

# Effect of Rotation on Rayleigh-Bénard-Marangoni Convection in a Relatively Hotter or Cooler Layer of Liquid

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**ABSTRACT:** The effect of uniform rotation on the onset of steady Rayleigh-Bénard- Marangoni convection in a relatively hotter or cooler layer of liquid is studied theoretically by means of modified linear stability theory and a normal mode analysis. The top surface of layer is non-deformable free where surface tension gradients arise on account of variation of temperature, and the bottom surface is rigid and thermally conducting. Both mechanisms causing instability namely, surface tension induced and buoyancy induced are taken into consideration. A Fourier series method is used to obtain the eigenvalue equation analytically which is then computed numerically. Numerical results obtained are found to be significant from both qualitative and quantitative points of view. It is shown that uniform rotation in a relatively hotter layer of liquid is relatively more stable than a cooler layer of the same liquid under identical conditions. When speed of rotation becomes sufficiently large, it is found that the two mechanisms causing instability become less tight as compared to the case when no rotation is present. The asymptotic behavior of the rotation for the large values of the Taylor number is also obtained.

**KEYWORDS:** Buoyancy, Convection, Linear stability, Surface tension, Taylor number

## I. INTRODUCTION

The mechanism of the onset of surface tension induced convection in a thin horizontal fluid layer heated from below with free upper surface was first reported experimentally by Block [5] and explained mathematically by Pearson [13]. They established that the patterned hexagonal cells observed by Bénard [3, 4] and explained by Rayleigh [14] in terms of buoyancy, were in fact due to temperature dependent surface tension. Convection driven by surface tension gradients is now commonly known as Bénard-Marangoni convection in contrast to the buoyancy driven Rayleigh-Bénard convection. Quantitative disagreement between experiment and theory has indicated that gravity was present in Bénard's experiments as well as in other experiments involving convection in a fluid layer with free surface in a laboratory on the earth, therefore, Nield [11] considered the combined effects of both the surface tension and buoyancy on the onset of convection in a liquid layer heated from below with free upper surface, called Rayleigh-Bénard-Marangoni convection, and found that the two effects causing instability are tightly coupled. For a detail study of convection one may be referred to the work of Chandrasekhar [6], Normand et al. [12] and Koschmieder [9].

## II. RELATED WORK

Since the process of controlling convection in a fluid has become important in material processing and because of its applications extending from producing large crystals of uniform properties to manufacturing new materials with unique properties. The stabilizing effect of Coriolis force on buoyancy driven or/and surface tension driven convection in a liquid layer heated from below has been well established, notably by Chandrasekhar [6], Vidal and Acrivos [15], Namikawa et al. [10] and Hashim and Sharma [8]. Recently, Gupta et al. [7] studied the Rayleigh-Bénard-Marangoni convection in a relatively hotter or cooler layer of liquid and established that irrespective of the nature of driving force (surface tension or buoyancy or both) the hotter layer with its heat diffusivity apparently increased as a consequent of

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actual decrease in its specific heat at constant volume, must exhibit convection at a higher temperature difference, hence more stable, than a cooler layer of the same liquid under identical conditions otherwise.

In the present investigation, we wish to study the influence of rotation on the onset of Rayleigh-Bénard-Marangoni convection in a relatively hotter or cooler layer of liquid being important in controlling and understanding qualitatively the mechanism of convective instability problems encountered in geophysics, oceanography, atmospheric sciences and chemical engineering of paints and detergents. This analysis extends the work of Gupta et al. [7] to include the effect of Coriolis force. The Fourier series method is used to obtain the eigenvalue equation analytically. The numerical results are obtained for a wide range of parameters relevant to the problem in the present context. The results of this analysis indicate that the critical eigenvalues in the presence of Coriolis force are greater in a relatively hotter layer of liquid than a cooler one under identical conditions otherwise. The vital role that a relatively hotter layer of liquid has stabilizing influence in the presence of rotation as compared to that which is relatively cooler under almost identical conditions is illustrated through neutral stability curves. In addition, we find that when speed of rotation becomes sufficiently large, the two mechanisms causing instability become less tight as compared to the case when there is no rotation present. The asymptotic behavior of the rotation for large values of Taylor number is also obtained and discussed.

### III. MATHEMATICAL FORMULATION OF THE PROBLEM

The physical configuration of the problem consists of an infinite horizontal layer of viscous fluid of uniform thickness  $d$  heated from below and which is kept rotating with a constant angular velocity  $\Omega$  about an axis parallel to the direction of gravity. The lower rigid boundary of the layer maintained at constant temperature  $T_0$  and the upper free surface is open to the atmosphere at temperature  $T_1 (<T_0)$  subject to general heat radiative condition. We choose a Cartesian coordinate system  $Oxyz$  so that  $Oz$  coincides with the axis of rotation and  $Ox$  and  $Oy$  lie in the plane of the lower surface of the layer so that the fluid layer is confined between the planes at  $z = 0$  and  $z = d$ . The surface tension on the upper free surface of fluid layer is regarded as a function of temperature only which is given by the simple linear law

$$\tau = \tau_1 - \sigma(T - T_1) \tag{1}$$

where the constant  $\tau_1$  is the unperturbed value of  $\tau$  at the unperturbed surface temperature  $T = T_1$  and  $-\sigma = (\partial\tau / \partial T)_{T=T_1}$  represents the rate of change of surface tension with temperature, evaluated at temperature  $T_1$ , and surface tension being a monotonically decreasing function of temperature,  $\sigma$  is positive.

