \in C^{(4)}(I)$ .

Almost quartic-spline interpolant  $(0, 2, 4)^*$  is a piecewise polynomial of degree 4 in each subinterval except in the last one, where it is a polynomial of degree 5. In this case, we have

$$(3.1) \quad S_{2,\Delta}^*(x) = S_{2,k}^*(x) = \sum_{j=0}^4 \frac{S_{k,j}^{*(2)}}{j!} (x - x_k)^j, \quad x_k \leq x \leq x_{k+1}, \quad k = 0(1)n - 2$$

$$= \sum_{j=0}^5 \frac{S_{n-1,j}^{*(2)}}{j!} (x - x_{n-1})^j, \quad x_{n-1} \leq x \leq x_n, \quad k = n - 1$$

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The coefficients  $S_{k,j}^{*(2)}$  are explicitly given in terms of the data.

In particular, for  $K=O(1) n-1$ , we prescribe

$$(3.2) \quad S_{k,j}^{*(2)} = f_k^{(j)}, \quad j = 0, 2, 4.$$

For  $k = 0(1)n-2$  and  $j = 1, 3$ ,  $S_{k,j}^{*(2)}$  are given by

$$(3.3) \quad S_{k,1}^{*(2)} = \frac{3}{2h} \left[ (f_{k+1} - f_k - \frac{h^2}{2!} f_k^{(2)}) - \frac{h}{3} (f_{k+1}' - h f_k^{(2)} - \frac{h^3}{3!} f_k^{(4)}) \right]$$

and

$$(3.4) \quad S_{k,3}^{*(2)} = \frac{3}{h^3} \left[ h (f_{k+1}' - h f_k^{(2)} - \frac{h^3}{3!} f_k^{(4)}) - (f_{k+1} - f_k - \frac{h^2}{2!} f_k^{(2)} - \frac{h^4}{4!} f_k^{(4)}) \right]$$

For  $k=n-1$  and  $j=1, 3$  and  $5$ , we have

$$(3.5) \quad S_{n-1,5}^{*(2)} = \frac{1}{h} (f_n^{(4)} - f_{n-1}^{(4)})$$

$$(3.6) \quad S_{n-1,1}^{*(2)} = \frac{1}{h} \left[ (f_n - f_{n-1} - \frac{h^2}{2!} f_{n-1}^{(2)} - \frac{h^4}{4!} f_{n-1}^{(4)}) - \frac{h^3}{3!} (f_n^{(2)} - f_{n-1}^{(2)} - \frac{h^2}{2!} f_{n-1}^{(4)}) + \frac{7}{3.5!} h^4 (f_n^{(4)} - f_{n-1}^{(4)}) \right]$$

and

$$(3.7) \quad S_{n-1,3}^{*(2)} = \frac{6}{h^3} \left[ (f_n - f_{n-1} - h S_{n-1,1}^{*(2)} - \frac{h^2}{2!} f_{n-1}^{(2)} - \frac{h^4}{4!} f_{n-1}^{(4)} - \frac{h^5}{5!} S_{n-1,5}^{*(2)}) \right]$$

(3.3) and (3.4) are obtained from the condition.

$$(3.8) \quad S_{2,\Delta}^* \in C^{(1)} [I],$$

While (3.5)-(3.7) are determined from interpolatory conditions (3.2) for  $k = n-1$  in (3.1).

Analogous to (2.6) for  $f \in C^{(4)} [I]$ , one can establish

$$(3.9) \quad |S_{k,j}^{*(2)} - f_k^{(j)}| \leq C_{k,j}^{*(2)} h^{4-j} \omega(f^{(4)}, h),$$

Where the constants  $C_{k,j}^{*(2)}$  are given by

$$C_{k,j}^{*(2)} = \begin{cases} \frac{1}{12}, & j = 1 \\ \frac{5}{8}, & j = 3 \end{cases}; \quad k=0(1)n-2$$

$$C_{k,j}^{*(2)} = \begin{cases} \frac{13}{90}, & j = 1 \\ \frac{7}{6}, & j = 3 \end{cases}; \quad \text{for } k=n-1$$

Finally, similar to theorem 2.1, we have

**Theorem 3.1** Let  $f \in C^{(4)} [I]$  and  $S_{2,\Delta}^*$  be the unique almost quartic spline interpolant  $(0, 2, 4)^*$ , given by (3.1), then

$$(3.10) \quad \|D^{(j)}(f - S_{2,\Delta}^*)\|_{L_\infty[x_k, x_{k+1}]} \leq C_{2,k}^{*(j)} h^{4-j} \omega(f^{(4)}, h),$$

Where the constants  $C_{2,k}^{*(j)}$  are given by :

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	$C_{2,k}^{*(0)}$	$C_{2,k}^{*(1)}$	$C_{2,k}^{*(2)}$	$C_{2,k}^{*(3)}$	$C_{3,k}^{*(4)}$
$k=0(1)n-2$	$\frac{23}{24}$	$\frac{9}{16}$	$\frac{9}{8}$	$\frac{13}{8}$	1
$K=n-1$	$\frac{37}{180}$	$\frac{161}{180}$	$\frac{13}{12}$	$\frac{19}{12}$	1

## IV. CONCLUSION

In this paper, we have studied the existence and uniqueness of the quinticlacunary g-splines and almost quartic spline interpolant (0,2,4). Also we conclude that this new technique which we have used in the proving of two important theorems which are solutions of (0,2,4)- interpolation and obtained their local approximations with functions belonging to  $C^{(4)}(I)$  and  $C^{(5)}(I)$ . Our methods are of lower degree having better convergence property than the earlier investigations.

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