

Numerical Solution of Imbibition Phenomenon in a Cracked Porous Medium with Magnetic Fluid

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ABSTRACT: In this paper, we have discussed imbibition phenomenon in double phase flow through cracked porous media. The mathematical formulation leads to non linear partial differential equation governing the phenomenon in a cracked porous medium with magnetic fluid. The mathematical solution has been obtained by finite element method with appropriate initial and boundary condition. Finite element method is a numerical method for finding an approximation solution of differential equation in finite region or domain. We obtained graphical representation of the solution using a Matlab coding.

KEYWORDS: Cracked Porous media, fluid flow, magnetic fluid, finite element method.

I. INTRODUCTION

It is well known that when a porous medium filled with some fluid is brought into contact with another fluid which preferentially wets the medium, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. Such a phenomena arising due to difference in wetting abilities is called counter-current imbibition. These phenomena have been formally discussed by Brownscombe and Dyes [4], Enright [5], Braham and Richarson [6], Rijik [7], Rijik et al [9], for homogeneous porous medium.

Bokserman, Zhelton and Kocheshkev [10] have described the physics of oil-water flow in a cracked media and Verma [11, 12] has investigated two specific oil-water displacement process from analytical point of view. Verma has obtained solution by performing a perturbation technique. He has also investigated this problem in the presence of randomly oriented pores in the fractured medium renders the differential equation highly non-linear due to an additional non-linear term. He has also considered the presence of heterogeneity in the medium marginally.

II. STATEMENT OF THE PROBLEM

We consider here a cylindrical mass of porous matrix of length $L (=1)$ containing a viscous oil, is completely surrounded by an impermeable surface except for one end of the cylinder which is labeled as the imbibitions phase and this end is exposed to an adjacent formation of 'injected' water. It is assumed that the injected water and the viscous oil are two immiscible liquids of different salinities with small viscosity difference; the former represents the preferentially wetting and less viscous phase. This arrangement gives rise to the phenomenon of linear counter-current imbibitions, that is, a spontaneous linear flow of water into the porous medium and a linear counter flow of oil from the medium.

III. MATHEMATICAL FORMULATION OF THE PROBLEM

In cracked porous media, water penetrating the seam along the cracks is sucked into blocks of rock under the action of capillary forces and the amount of water entering the blocks in an elementary volume of the seam is designated as the impregnation function $\phi(t)$, where t denotes time as in figure.

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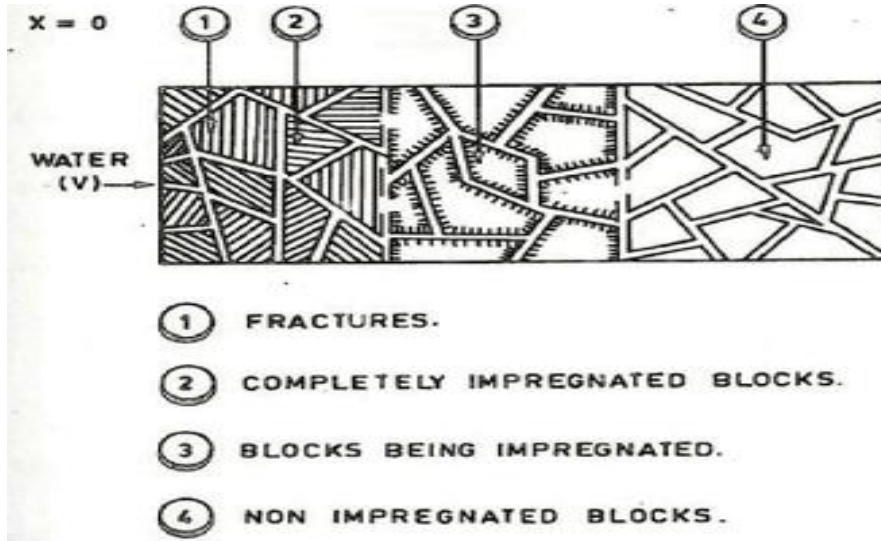


Figure 1. Impregnation of a one-dimensional fractured porous medium with water

Considering the balance of water sucked into the blocks of rock per unit time and employing the result of Mattan and Kyta [21] and Vezirov and Kocheshkov [22] for the analytical value of $\phi(t)$, we may write,

$$\phi(T-\tau(u)) = \frac{D}{\sqrt{T-Rx^2}} \quad \tau \leq t, \quad \dots\dots\dots (1) \quad u = \frac{x}{L_M}, \quad T = \varepsilon t, \quad R = \frac{a}{L_M^2}, \quad D = \frac{A}{2} m_B g K \left(\frac{\delta \cos \theta + S^2 (k/m_B)^{1/2}}{\gamma_o} \right)$$

$$\varepsilon = \frac{\delta \cos \theta S^2 (k/m_B)^{1/2}}{\gamma_o} \quad a = \left[\frac{\pi A S^2 \delta \cos \theta m_B g K (k/m_B)^{1/2}}{4q \gamma_o} \right]^2$$