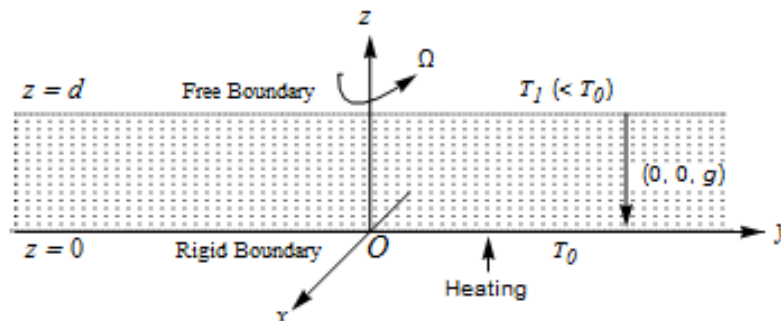


FIGURE 1. Schematic representation of the physical configuration of the problem

Following Banerjee et al.[2], the modified linearized perturbation equations governing the system under rotation are given as

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$$\left(\frac{\partial}{\partial t} - \nu \nabla^2\right) \nabla^2 w = g \alpha \nabla_1^2 \theta - 2\Omega \frac{\partial \zeta}{\partial z}, \quad (2)$$

$$(1 - \alpha_2 T_0) \left(\frac{\partial}{\partial t} - \kappa \nabla^2\right) \theta = \beta w, \quad (3)$$

$$\left(\frac{\partial}{\partial t} - \nu \nabla^2\right) \zeta = 2\Omega \frac{\partial w}{\partial z}, \quad (4)$$

where  $w$  is the perturbation velocity,  $\theta$  the perturbation temperature,  $\zeta$  is the  $z$ -component of vorticity. The adverse uniform temperature gradient  $\beta = (T_0 - T_1)/d > 0$ , the gravitational acceleration  $g$ , the coefficient of volume expansion  $\alpha$ , the kinematic viscosity  $\nu$ , the thermal diffusivity  $\kappa$  are each assumed constant,  $\nabla^2$  and  $\nabla_1^2$  represent respectively

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \text{ and } \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},$$

and  $t$  represents time. Further, the coefficient  $\alpha_2$  (due to variation in specific heat at constant volume on account of variations in temperature) lies in the range from 0 to  $10^{-4}$  and that range of the dimensionless parameter  $\alpha_2 T_0$  covering the usual laboratory conditions is  $0 \leq \alpha_2 T_0 < 1$  for liquids with which we are mostly concerned. In this range, any given value of  $\alpha_2 T_0 (\neq 0)$  corresponds to the layer of liquid which is relatively hotter compared to that associated with its value (including  $\alpha_2 T_0 = 0$ ) less than the given one. A detailed account of this has been given in the research monograph by Banerjee and Gupta [1].

Equations (2) – (4), must be solved subject to appropriate boundary conditions. For the rigid and perfectly thermally conducting (the perturbation temperature is thus zero) lower boundary at  $z = 0$ , the boundary conditions are

$$w = 0, \quad \frac{\partial w}{\partial z} = 0, \quad \theta = 0, \quad \zeta = 0. \quad (5a,b,c, d)$$

Since the tangential viscous stress experienced by the liquid at the upper free surface is balanced by the traction due to variation with temperature of surface tension, and the general heat radiation condition at  $z = d$ , the boundary conditions are

$$w = 0, \quad \rho \nu \frac{\partial^2 w}{\partial z^2} = \sigma \nabla_1^2 \theta, \quad -k \frac{\partial \theta}{\partial z} = q\theta, \quad \frac{\partial \zeta}{\partial z} = 0, \quad (6a,b,c, d)$$

where  $k$  is the heat conductivity and  $q$  is the rate of change with temperature of the time rate of heat loss per unit area from the upper surface.

We now analyze an arbitrary disturbance in terms of normal modes assuming that the perturbations  $w$ ,  $\theta$  and  $\zeta$  are of the form

$$[w(x, y, z, t), \theta(x, y, z, t), \zeta(x, y, z, t)] = [W(z), \Theta(z), Z(z)] \exp\{i(a_x x + a_y y) + st\}$$

where  $a_x$  and  $a_y$  are components of the horizontal wave number  $a (= \sqrt{a_x^2 + a_y^2})$  of the disturbance, and  $s$  is the time growth rate (a complex number in general). Using the above expressions for  $w$ ,  $\theta$  and  $\zeta$  in equations (2) – (4) and then making the resulting equations dimensionless by choosing  $d$ ,  $d^2/\nu$ ,  $\nu/d$ ,  $\beta d(\nu/k)$  as units of length, time, velocity and temperature scales respectively, we obtain

$$(D^2 - a^2)(D^2 - a^2 - p)W = Ra^2\Theta + T^{\frac{1}{2}}DZ, \quad (7)$$

$$(D^2 - a^2 - (1 - \alpha_2 T_0)pPr)\Theta = -(1 - \alpha_2 T_0)W, \quad (8)$$

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$$(D^2 - a^2 - p)Z = -T^{\frac{1}{2}}DW. \tag{9}$$

Here  $R(= g\alpha\beta d^4 / \kappa\nu)$  is the Rayleigh Number,  $P_r(= \nu / \kappa)$  is the Prandtl Number,  $T(= 4\Omega^2 d^4 / \nu^2)$  is the Taylor Number.

