

Interaction Of Higher Order Dromion in (2+1) Dimensional Long Dispersive Wave Equation Using PTA

P.Myvizhi, R.Velavan

Department of Physics, Bharath Institute of Higher Education and Research, Chennai, India

Department of Physics, Bharath University, Chennai , India

ABSTRACT: Using the methodology of Painleve Truncation Approach (PTA), an infinite integrable non linear dynamical system named long dispersive wave equation (2LDW) has been solved and higher order dromion solution is obtained from generalized periodic solution. Also, interaction between higher order (2,1) dromion can be obtained in terms of their arbitrary solutions.

KEYWORDS: Painleve, trilinear, dromions. phase shift.

I. INTRODUCTION

The identification of a large number of arbitrary functions in the solutions of (2+1) dimensional integrable models have only added to the richness in the structure of them and hence construction of localized excitations in (2+1) dimensions continues to be challenging and rewarding contemporary problem.

In this equation we developed a very simple and straight forward procedure to generate a rather extended class of generic solutions of physical interest. In this 2LDW equation we utilize the local Laurent expansion of the general solution and truncate it at the constant level term (painleve truncation approach) and obtain solutions in terms of arbitrary functions. Through this procedure we generate various periodic and exponentially localized solutions. The novelty here is that the solution is generated through a simple procedure. But the solution obtained is rich in structure because of the arbitrary functions [9-14].

II. PAINLEVE TRUNCATION APPROACH

If the solution of a nonlinear differential equation is free from movable (integration constant/initial condition dependent) critical singular points (branch points and essential singularities) it is said to possess the painleve property. In this case the system is expected to be integrable.

Painleve singularity structure analysis is used in which the solution of the given equation of motion of a dynamical system is analyzed for the presence or absence of movable critical singular points and it can be used as a powerful test to identify and isolate integrable infinite dimensional nonlinear dynamical system. It has been pointed out that there exists a large number of solitons possessing completely integrable infinite dimensional dynamical systems.

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Painleve truncation approach is a methodology in which we are going to prove that the (2+1) dimensional long dispersive wave equation is a completely integrable infinite dimensional nonlinear dynamical system.

The recent interest in the investigation of integrable models in (2+1) dimensions can be attributed to the identification of dromions in the Davey-Stewartson I equation [3]. These dromions which originate at the cross point of the intersection of two nonparallel ghost solitons decay exponentially in all directions and are driven by lower dimensional boundaries or velocity potentials (arbitrary functions) in the system. It must be mentioned that the presence of these so called boundaries enriches the structure of (2+1) dimensional integrable models leading to the formation of localized solutions like dromions, positions, etc. [5]. Thus a judicious harnessing of these lower dimensional arbitrary functions [9-14] of space and time may give rise to new spatio-temporal patterns in higher dimensions besides throwing more light on their integrability.

In this way, a simple procedure is used to solve the (2+1) dimensional system namely Painleve Truncation Approach (PTA) [5]. This approach converts **the given evolution equation into a multi linear equation in terms of non characteristic manifold. This multi linear equation can be solved in terms of lower dimensional arbitrary functions of space and time.** Using this approach one can generate various generalized periodic and localized solutions. Through PTA (2+1) dimensional long dispersive wave equation can be trilinearized. A four parameter special class of solutions involving three arbitrary functions in the variables naturally arises for this **trilinear** equation. Thus, there exists a possibility of obtaining more general solutions.

III.DROMIONS

Dromions which originate at the crosspoint of the intersection of two nonparallel ghost solitons decay exponentially in all directions and are driven by lower dimensional boundaries or velocity potentials (arbitrary functions) in the system. It must be mentioned that the presence of these so called boundaries enriches the structure of (2 +1)dimensional integrable models leading to the formation of localized solutions like dromions.

IV.PHASE SHIFT

In phase shift,nonlinearity dominates. Dromions (2+1) dimensional of different amplitude, interacts with time as $t \rightarrow \infty$, two waves travels, as the solution develops a train of well defined waves with different amplitude with the faster (taller) waves catching up and overtaking the slower (short) waves. These nonlinear waves interact strongly and then continue thereafter, as if there had been no interaction at all. Wave solution regain their initial shape and their velocities with a shift in phase.

