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Some Applications of Mathematical Analysis

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ABSTRACT: In this paper, we study some application of mathematical analysis using closed sets, isolated points, accumulation points.

KEYWORDS: accumulation points, closed sets.

I. INTRODUCTION

This concepts related in some way to real numbers, so we begin our study of analysis with a discussion of the real-number system. Several methods are used to introduce real numbers. One method starts with positive integers 1,2,3,... as undefined concepts and uses them to build a largest system, the prime, Mersenne prime, Fermat prime & Fibonacci numbers. The set of all points between a and b is called an *interval*[3]. Sometimes it is important to distinguish between intervals which include their endpoints and intervals which do not. The notation $\{x: x \text{ satisfies } P\}$ will be used to designate the set of all real numbers x satisfy property p. Assume $a < b$. the *open interval* (a,b) is defined to be the set $(a,b) = \{x: a < x < b\}$. The *closed interval* [a,b] is the set $\{x: a \leq x \leq b\}$. This section describes the applications of closed sets, isolated points ,accumulation points[2].

Definition 1.1. Let S be a set in E_1 and x a point in E_1 , x not necessarily in S. Then x is called an accumulation point of S, provided every neighbourhood of x contains at least one point of S distinct from x. If x is an accumulation point of S, then every neighbourhood $N(x)$ contains infinitely many points of S[1].

Definition 1.2. Closed sets in E_1 A set is called closed if it contains all its accumulation points. Thus a closed interval is a closed set. An open interval, however, is not a closed set because it does not contain its endpoints, both of which are accumulation points of the set. A set which has no accumulation points is automatically closed and, in particular, every finite set is closed[5]. The empty set is also closed, as is the whole real line E_1 .

A set which is not closed need not be open, as, for example, the half-open interval (a, b], which is neither open nor closed.

Definition 1.3 If S is open, then the complement $E_1 - S$ is closed. Conversely, if S is closed, then $E_1 - S$ is open. It follows from the above that the union of a finite collection of closed sets is closed and that the intersection of an arbitrary collection of closed sets is closed[7].

Definition 1.4 A point x in E_n is said to be a **condensation** point of S if every neighbourhood $N(x)$ has the property that $N(x) \cap S$ is not countable. Show that if S is not countable, then there exists a point x in S such that x is a condensation point of S.

II. MAIN RESULTS

In this section. We prove some applications on countable, closed sets, isolated points.

Theorem 2.1 Prove that if S is not countable, then there exists a point x in S such that x is a condensation point of S[5].

Proof. Divide the set S up into n (possibly partially overlapping) sets S_i , where n is a finite number; $i = 1, 2, \dots, n$; and $\bigcup_i S_i = S$. If the set S is not countable, then at least one of these 'regions' will have to be uncountable as well, or else we will have a finite collection of countable sets and thus S will be countable — and we will have a contradiction. Choose one of the sets S_i which is uncountable, and repeat the above with this set (i.e. divide it up into a finite collection of smaller sets, at least one of which will be uncountable). If we continue to do this, then this process will converge onto a point $z \in S$ which will be the condensation point we are looking for. Justification: with every subdivision, we find a

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smaller and smaller "region" which is uncountable [7]. In the limit of this process, every neighbourhood of the point z we converge onto will contain one of the regions which we know to be uncountable (i.e. some region $S_j \subset N(z)$). Therefore, every neighbourhood of this point z has the property that $N(z) \cap S$ is not countable, i.e. z is a condensation point. Conclusion: given an uncountable set S , we can always find a condensation point $z \in S$.

Theorem 2.2 Assume that $S \setminus C E_n$ and assume that S is not countable. Let T denote the set of condensation points of S . Show that

- (a) $S \setminus T$ is countable, (b) $S \cap T$ is not countable,
(c) T is a closed set, (d) T contains no isolated point

Proof.

(a) If $S \setminus T$ was not countable, then it would contain a condensation point. But if T is the set of condensation points of S , then $S \setminus T$ contains no condensation points. It follows (by contradiction) that $S \setminus T$ is countable.

(b) The set $S \cap T$ is the set of condensation points which are in S . We must therefore prove that the set of condensation points in S is not countable. To do this, we use three pieces of information: (1) From set theory, we know that $S \cap T = S \setminus T_c$; (2) We know that S is not countable; and (3) We know that $S \setminus T$ is countable. If $S \setminus T$ is countable, and if S is not countable, then $S \setminus T_c$ must be non countable. Using (1), it follows that $S \cap T$ must also be non countable.

(c) By definition, T is closed if it contains all its accumulation points. We know that every point $x \in T$ is a condensation point, so that every neighbourhood $N(x)$ of a point $x \in T$ is not countable. This implies that the neighbourhood contains another condensation point $y \in T$ ($N(x) \cap S$ uncountable $\Rightarrow \exists$ a condensation point $y \in N(x) \cap S$), so that the point x is an accumulation point of the set T . This applies for all condensation points, so that every point $x \in T$ is an accumulation point [9].

It remains to show that T has no accumulation points that are not in T . Assume that z is an accumulation point of the set T — with $z \notin T$. Because z is not a condensation point, it follows that not all neighbourhoods $N(z)$ are uncountable. It follows that all the neighbourhoods of the point z must be uncountable, and so we must have $z \in T$, and so T has no accumulation points that are not in T .

(d) Because we concluded in (c) that every point in T is an accumulation point, it follows that no point in T is an isolated point, i.e. not an accumulation point [8].

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