We restrict our analysis to the case when the principle of exchange of stabilities is valid for the present problem so that instability first sets in as stationary convection. In that case, the marginal state is characterized by  $p=0$ , and equations (7) – (9) reduces to

$$(D^2 - a^2)^2 W = Ra^2\Theta + T^{\frac{1}{2}}DZ, \tag{10}$$

$$(D^2 - a^2)\Theta = -(1 - \alpha_2 T_0)W, \tag{11}$$

$$(D^2 - a^2)Z = -T^{\frac{1}{2}}DW. \tag{12}$$

In terms of new variables, the non-dimensional form of boundary conditions (5a, b, c, d) – (6a, b, c, d) can be written as

$$W(0) = 0, \quad DW(0) = 0, \quad \Theta(0) = 0, \quad Z(0) = 0, \tag{13a, b, c, d}$$

evaluated on the lower boundary  $z = 0$ , and

$$W(1) = 0, \quad D^2W(1) + a^2M\Theta(1) = 0, \quad D\Theta(1) + L\Theta(1) = 0, \quad DZ(1) = 0. \tag{14a, b, c, d}$$

evaluated on the upper boundary  $z = 1$ . Here  $M = \sigma\beta d^2 / \rho\kappa\nu$  is the Marangoni number and  $L = qd / k$  is the Biot number.

Equations (10) – (12) together with boundary conditions (13a, b, c, d) – (14a, b, c, d) constitute an eigenvalue problem of order eight with  $M$  as an eigenvalue for given values of the remaining parameters.

## IV. SOLUTION OF THE PROBLEM

The Fourier series method as presented by Nield [11] is convenient for the problem under consideration. We let

$$W(z) = \sum_{n=1}^{\infty} \left[ A_n - \frac{2}{n^3\pi^3} \left\{ D^2W(0) - (-1)^n D^2W(1) \right\} \right] \sin n\pi z, \tag{15}$$

$$\Theta(z) = \sum_{n=1}^{\infty} \left[ B_n - (-1)^n \frac{2}{n\pi} \Theta(1) \right] \sin n\pi z, \tag{16}$$

$$Z(z) = C_0 + \sum_{n=1}^{\infty} \left[ C_n - \frac{2}{n^2\pi^2} DZ(0) \right] \cos n\pi z + \frac{1}{3} DZ(0), \tag{17}$$

where the boundary conditions (13a, c) and (14a, d) have already been used while writing equations (15)-(17). The differential equations (10)-(12) are satisfied by substituting  $W(z)$ ,  $\Theta(z)$ ,  $Z(z)$  and their derivatives, we obtain the system of equations which can be solved for  $A_n, B_n$  and  $C_n$ . Substitution of  $A_n, B_n$  and  $C_n$  in the remaining boundary conditions we get three linear homogeneous algebraic equations. Finally, elimination of the unknown constants  $DW(0)$ ,  $D^2W(1)$  and  $DZ(1)$  yields the eigenvalue equation as

$$\begin{vmatrix} \sum_{n=1}^{\infty} \frac{F_n}{H_n} & M \sum_{n=1}^{\infty} \frac{(-1)^n F_n}{H_n} + R \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{H_n} & T \sum_{n=1}^{\infty} \frac{E_n}{H_n} \\ \sum_{n=1}^{\infty} \frac{E_n}{H_n} & M \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{H_n} + R \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{(n^2\pi^2 + a^2)H_n} & T \sum_{n=1}^{\infty} \frac{E_n}{(n^2\pi^2 + a^2)H_n} - H \\ \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{H_n} & M \sum_{n=1}^{\infty} \frac{E_n}{H_n} + R \sum_{n=1}^{\infty} \frac{1}{H_n} - \frac{1}{a^2(1 - \alpha_2 T_0)} \left( \frac{L+1}{2} + \sum_{n=1}^{\infty} \frac{G_n}{H_n} \right) - \frac{T}{(1 - \alpha_2 T_0)} \sum_{n=1}^{\infty} \frac{E_n}{(n^2\pi^2 + a^2)H_n} & T \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{(n^2\pi^2 + a^2)H_n} \end{vmatrix} = 0 \tag{18}$$

