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Some More Properties of Euler Method in Non-Archimedean Fields

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ABSTRACT: The aim of this paper is to introduce sequence spaces for Euler method and prove a theorem for such a method in a Complete, Non-trivially valued, Non-Archimedean fields.

KEYWORDS: Euler matrix, Non-Archimedean fields, Regular.

I. INTRODUCTION

Throughout this paper, K denotes complete, non-trivially valued, non-archimedean fields. Infinite matrices and sequences considered in the sequel have entries in K . A general reference for the study of summability methods in classical one is [9]. For analysis, in non-archimedean fields, see [3].

If $A = (a_{n,k})$, $a_{n,k} \in K$, $n, k = 0, 1, 2, \dots$ is an infinite matrix, then the A -transform of a sequence $x = \{x_k\}$, $x_k \in K$, $k = 0, 1, 2, \dots$, is the sequence, where

$$(Ax)_n = \sum_{k=0}^n a_{nk} x_k, \quad n = 0, 1, 2, \dots, \text{ it being assumed that the series on the right converge [17].}$$

In the sequel, the following definitions and theorems are needed.

Definition 1. Let $\{x_k\}$ be a sequence in K and $x \in K$. We say that $\lim_{k \rightarrow \infty} x_k = x$ if for each $\epsilon > 0$, the set $\{|x - x_k| \geq \epsilon\}$ is finite. In such a case, we say that x is the limit of $\{x_k\}$.

Definition [15] 2. Let $A = (a_{n,k})$, $a_{n,k} \in K$, $n, k = 0, 1, 2, \dots$, if $\lim_{n \rightarrow \infty} (Ax)_n$ exists whenever $\lim_{n \rightarrow \infty} x_k$ exists, then A is said to be convergence preserving. Further, if $\lim_{n \rightarrow \infty} (Ax)_n = s$ whenever $\lim_{n \rightarrow \infty} x_k = s$, we say that A is regular.

Theorem 1. $A = (a_{n,k})$ is a regular if and only if

(i) $\sup_{n,k} |a_{n,k}| < \infty$

(ii) $\lim_{n \rightarrow \infty} a_{n,k} = 0, \quad k = 0, 1, \dots,$

(iii) $\lim_{n \rightarrow \infty} \sum_{k=0}^{\infty} a_{n,k} = 1$

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In 1969, Maddox [4] studied Continuous and Kothe-Toeplitz duals of certain sequence spaces and later he worked with Lascarides to determine the matrices of the classes [5] $(\ell(p), \ell_\infty)$, $(\ell(p), c)$ and $(\ell_\infty, \ell_\infty)$. Nanda [8] studied some sequence spaces over valued fields.

If $x = \{x_k\}$ is a sequence defined over \mathbf{K} , then the sequence spaces are [16]:

$$c = \left\{ x = \{x_k\} \in K : \lim_{k \rightarrow \infty} x_k \text{ exists} \right\}, \ell_\infty = \left\{ x = \{x_k\} \in K : \sup_k |x_k| < \infty \right\} \text{ and}$$

$$\ell_p = \left\{ x = \{x_k\} \in K : \sum_{k=0}^n |x_k|^p < \infty, p \geq 1 \right\}$$

In 1970, Somasundaram [11] defined and studied the sequence spaces $r(F)$ and $r^*(F)$ and Mursaleen et al [7] generalize the spaces $r(F)$ and $r^*(F)$ by considering an arbitrary sequence $\{u_k\}$, $k=1,2,\dots$, and characterize some classes of infinite matrices by considering these new classes of sequences in non-archimedean fields.

Wang [12] studied sequence spaces for Nörlund (1978), In 1981, Ahmad and Saraswat [10] obtained the necessary and sufficient conditions to characterize the matrices of the classes C_0 , c , ℓ_∞ and ℓ_p [14]. Altay et. al. [1] studied the

Euler sequence spaces which include the spaces ℓ_p and ℓ_∞ .

Deepa and Ganesan [2] studied some properties of Euler method of summability in a complete, non-trivially valued, non-archimedean fields. Now, we introduce sequence spaces for Euler method and prove a theorem for such a method in a complete, non-trivially valued, non-archimedean fields [13].

Definition 3. Let $p \in K$ be such that $|1-p| < 1$. The Euler method $[E, p]$ is given by the matrix $(e_{n,k}^{(p)})$ as follows:

If $p \neq 1$

$$e_{n,k}^{(p)} = \begin{cases} n C_k p^k (1-p)^{n-k}, & k \leq n; \\ 0, & k > n, \end{cases}$$

If $p = 1$

$$e_{n,k}^{(1)} = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases}$$

$e_{n,k}^{(p)}$ is called the $[E, p]$ matrix [12].

Theorem 2. $A = (a_{n,k})$ is (c, c_0) if and only if

(i) $\sup_{n,k} |a_{n,k}| < \infty$

(ii) $\lim_{n \rightarrow \infty} a_{n,k} = 0, \quad k = 0, 1, \dots$

Theorem 3. The Euler method $[E, p]$ is regular if and only if $[E, p]$ is (c, c_0)

Proof. Suppose $[E, p]$ is (c, c_0) . Then if the infinite matrix $e_{n,k}^{(p)}$ defines $[E, p]$ method, [11] $\lim_{n \rightarrow \infty} e_{n,k}^{(p)} = 0$.

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In particular, $\lim_{n \rightarrow \infty} e_{n,0}^{(p)} = 0, n=0,1,2,\dots$

(i.e.) $\lim_{n \rightarrow \infty} (1-p)^n = 0 \Rightarrow |1-p| < 1.$

$\therefore [E, p]$ is regular.

Conversely, let $[E, p]$ be regular. Then it defines matrix $(e_{n,k}^{(p)})$ satisfies the condition

$\lim_{n \rightarrow \infty} e_{n,k}^{(p)} = 0, k = 0,1,2,\dots$ and $\sup_{n,k} |e_{n,k}^{(p)}| < \infty.$ This implies that $[E, p]$ is (c.c₀).

This completes the proof of the theorem [10].

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