Where,

m_B = Porosity of the blocks., g_k = saturation of the block with water at the moment t_k , T = time,
 δ = surface tension, θ = angle of wetting, S = mean specific surface area of the blocks,
 γ_o = viscosity of native water (as oil), K = permeability of crack system, L_M = mean block size,
 q = constant rate of the delivery of injected water per unit surface area perpendicular to the direction of flow.
 The equation of continuity for native and injected liquid, (neglecting variation of phase density), are

$$P \left(\frac{\partial S_w}{\partial t} \right) + \left(\frac{\partial V_w}{\partial x} \right) + \phi(T-\tau(u)) = 0 \quad \dots\dots\dots (2)$$

where $\phi(T-\tau(u))$ is the impregnation function.

$$P \left(\frac{\partial S_o}{\partial t} \right) + \left(\frac{\partial V_o}{\partial x} \right) = 0$$

where P = porosity of the medium. S_w, S_o = Saturation of the injected and native liquid respectively. From the definition of phase saturation, it is obvious that $S_w + S_o = 1$.

Also $V_o = -V_w$ for imbibition phenomenon. (3)

Now equation (2) becomes,

$$P \left(\frac{\partial S_w}{\partial t} \right) + \frac{\partial}{\partial x} \left(\left(\frac{K_o / \delta_o \cdot K_w / \delta_w}{K_o / \delta_o + K_w / \delta_w} \right) K \left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x} \right] \right) + \phi(T-\tau(u)) = 0 \quad \dots\dots\dots (4)$$

which is the desired non linear differential equation of motion which describes the linear counter current imbibition phenomena in a cracked porous medium with effect of magnetic fluid.

International Journal of Innovative Research in Science, Engineering and Technology

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Substituting the value of $\phi(T - \tau(u))$ from equation (1) into equation (4) we get,

$$\varepsilon P \left(\frac{\partial S_w}{\partial T} \right) + \frac{\partial}{\partial x} \left(\left(\frac{K_o / \delta_o \cdot K_w / \delta_w}{K_o / \delta_o + K_w / \delta_w} \right) K \left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x} \right] \right) = - \frac{D}{\sqrt{T - Rx^2}} \quad \dots\dots\dots (5)$$

At this state, for definiteness of the mathematical analysis, we assume standard relationship between phase saturation and relative permeability.

Assuming $\frac{K_o K_w}{K_o \delta_w + K_w \delta_o} \approx \frac{K_o}{\delta_o}$, and equation (5) becomes,

$$\varepsilon P \left(\frac{\partial S_w}{\partial T} \right) + \frac{K}{\delta_o} \frac{\partial}{\partial x} \left((1 - \alpha S_w) \left[-\beta_o \frac{\partial S_w}{\partial x} - \gamma H \frac{\partial H}{\partial x} \right] \right) = - \frac{D}{\sqrt{T - Rx^2}} \quad \dots\dots\dots (6)$$

Considering the magnetic fluid H in the x-direction only, we may write [23], $H = \frac{\Lambda}{x^n}$ where Λ is a constant parameter and n is an integer. Using the value of H for n = -1 in equation (6), we get,

$$C_1 \left(\frac{\partial S_w}{\partial t} \right) - C_2 \frac{\partial}{\partial x} \left((1 - \alpha S_w) \frac{\partial S_w}{\partial x} \right) - C_3 \frac{\partial}{\partial x} \left((1 - \alpha S_w) x \right) = - \frac{D}{\sqrt{T - Rx^2}} \quad \dots\dots\dots (7) \text{ where,}$$

$$C_1 = P, \quad C_2 = \frac{K \beta_o}{\delta_o}, \quad C_3 = \frac{\gamma K \Lambda^2}{\delta_o}$$

A set of suitable boundary conditions associated to problem (7) are

$$S_w(x, 0) = 0 \quad \text{for all } x > 0; \quad \dots\dots\dots (8)$$

$$S_w(0, t) = S_{w0}; \quad S_w(L, t) = S_{w1} \quad \text{for all } t \geq 0 \quad \dots\dots\dots (9)$$

Equation (9) is reduced to dimensionless form by setting

$$x^* = x/L, \quad T^* = \frac{T}{L^2 (C_1 / C_2)}, \quad (1 - \alpha S_w(x, T)) = S_w^*(x^*, T^*)$$

so that $\frac{\partial S_w}{\partial T} \frac{\partial}{\partial x} \left(S_w \frac{\partial S_w}{\partial X} \right) - C_o \frac{\partial}{\partial x} (S_w x) = \frac{D \alpha L}{C_2 \sqrt{R}} \frac{1}{\sqrt{\frac{C_1 T}{C_2 R} - x^2}} \quad \dots\dots\dots (10) \text{ where } C_o = \frac{-C_3 L^2 \alpha}{C_2}$