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where

$$\left. \begin{aligned} E_n &= n^2 \pi^2 \\ F_n &= n^2 \pi^2 (n^2 \pi^2 + a^2) \\ G_n &= a^2 (n^2 \pi^2 + a^2)^2 \\ H_n &= (n^2 \pi^2 + a^2)^3 - a^2 R (1 - \alpha_2 T_0) + n^2 \pi^2 T \\ H &= \frac{1}{2a} \coth a \end{aligned} \right\} \quad (19)$$

We can obtain  $M$  from the eigenvalue equation (18) in terms of  $a, R, L, Q$  and  $\alpha_2 T_0$  as the ratio of two determinants given by

$$M = \frac{\begin{vmatrix} \sum_{n=1}^{\infty} \frac{F_n}{H_n} & -R \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{H_n} & T \sum_{n=1}^{\infty} \frac{E_n}{H_n} \\ \sum_{n=1}^{\infty} \frac{E_n}{H_n} & -R \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{(n^2 \pi^2 + a^2) H_n} & T \sum_{n=1}^{\infty} \frac{E_n}{(n^2 \pi^2 + a^2) H_n} - H \\ \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{H_n} & -R \sum_{n=1}^{\infty} \frac{1}{H_n} + \frac{1}{a^2 (1 - \alpha_2 T_0)} \left( \frac{L+1}{2} + \sum_{n=1}^{\infty} \frac{G_n}{H_n} \right) + \frac{T}{(1 - \alpha_2 T_0)} \sum_{n=1}^{\infty} \frac{E_n}{(n^2 \pi^2 + a^2) H_n} & T \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{(n^2 \pi^2 + a^2) H_n} \end{vmatrix}}{\begin{vmatrix} \sum_{n=1}^{\infty} \frac{F_n}{H_n} & \sum_{n=1}^{\infty} \frac{(-1)^n F_n}{H_n} & T \sum_{n=1}^{\infty} \frac{E_n}{H_n} \\ \sum_{n=1}^{\infty} \frac{E_n}{H_n} & \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{H_n} & T \sum_{n=1}^{\infty} \frac{E_n}{(n^2 \pi^2 + a^2) H_n} - H \\ \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{H_n} & \sum_{n=1}^{\infty} \frac{E_n}{H_n} & T \sum_{n=1}^{\infty} \frac{(-1)^n E_n}{(n^2 \pi^2 + a^2) H_n} \end{vmatrix}}, \quad (20)$$

## V. NUMERICAL RESULTS AND DISCUSSION

The numerical calculations may be carried out as follows. For fixed values of  $T, \alpha_2 T_0$  and  $L$  the expression (20) determines the Marangoni number  $M$  as a function of the wave number  $a$ . The minimum value of  $M$  is the critical Marangoni number  $M_c$  and the value of  $a$  at which  $M$  attains the minimum is the critical wave number  $a_M$ . Alternatively, for fixed values of  $M, T, \alpha_2 T_0$  and  $L$ , we may determine the Rayleigh number  $R$  as a function of the wave number  $a$ . The minimum value of  $R$  is the critical Rayleigh number  $R_c$  and the value of  $a$  at which  $R$  attains the minimum is the critical wave number  $a_R$ . By either means we can plot the  $(R, M)$ -curve for fixed values of  $T, \alpha_2 T_0$  and  $L$  corresponding to the marginal stability.

According to the procedure as described above, numerical values are computed with the aid of symbolic algebraic package Mathematica 5.2. The numerical values of  $M_c$  and  $a_M$  ( $R = 0$ ) computed from expression (20) when  $L = 0$  and  $L = 10^3$  are respectively presented in Table 1 and Table 2 for fixed values of  $\alpha_2 T_0$  and  $T$ . The values of  $T$  are chosen so as to compare the results obtained by us with the results of Namikawa et al. [10]. We find that when  $\alpha_2 T_0 = 0$  values of  $M_c$  and  $a_M$  for various values of  $T$  obtained here are in close agreement with corresponding values obtained

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by Namikawa et al. [10]. When  $\mathcal{T} = 0$  the results obtained here for various values of  $\alpha_2 T_0$  agree precisely with corresponding values obtained by Gupta et al. [7].

TABLE 1. Values of  $M_c$  and  $a_M$  for various values of  $\alpha_2 T_0$  when  $L = 0$

$\mathcal{T}$	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.1$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	$M_c$	$a_M$	$M_c$	$a_M$	$M_c$	$a_M$	$M_c$	$a_M$
0	79.61	1.99	88.45	1.99	113.72	1.99	159.21	1.99
1	79.73	2.00	88.59	2.00	113.90	2.00	159.47	2.00
10	80.86	2.01	89.84	2.01	115.51	2.01	161.72	2.01
$10^2$	91.35	2.17	101.50	2.17	130.51	2.17	182.71	2.17
$10^3$	163.41	2.97	181.56	2.97	233.44	2.97	326.81	2.97
$10^4$	456.92	5.01	507.69	5.01	652.75	5.01	913.85	5.01
$10^5$	1401.76	8.63	1557.51	8.63	2002.51	8.63	2803.52	8.63
$10^6$	4405.48	15.11	4894.98	15.11	6293.54	15.11	8810.96	15.11
$10^7$	13925.70	26.80	15473.00	26.80	19893.86	26.80	27851.40	26.80
$10^8$	44036.70	47.65	48929.67	47.65	62909.57	47.65	88073.40	47.65