After solving the physical system such as 2LDW equation,
Considering the long dispersive wave equation (2LDW) [15,16]

$$\lambda_{qt} + q_{xx} - 2_{qv} = 0 \tag{1a}$$

$$\lambda_{rt} - r_{xx} + 2_{rv} = 0 \tag{1b}$$

$$(qr)_x = V_\eta \tag{1c}$$

the trilinear equation obtained is as follows.

$$V_{1\eta} = \Phi_{x\eta}^2 + (\Phi_x \Phi_{x\eta\eta} / \Phi_\eta) - (\Phi_x \Phi_{x\eta} \Phi_{\eta\eta} / \Phi_\eta^2) - (\Phi_{xx\eta} \Phi_{x\eta}^2 / \Phi_\eta) + \Phi_x \Phi_{x\eta\eta} \Phi_\eta - \Phi_x \Phi_{x\eta} \Phi_{\eta\eta} - \Phi_\eta^2 \Phi_{xx\eta} = 0 \tag{2}$$

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V.NOVEL EXACT SOLUTIONS OF 2LDW EQUATION

Now we can make use of the truncated Laurent expansion solution to obtain exact solutions of the 2LDW equation (1) for the variables $q(x, \eta, t)$ and $V(x, \eta, t)$. Taking into account our notation in equation (1). From the results, we find that the solution of the original variable takes the form

$$q = [F_1(\eta) F_2(t) (\Phi_{1x})^{\frac{1}{2}} / \Phi_1(x, t) + \Phi_{21}(\eta) + \Phi_{22}(t)] * e^{\int -1/2\lambda(\Phi_1 t + \Phi_{22}, t) / \Phi_{1x} dx} \quad (3)$$

The form of $V(x, \eta, t)$ remains the same as given in truncated Laurent expansion with the above general form of the solutions, we now identify interesting classes of exact solutions to the original equation including periodic and localized solutions by giving specific forms for the three arbitrary functions $\Phi_1(x, t)$, $\Phi_{21}(\eta)$, $\Phi_{22}(t)$

A. Periodic solution and localized dromion solution

Let us now choose the arbitrary functions Φ_1 and $F_{1\eta}$ and F_{2t} to be Jacobian elliptic functions namely sn or cn functions. The motivation behind this choice of arbitrary functions stems from the fact that the limiting forms of these functions happen to be localized functions. Hence a choice of sn or cn functions can yield periodic solutions [16-18] which are more general than exponentially localized solutions (dromions). We choose for example

$$\Phi_1(x, t) = n_1 + \text{sn}(ax + b\eta + c; m_1), \quad (4a)$$

$$\Phi_{21}(\eta) = \text{sn}(d\eta; m_2), \quad (4b)$$

$$\Phi_{22}(t) = \text{sn}(lt; m_3), \quad (4c)$$

$$F_1(\eta) = \text{cn}(g\eta; m_4) \quad (4d)$$

$$F_2(t) = \text{cn}(ht; m_5) \quad (4e)$$

$$q(x, \eta, t) =$$

$$[\text{cn}(g\eta; m_4)\text{cn}(ht; m_5)] [\sqrt{\text{acn}(u_1; m_1)\text{dn}(u_1; m_1)}] / (n_1 + \text{sn}(u_1; m_1) + \text{sn}(d\eta; m_2) + \text{sn}(lt; m_3)) \quad (4f)$$

where $u_1 = (ax + b\eta + c)$.

In (4f) the quantities m_1 and m_2 are the modulus parameters of the respective Jacobian elliptic functions while a, b and c are arbitrary constants. The corresponding expression for $|s(x, \eta, t)|^2$ takes the form

$$|q|^2 = [\text{cn}(g\eta; m_4)\text{cn}(ht; m_5)]^2 [\text{acn}(u_1; m_1)\text{dn}(u_1; m_1)] / [4 + \text{sn}(d\eta; m_2) + \text{sn}(lt; m_3)]^2 \quad (4g)$$

VI. MORE GENERAL PERIODIC SOLUTIONS AND HIGHER DROMION SOLUTIONS

A. Periodic solution and (2,1) dromion

Next we obtain a more general periodic solution by choosing further general forms for the arbitrary functions. As an example we choose [18-20]

$$\Phi_1(x, t) = n_1 + d_1 \text{sn}(c_1 + a_1 x + b_1 \eta; m_1) + d_2 \text{sn}(c_2 + a_2 x + b_2 t; m_2) \quad (5a)$$

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$$\Phi_{21}(\eta) = \text{sn}(d\eta : m2) \tag{5b}$$

$$\Phi_{22}(\eta) = \text{sn}(lt : m3) \tag{5c}$$

$$F_1(\eta) = \text{cn}(g\eta : m4) \tag{5d}$$

$$F_2(\eta) = \text{cn}(ht : m5) \tag{5e}$$

where a_i, b_i, c_i, d_i are arbitrary constants and m_i 's are modulus parameters ($i=1,2,3$).

