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## $\psi g^{**}$ - Closed Sets In Topological Spaces

N. Balamani<sup>1</sup>, A.Parvathi<sup>2</sup>

Assistant Professor of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women  
University, Coimbatore, Tamil Nadu, India<sup>1</sup>

Professor of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women  
University, Coimbatore, Tamil Nadu, India<sup>2</sup>

**ABSTRACT:** In this paper we introduce and study a new class of generalized closed sets called  $\psi g^{**}$ -closed sets in topological spaces and investigate some of the basic properties. We analyse the relation between  $\psi g^{**}$ -closed sets with already existing closed sets. The class of  $\psi g^{**}$ -closed sets is properly placed between the classes of  $\psi$ -closed sets and the class of  $\psi g$ -closed sets.

**KEYWORDS:**  $g$ -closed sets,  $g^{**}$ -closed sets,  $\psi g$ -closed sets and  $\psi g^{**}$ -closed sets

### I. INTRODUCTION

Levine [6] introduced generalized closed sets (briefly  $g$ -closed sets) in topological spaces and studied their basic properties. Veerakumar [15] introduced and studied  $\psi$ -closed sets. Veerakumar [16] introduced  $g^*$ -closed sets in topological spaces and studied their properties. Pauline Mary Helan et.al [11] introduced and studied  $g^{**}$ -closed sets in topological spaces. The aim of this paper is to introduce a new class of generalized closed sets called  $\psi g^{**}$ -closed sets.

### II. PRELIMINARIES

**Definition 2.1** A subset  $A$  of a topological space  $(X, \tau)$  is called

- (i) Regular open set [14] if  $A = \text{int}(\text{cl}(A))$
- (ii) Semi-open set [5] if  $A \subseteq \text{cl}(\text{int}(A))$
- (iii)  $\alpha$ -open set [10] if  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$
- (iv) Pre-open set [9] if  $A \subseteq \text{int}(\text{cl}(A))$
- (v) semi pre-open set [1] if  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$

The complements of the above mentioned sets are called regular closed, semi-closed,  $\alpha$ -closed, pre-closed and semi pre-closed sets respectively.

The intersection of all regular closed (resp. semi-closed,  $\alpha$ -closed, pre-closed and semi pre-closed) subsets of  $(X, \tau)$  containing  $A$  is called the regular closure (resp. semi-closure,  $\alpha$ -closure, pre-closure and semi pre-closure) of  $A$  and is denoted by  $\text{rcl}(A)$  (resp.  $\text{scl}(A)$ ,  $\alpha\text{cl}(A)$ ,  $\text{pcl}(A)$  and  $\text{spcl}(A)$ )

**Definition 2.2** A subset  $A$  of a topological space  $(X, \tau)$  is called

- (a) generalized closed set (briefly  $g$ -closed) [6] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- (b) generalized semi-closed set (briefly  $gs$ -closed) [2] if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$

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- (c) semi-generalized closed set (briefly sg-closed) [3] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$
- (d) generalized  $\alpha$ -closed set (briefly  $g\alpha$ -closed [7] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $(X, \tau)$
- (e)  $\alpha$ -generalized closed set (briefly  $\alpha g$ -closed) [8] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- (f) generalized semi-pre -closed set (briefly gsp-closed ) [ 4] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- (g)  $\psi$ -closed set [15] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is sg-open in  $(X, \tau)$
- (h)  $\psi g$ -closed set [13] if  $\psi cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- (i)  $g^*$ -closed set [16] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$
- (j)  $\hat{g}$ -closed set [17] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$
- (k)  $g^\#$ -closed set [18] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha g$ -open set in  $(X, \tau)$
- (l)  $wg\alpha$ -closed set [20] if  $\alpha cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $(X, \tau)$
- (m)  $w\alpha g$ -closed set [20] if  $\alpha cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- (n)  $g^*$ s closed set [12] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $gs$ -open in  $(X, \tau)$
- (o)  $g^{**}$ -closed set [11] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g^*$ -open in  $(X, \tau)$
- (p)  $\alpha^*$ -closed set [19] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha^*$ -open in  $(X, \tau)$

The complements of the above mentioned sets are called their respective open sets.

### Remark 2.3

$r$ -closed( $r$ -open)  $\rightarrow$  closed (open)  $\rightarrow$   $\alpha$ -closed( $\alpha$ -open)  $\rightarrow$   $\psi$ -closed ( $\psi$ -open)  $\rightarrow$  semi pre-closed(semi pre-open)

### III $\psi g^{**}$ - CLOSED SETS

**Definition: 3.1** A subset  $A$  of a topological space  $(X, \tau)$  is said to be  $\psi g^{**}$ -closed set if  $\psi cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g^{**}$ -open in  $(X, \tau)$

The class of all  $\psi g^{**}$ -closed sets of  $(X, \tau)$  is denoted by  $\psi g^{**}C(X, \tau)$

**Proposition 3.2** Every closed set in  $(X, \tau)$  is  $\psi g^{**}$ -closed.

**Proof:** Let  $A$  be a closed set of  $(X, \tau)$ . Let  $U$  be any  $g^{**}$ -open set containing  $A$  in  $X$ . By Remark 2.3 every closed set is  $\psi$ -closed,  $\psi cl(A) \subseteq cl(A) = A \subseteq U$ . Therefore  $A$  is  $\psi g^{**}$ -closed.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.3** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ . Then the subset  $\{b\}$  is  $\psi g^{**}$ -closed but not closed in  $(X, \tau)$ .