TABLE 2. Values of  $M_c$  and  $a_M$  for various values of  $\alpha_2 T_0$  when  $L = 10^3$

$\mathcal{T}$	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.1$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	$M_c$	$a_M$	$M_c$	$a_M$	$M_c$	$a_M$	$M_c$	$a_M$
0	$10^3 \times 32.17$	3.01	$34.4410^3 \times$	3.01	$45.9610^3 \times$	3.01	$64.3410^3 \times$	3.01
1	$10^3 \times 32.19$	3.01	$35.7710^3 \times$	3.01	$45.9910^3 \times$	3.01	$64.3810^3 \times$	3.01
10	$32.4110^3 \times$	3.03	$36.0110^3 \times$	3.03	$46.3010^3 \times$	3.03	$64.8210^3 \times$	3.03
$10^2$	$34.3310^3 \times$	3.23	$38.1410^3 \times$	3.23	$49.0410^3 \times$	3.23	$68.6610^3 \times$	3.23
$10^3$	$45.7110^3 \times$	4.29	$50.7910^3 \times$	4.29	$65.3010^3 \times$	4.29	$91.4210^3 \times$	4.29
$10^4$	$77.0910^3 \times$	7.06	$152.4010^3 \times$	7.06	$110.1310^3 \times$	7.06	$154.1810^3 \times$	7.06
$10^5$	$137.1610^3 \times$	12.34	$273.5610^3 \times$	12.34	$195.9410^3 \times$	12.34	$274.3210^3 \times$	12.34
$10^6$	$246.2010^3 \times$	21.86	$494.5310^3 \times$	21.86	$351.7110^3 \times$	21.86	$492.4010^3 \times$	21.86
$10^7$	$445.0810^3 \times$	38.63	$904.7410^3 \times$	38.63	$635.8310^3 \times$	38.63	$890.1610^3 \times$	38.63
$10^8$	$814.2710^3 \times$	68.00	$814.2710^3 \times$	68.00	$1163.2510^3 \times$	68.00	$1628.5410^3 \times$	68.00

The neutral stability curves for the limiting cases namely, Bénard-Marangoni convection ( $R=0$ ) and for Rayleigh-Bénard convection ( $M=0$ ) are plotted respectively in Fig. 2 and Fig. 3 for various values of  $\mathcal{T}$  and  $\alpha_2 T_0$  when  $L = 0$ . Points below a curve represent stable state.

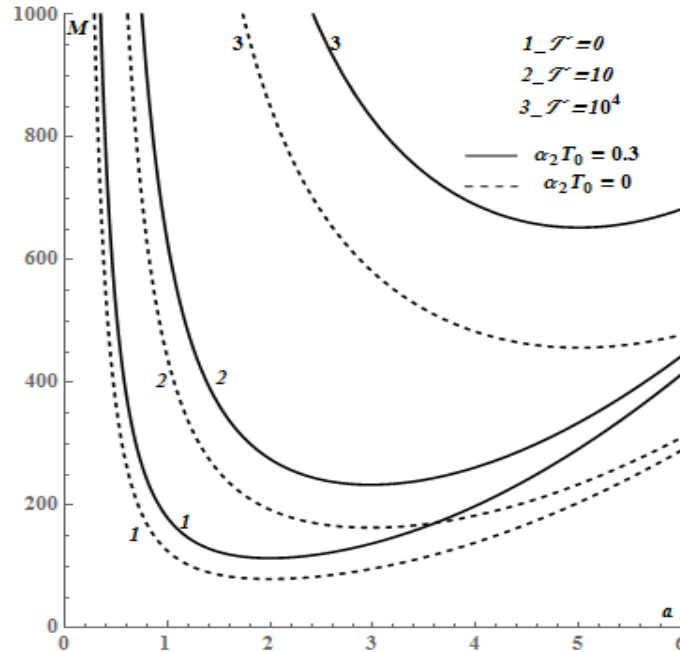


FIGURE 2. Bénard-Marangoni neutral stability curves for various values of  $\mathcal{T}$  and  $\alpha_2 T_0$  when  $L = 0$ .