Then the profile of the solution for the parametric choice $a = d = d1 = h1 = 2, b = g = g1 = 3, h = 4, c = 1, m1, m2, m4, m5 = 1$.

The solution degenerates into a (2,1) dromion solution given by

$$|q|^2 = (a_1 d_1 \text{sech}^2(a_1 x + b_1 \eta + c_1) + a_2 d_2 \text{sech}^2(a_2 x + b_2 \eta + c_2)) d_3 \text{sech}^2(g\eta) d_4 \text{sech}^2(ht) / [n_1 + \tanh(a_1 x + b_1 \eta + c_1) + \tanh(a_2 x + b_2 \eta + c_2) + d_3 \tanh(d\eta) + d_4 \tanh(lt)]^2$$

where $u_1 = c+a_1x+b_1\eta, u_2 = c+a_2x+b_2\eta, u_3 = g\eta$ and $u_4 = ht$. The dromion interaction for the parametric choice $a = d = d1 = h1 = 2, b = g = g1 = 3, h = 4, c = 1$ is shown in figures 4.4.1 (b,c,d) for different time intervals.

Here both the dromions travel with equal velocity but along opposite diagonals in the x-y plane. The interaction is elastic for this choice.[21]

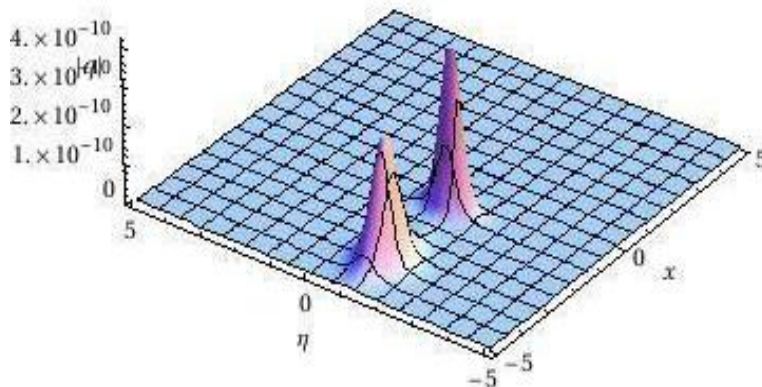


Figure 4.4.1 (b) - (2,1) Dromion Solution - Before Interaction

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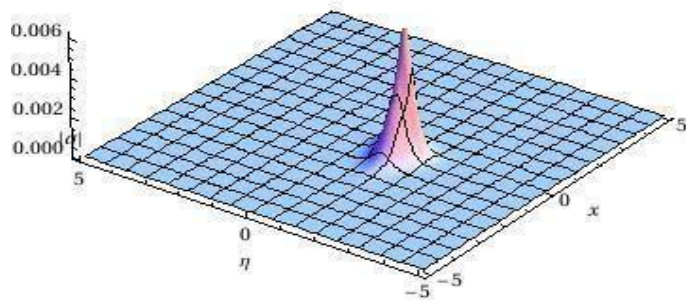


Figure 4.4.1 (c) - (2,1) Dromion Solution - During Interaction

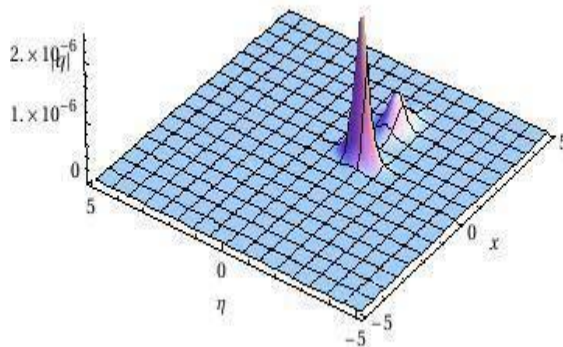


Figure 4.4.1 (d) - Dromion Solution After Interaction

VII.CONCLUSION

In this paper, I had chosen (2+1) dimensional long dispersive wave equation which is solved already in terms of the arbitrary functions using the **Painleve Truncation Approach (PTA)** method. More general higher order dromion

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solution such as limiting form of **(2,1) dromion** is generated from the trilinear equation. Interactions between dromions is shown clearly in the form of graphical representation. Dromions (2+1) dimensional of different amplitude, interacts with time as $t \rightarrow \infty$, two waves travels, as the solution develops a train of well defined waves .It will enrich the existence of lower dimensional arbitrary functions, which will help us further to construct further higher dromion solutions such as (M,N)dromions in future.

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