**Proposition 3.4** Every regular closed set in  $(X, \tau)$  is  $\psi g^{**}$ -closed

**Proof:** The proof follows from the result that every regular closed set is closed and by proposition 3.2.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.5** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ . Then the subset  $\{c\}$  is  $\psi g^{**}$ -closed but not regular closed in  $(X, \tau)$

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**Proposition 3.6** Every  $\alpha$ -closed set in  $(X, \tau)$  is  $\psi g^{**}$ -closed.

**Proof:** Let  $A$  be an  $\alpha$ -closed set of  $(X, \tau)$ . Let  $U$  be any  $g^{**}$ -open set containing  $A$  in  $X$ . Since  $A$  is  $\alpha$ -closed,  $\alpha cl(A) = A$ . By Remark 2.3 every  $\alpha$ -closed set is  $\psi$ -closed,  $\psi cl(A) \subseteq \alpha cl(A) = A \subseteq U$ . Therefore  $A$  is  $\psi g^{**}$ -closed.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.7** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a, b\}\}$ . Then the subset  $\{b, c\}$  is  $\psi g^{**}$ -closed but not  $\alpha$ -closed in  $(X, \tau)$ .

**Proposition 3.8** Every  $\psi$ -closed set in  $(X, \tau)$  is  $\psi g^{**}$ -closed.

**Proof:** Let  $A$  be a  $\psi$ -closed set of  $(X, \tau)$ . Let  $U$  be any  $g^{**}$ -open set containing  $A$  in  $X$ . Since  $A$  is  $\psi$ -closed,  $\psi cl(A) = A$ . Therefore  $\psi cl(A) = A \subseteq U$  and hence  $A$  is  $\psi g^{**}$ -closed.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.9** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ . Then the subset  $\{a, c\}$  is  $\psi g^{**}$ -closed but not  $\psi$ -closed in  $(X, \tau)$ .

**Proposition 3.10** Every  $g^*$ -closed set in  $(X, \tau)$  is  $\psi g^{**}$ -closed.

**Proof:** Let  $A$  be a  $g^*$ -closed set of  $X$ . Let  $U$  be any  $g^{**}$ -open set containing  $A$  in  $X$ . Since every  $g^{**}$ -open set is  $g$ -open and  $A$  is  $g^*$ -closed,  $cl(A) \subseteq U$ . For every subset  $A$  of  $X$ ,  $\psi cl(A) \subseteq cl(A)$  and so  $\psi cl(A) \subseteq U$ . Hence  $A$  is  $\psi g^{**}$ -closed set in  $(X, \tau)$ .

The converse of the above theorem need not be true as seen from the following example.

**Example 3.11** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ . Then the subset  $\{b\}$  is  $\psi g^{**}$ -closed but not  $g^*$ -closed in  $(X, \tau)$ .

**Proposition 3.12** Every  $g^\#$ -closed set in  $(X, \tau)$  is  $\psi g^{**}$ -closed.

**Proof:** Let  $A$  be a  $g^\#$ -closed set of a topological space  $(X, \tau)$ . Let  $U$  be any  $g^{**}$ -open set containing  $A$  in  $X$ . Since every  $g^{**}$ -open set is  $\alpha g$ -open and  $A$  is  $g^\#$ -closed,  $cl(A) \subseteq U$ . For every subset  $A$  of  $X$ ,  $\psi cl(A) \subseteq cl(A)$  and so  $\psi cl(A) \subseteq U$  and hence  $A$  is  $\psi g^{**}$ -closed.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.13** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a\}\}$ . Then the subset  $\{b\}$  is  $\psi g^{**}$ -closed but not  $g^\#$ -closed in  $(X, \tau)$ .

**Proposition 3.14** Every  $\psi g^{**}$ -closed set in  $(X, \tau)$  is  $\psi g$ -closed.

**Proof:** Let  $A$  be a  $\psi g^{**}$ -closed set and  $U$  be any open set containing  $A$  in  $X$ . Since every open set is  $g^{**}$ -open,  $\psi cl(A) \subseteq U$ . Hence  $A$  is  $\psi g$ -closed.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.15** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a\}\}$ . Then the subset  $\{a, b\}$  is  $\psi g$ -closed but not  $\psi g^{**}$ -closed in  $(X, \tau)$ .