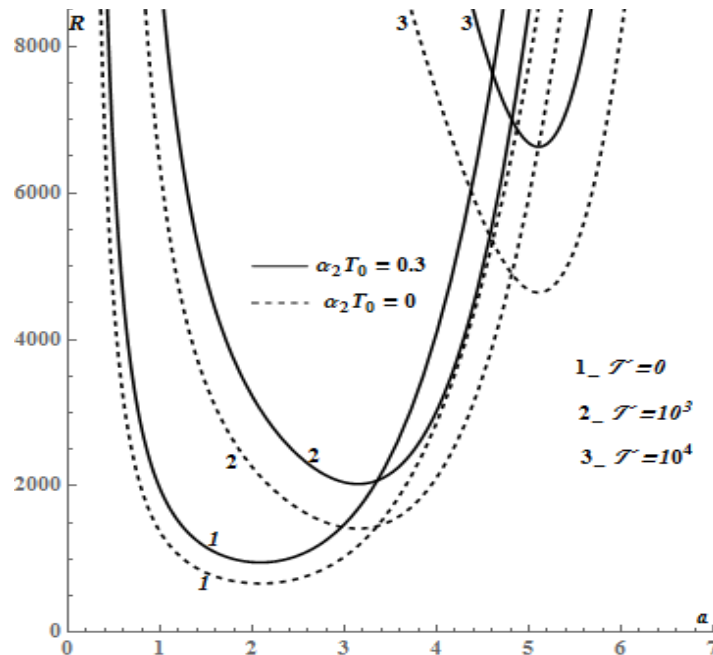


FIGURE 3. Rayleigh-Bénard neutral stability curves for various values of  $\mathcal{T}$  and  $\alpha_2 T_0$  when  $L = 0$

The upward moving neutral stability curves in Fig 2 and 3, illustrate that increasing values of either the Taylor number  $\mathcal{T}$  (for fixed value of  $\alpha_2 T_0$ ) or the  $\alpha_2 T_0$  (for a fixed value of  $\mathcal{T}$ ) have the stabilizing effect on the onset of convection in each case. In other words, Tables 3 to 4 and fig. 2 and 3 are showing clearly the vital role that a relatively hotter

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layer of liquid is relatively more stable in the presence of rotation compared to a relatively cooler one under almost identical conditions in each case.

The numerical values of both  $R_c$  and  $a_R$  ( $M = 0$ ) computed from expression (18) when  $L = 0$  and  $L=10^3$  are respectively presented in Table 3 and Table 4 for various given values of  $\alpha_2 T_0$  and  $\mathcal{T}$ . When  $\alpha_2 T_0 = 0$  values of  $R_c$  and  $a_R$  for various values of  $\mathcal{T}$  obtained here agree precisely with corresponding values as obtained by Namikawa et al. [10].

For fixed values of  $\alpha_2 T_0$  and  $\mathcal{T}$ , it may be mentioned here that as  $L$  increases values of both  $M_c$  and  $R_c$  increase. In the limit  $L \rightarrow \infty$ , we find that value of  $M_c$  becomes asymptotically proportional to  $L$  while that of  $R_c$  tends to a finite limit. In either case, the corresponding critical wave number remains finite. The case when  $L$  increases from 0 to  $\infty$  corresponds to the situation wherein thermal boundary conditions at the free surface changes from constant heat flux ( $D\Theta(1) = 0$ ) to constant temperature ( $\Theta(1) = 0$ ). Therefore, when  $L$  is small it is easier for temperature perturbations to be set-up, but when  $L$  is large, any temperature variations across the free surface decay rapidly. Thus as  $L$  becomes large, the values of  $M_c$  tend to infinity since it becomes difficult for the surface tension to be operative, while those of  $R_c$  remain finite since the buoyancy force is still operative.

TABLE 3. Values of  $R_c$  and  $a_R$  for various values of  $\alpha_2 T_0$  when  $L = 0$

$\mathcal{T}$	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.1$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	$R_c$	$a_R$	$R_c$	$a_R$	$R_c$	$a_R$	$R_c$	$a_R$
0	669.00	2.09	743.33	2.09	955.71	2.09	1337.99	2.09
10	679.70	2.11	755.22	2.11	971.00	2.11	1359.39	2.11
$10^2$	769.87	2.28	855.41	2.28	1099.81	2.28	1539.74	2.28
$10^3$	1418.95	3.15	1576.61	3.15	2027.07	3.15	2837.90	3.15
$10^4$	4640.20	5.11	5155.78	5.11	6628.86	5.11	9280.40	5.11
$10^5$	18695.70	7.94	20773.00	7.94	26708.14	7.94	37391.40	7.94
$10^6$	81478.40	11.96	90531.56	11.96	116397.71	11.96	162956.80	11.96
$10^7$	369697.00	17.83	410774.44	17.83	528138.57	17.83	739394.00	17.83
$10^8$	1711750.00	26.49	1901944.44	26.49	2445357.14	26.49	3423500.00	26.49

TABLE 4. Values of  $R_c$  and  $a_R$  for various values of  $\alpha_2 T_0$  when  $L = 10^3$

$\mathcal{T}$	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.1$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	$R_c$	$a_R$	$R_c$	$a_R$	$R_c$	$a_R$	$R_c$	$a_R$
0	1099.29	2.68	1221.43	2.68	1570.41	2.68	2198.58	2.68
10	1112.18	2.70	1235.76	2.70	1588.83	2.70	2224.36	2.70
$10^2$	1202.17	2.85	1335.74	2.85	1717.39	2.85	2404.34	2.85
$10^3$	1879.47	3.63	2088.30	3.63	2684.96	3.63	3758.94	3.63
$10^4$	5137.39	5.36	5708.21	5.36	7339.13	5.36	10274.78	5.36
$10^5$	19120.10	8.05	21244.56	8.05	27314.43	8.05	38240.20	8.05
$10^6$	81806.90	12.01	90896.56	12.01	116867.00	12.01	163613.00	12.01
$10^7$	361823.00	17.85	402025.56	17.85	516890.00	17.85	723646.00	17.85
$10^8$	1702100.00	26.50	1891222.22	26.50	2431571.43	26.50	3404200.00	26.50

The asymptotic behavior of  $M_c$  as  $Q \rightarrow \infty$  depends critically on both  $\alpha_2 T_0$  and  $L$ , whereas the asymptotic behavior of  $a_M$  depends only on  $L$ . When  $L=0$ , we find numerically from equation (20) that



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$$M_c \approx \frac{4.40}{1 - \alpha_2 T_0} \mathcal{T}^{\frac{1}{2}} \text{ and } a_M \approx 0.48 \mathcal{T}^{\frac{1}{4}}$$

Which are in close agreement with the values  $M_c \approx 4.42 \mathcal{T}^{\frac{1}{2}}$  and  $a_M \approx 0.5 \mathcal{T}^{\frac{1}{4}}$  as obtained by Vidal and Acrivos [15] (when  $\alpha_2 T_0 = 0$ ) by means of asymptotic analysis.

When  $L = 10^3$ , we obtain

$$\frac{M_c}{L} \approx \frac{0.81}{1 - \alpha_2 T_0} \mathcal{T}^{\frac{1}{2}} \text{ and } a_M \approx 0.69 \mathcal{T}^{\frac{1}{4}}$$

On the other hand, the asymptotic behaviour of  $R_c$  and  $a_R$  as  $\mathcal{T} \rightarrow \infty$  is independent of  $L$ . However, the asymptotic behavior of  $R_c$  critically depends on  $\alpha_2 T_0$ . In this case, we find that

$$R_c \approx \frac{7.95}{1 - \alpha_2 T_0} \mathcal{T} \text{ and } a_R \approx 1.23 \mathcal{T}^{\frac{1}{6}}$$

which are in close agreement with the values  $R_c \approx 8.69 \mathcal{T}^{\frac{2}{3}}$  and  $a_R \approx 1.31 \mathcal{T}^{\frac{1}{6}}$  as obtained by Vidal and Acrivos [15] (when  $\alpha_2 T_0 = 0$ ).

The  $(R, M)$ -loci corresponding to neutral stability curves for the combined surface tension and buoyancy effects, normalized for critical values of  $R_c$  and  $M_c$  are plotted in Fig 4 for various values of  $\mathcal{T}$  and  $\alpha_2 T_0$  for the limiting case when  $L=0$  (insulating boundary).

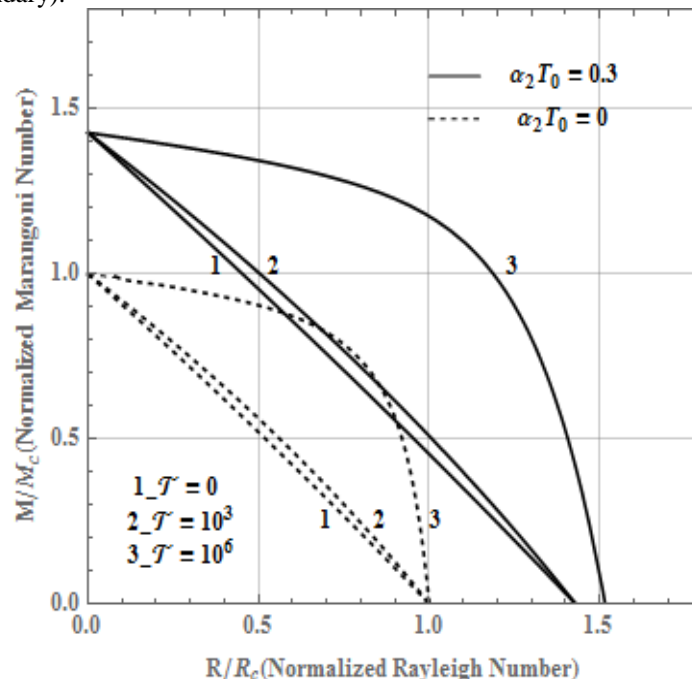


FIGURE 4.  $(R, M)$ - loci corresponding to Rayleigh-Bénard-Marangoni neutral stability curves (normalized to give unit intercepts) for various values of  $\mathcal{T}$  and  $\alpha_2 T_0$  when  $L=0$ .

Fig. 4 shows that the inhibiting effect of the Coriolis force remains unchanged. Comparison of curves in Fig. 4 illustrate that when  $\mathcal{T} = 0$  the coupling between the two agencies remain tight but not perfect. If there were perfect coupling, one would expect the locus to be the line  $R/R_c + M/M_c = 1/(1 - \alpha_2 T_0)$ . As  $\mathcal{T}$  becomes large the locus goes away from the

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line  $R/R_c + M/M_c = 1/(1 - \alpha_2 T_0)$ , illustrating that the coupling between the two mechanism causing instability becomes less tight.