**Proposition 3.16** Every  $g^*$ -closed set in  $(X, \tau)$  is  $\psi g^{**}$ -closed.

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**Proof:** Let  $A$  be  $g^*s$ -closed set of  $X$ . Let  $U$  be any  $g^{**}$ -open set containing  $A$  in  $X$ . Since every  $g^{**}$ -open set is  $gs$ -open and  $A$  is  $g^*s$ -closed,  $scl(A) \subseteq U$ . For every subset  $A$  of  $X$ ,  $\psi cl(A) \subseteq scl(A)$  and so  $\psi cl(A) \subseteq U$ . Hence  $A$  is  $\psi g^{**}$ -closed set in  $(X, \tau)$ .

The converse of the above theorem need not be true as seen from the following example.

**Example 3.17** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ . Then the subset  $\{a, b\}$  is  $\psi g^{**}$ -closed but not  $g^*s$ -closed in  $(X, \tau)$

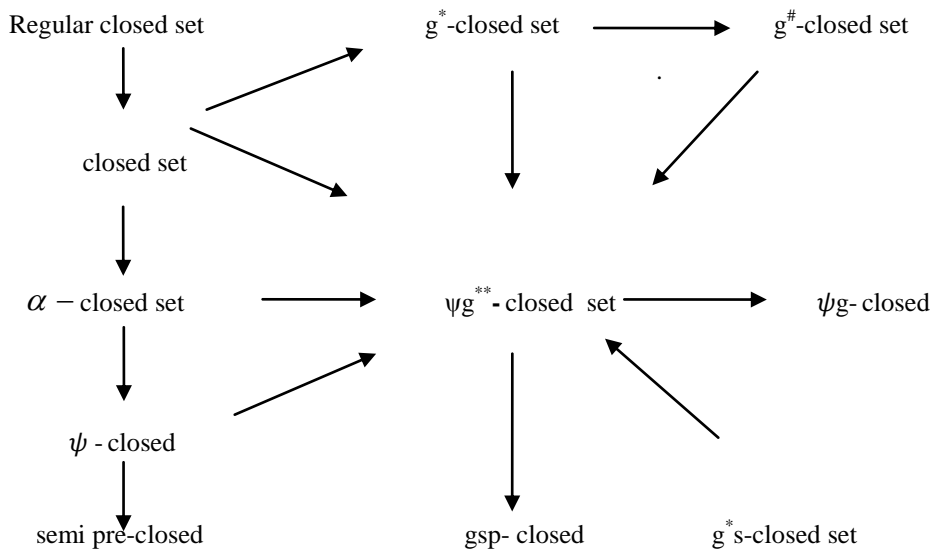
**Proposition 3.18** Every  $\psi g^{**}$ -closed set in  $(X, \tau)$  is  $gsp$ -closed.

**Proof:** Let  $A$  be a  $\psi g^{**}$ -closed set and  $U$  be any open set containing  $A$  in  $X$ . Since every open set is  $g^{**}$ -open,  $\psi cl(A) \subseteq U$ . For every subset  $A$  of  $X$ ,  $spcl(A) \subseteq \psi cl(A)$  and so  $spcl(A) \subseteq U$ . Hence  $A$  is  $\psi g^{**}$ -closed set in  $(X, \tau)$ .

The converse of the above theorem need not be true as seen from the following example.

**Example: 3.19** Let  $X = \{a, b, c\}$  with topology  $\tau = \{\phi, X, \{a, b\}\}$ . Then the set  $\{a\}$  is  $gsp$ -closed but not  $\psi g^{**}$ -closed set in  $(X, \tau)$ .

**Remark 3.20** The following diagram gives the dependence of  $\psi g^{**}$ -closed set with already existing closed sets.



**Remark 3.21** The following examples show that  $\psi g^{**}$ -closedness is independent from  $g$ -closedness,  $\hat{g}$ -closedness,  $g^{**}$ -closedness,  $\alpha g$ -closedness,  $g\alpha$ -closedness,  $w\alpha g$ -closedness,  $wg\alpha$ -closedness and  $\alpha^*$ -closedness.

**Example 3.22** Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ . In this topology the subset  $\{a\}$  is  $\psi g^{**}$ -closed but not  $g$ -closed,  $\hat{g}$ -closed,  $g^{**}$ -closed,  $\alpha g$ -closed,  $g\alpha$ -closed,  $w\alpha g$ -closed,  $wg\alpha$ -closed and  $\alpha^*$ -closed.

**Example 3.23** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ . In this topology the subset  $\{a, b\}$  is  $g$ -closed,  $\hat{g}$ -closed,  $g^{**}$ -closed,  $\alpha g$ -closed,  $g\alpha$ -closed,  $w\alpha g$ -closed,  $wg\alpha$ -closed and  $\alpha^*$ -closed but not  $\psi g^{**}$ -closed

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**Remark 3.24** The following examples show that  $\psi g^{**}$ -closedness is independent from  $sg$ -closedness and semi pre-closedness.

**Example 3.25** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ . In this topology the subset  $\{a, c\}$  is  $\psi g^{**}$ -closed but not  $sg$ -closed and semi pre-closed

**Example 3.26** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ . In this topology the subset  $\{a, b\}$  is  $sg$ -closed and semi pre-closed but not  $\psi g^{**}$ -closed

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