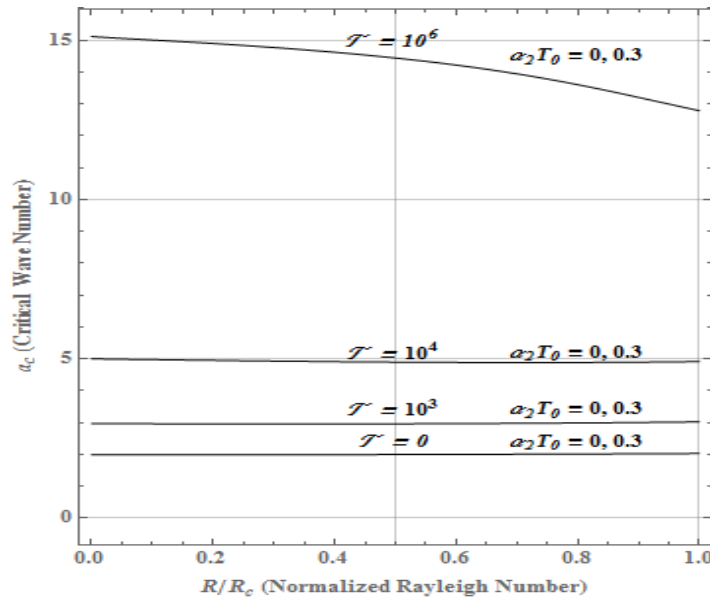


FIGURE 5. Wave number corresponding to Rayleigh-Bénard-Marangoni neutral stability versus normalized Rayleigh number.

Fig. 5 illustrate the variation of wave numbers corresponding to the Rayleigh-Bénard-Marangoni at the marginal stability. For large values of  $T$  the convective cells increase in size, independent of  $\alpha_2 T_0$ , at the onset of convection as the dominance of buoyancy effect increases.

## VI. CONCLUSION

The modified linear stability analysis of the Rayleigh-Bénard-Marangoni convection in a relatively hotter or cooler layer of liquid in the presence of rotation has been studied theoretically and we conclude that Coriolis force is relatively more stabilizing in a relatively hotter layer of liquid than a cooler one under identical conditions, irrespective of whether the driving force is either surface tension or buoyancy or both surface tension and buoyancy.

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## REFERENCES

- [1] Banerjee M.B. and Gupta J.R., "Studies in Hydrodynamic and Hydromagnetic Stability", Silver Line Publication, Shimla, 1991.
- [2] Banerjee M.B., Gupta J.R., Shandil R.G., Sharma K.C. and Katoch D.C., "A modified analysis of thermal and thermohaline instability of a liquid layer heated underside", J. Math. Phys. Sci., Vol. 17, pp.603-629, 1983.
- [3] Bénard H., "Les tourbillons cellulaires dans un liquide transportant de la chaleur par convection en régime permanent", Revue générale des Sciences pures et appliquées, Vol. 11, pp.1261-1271, 1900.
- [4] Bénard H., "Les tourbillons cellulaires dans un liquide transportant de la chaleur par convection en régime permanent", Ann. Chimie (Paris), Vol. 23, pp.62-144, 1901.
- [5] Block M.J., "Surface tension as the cause of Bénard cells and surface deformation in a liquid film". Nature, Vol. 178, pp.650-651, 1956.
- [6] Chandrasekhar S., "Hydrodynamic and Hydromagnetic Stability", Dover Publications, New York, 1961.
- [7] Gupta A.K., Shandil R.G., Kumar S. and Surya D., "On revisiting surface tension and buoyancy driven convection", Conference Proceeding, National Conference on Advances in Mathematics and its Applications, pp.157-162, 2013.

## International Journal of Innovative Research in Science, Engineering and Technology

*(An ISO 3297: 2007 Certified Organization)*

**Vol. 4, Issue 6, June 2015**

- [8] Hashim I. and Sarma W., "On the onset of Marangoni convection in a rotating fluid layer", J. Phy. Soc. Japan, Vol. 75(3), pp. 035001-2, 2006.
- [9] E.L. Koschmieder, "Bénard Cells and Taylor Vortices", Cambridge University Press, Cambridge, 1993.
- [10] Namikawa T., Takashima M. and Matsushita S., "The effect of rotation on convective instability induced by surface tension and buoyancy", J. Phy. Soc. Japan, Vol. 28 (5), pp. 1340-1349, 1970.
- [11] Nield D.A., "Surface tension and buoyancy effects in cellular convection", J. Fluid Mech., Vol. 19, pp.341-352, 1964.
- [12] Normand C., Pomeau Y. and Velarde M.G., "Convective instability: A physicist's approach", Rev. Mod. Phys., Vol. 49, pp.581-624, 1977.
- [13] Pearson J.R.A., "On convection cells induced by surface tension", J. Fluid Mech., Vol. 4, pp.489-500, 1958.
- [14] Rayleigh L., "On convection currents in a horizontal layer of fluid, when the higher temperature is on the underside", Phil. Mag., Vol. 32, pp.529-546, 1916.
- [15] Vidal A. and Acrivos A., "The influence of Coriolis force on surface-tension-driven convection", J. Fluid. Mech., Vol.26 (4), pp. 807-818, 1966.