The initial and boundary conditions (8) and (9) now becomes,

$$S_w(x, 0) = 1 \quad \forall \quad x > 0 \quad \dots\dots\dots (11)$$

$$S_w(0, t) = 1 - \alpha S_{w0}; \quad S_w(L, t) = 1 - \alpha S_{w1} \quad \text{for all } t \geq 0 \quad \dots\dots\dots (12)$$

Equation (10) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in homogeneous medium with effect of magnetic fluid. A Matlab Code is prepared and executed with $C_o = 4.046 \times 10^{-11}$, $c1 (= C_2 \sqrt{R} / D) = 0.3 \times 10^{11}$, $c2 (= C_1 / C_2 R) = 90$, $\alpha = 1.11$, $L = 1$, $h = 1/15$, $k = 0.002223$ for 225 time levels, $S_{w0} = 0.5$ and $S_{w1} = 0$. Curves indicating the behavior of Saturation of injected fluid with respect to various time period.

IV. FINITE ELEMENT METHOD

For the problem under consideration, the length variable x varies between 0 and L (figure 2(a)). This domain is divided into set of linear elements (figure 2(b)).

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

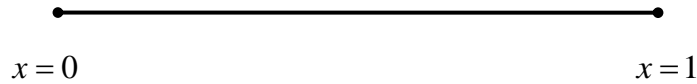


Figure 2(a)

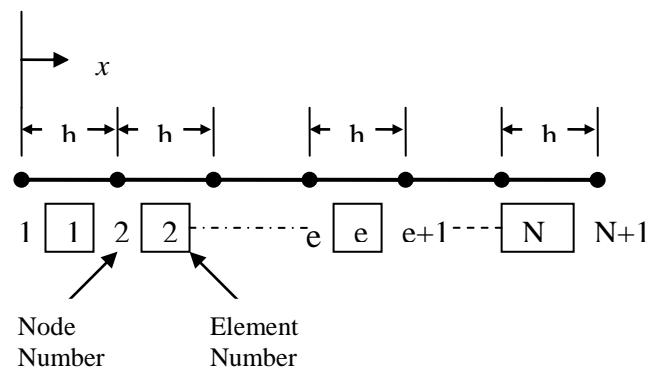


Figure 2(b)

Now, the variational form of given partial differential equation is,

$$J(S_w) = \frac{1}{2} \int_R \left[S_w \left(\frac{\partial S_w}{\partial x} \right)^2 + 2S_w \frac{\partial S_w}{\partial t} + C_0 x S_w \frac{\partial S_w}{\partial x} + \frac{\alpha}{c_1 \sqrt{c_2 T - x^2}} S_w \right] dx \quad \dots\dots\dots (13)$$

Choose an arbitrary linear element $R^{(e)} = [S_1^{(e)}, S_2^{(e)}]$ and obtain interpolation function for $R^{(e)}$ using Lagrange interpolation Method such as

$$S^{(e)}(x) = \sum_{j=1}^2 N_j(x) S_j^{(e)} = N^{(e)} \varphi^{(e)} = \varphi^{(e)T} N^{(e)T} \quad \dots\dots\dots (14)$$

where $N^{(e)} = [N_1 \ N_2]$ & $\varphi^{(e)} = [S_1 \ S_2]^T$

and N_1, N_2 are shape function for linear element.

Now, apply Variational Method to $R^{(e)}$, therefore equation (13) becomes,

$$J(S^{(e)}) = \frac{1}{2} \int_{R^{(e)}} \left[S^{(e)} \left(\frac{\partial S^{(e)}}{\partial x} \right)^2 + 2S^{(e)} \frac{\partial S^{(e)}}{\partial t} + C_0 x S^{(e)} \frac{\partial S^{(e)}}{\partial x} + \frac{\alpha}{c_1 \sqrt{c_2 T - x^2}} S^{(e)} \right] dx \quad \dots\dots\dots (15)$$

As, $S^{(e)}(x) = N^{(e)} \varphi^{(e)} = \varphi^{(e)T} N^{(e)T}$
 $\therefore \frac{\partial S^{(e)}}{\partial x} = \frac{\partial N^{(e)}}{\partial x} \varphi^{(e)} = \varphi^{(e)T} \frac{\partial N^{(e)T}}{\partial x}$ and $\left(\frac{\partial S^{(e)}}{\partial x} \right)^2 = \varphi^{(e)T} \frac{\partial N^{(e)T}}{\partial x} \frac{\partial N^{(e)}}{\partial x} \varphi^{(e)}$

Therefore equation (15) becomes

$$J(S^{(e)}) = \frac{1}{2} \int_{R^{(e)}} \varphi^{(e)T} \left[N^{(e)} \varphi^{(e)} \left(\frac{\partial N^{(e)T}}{\partial x} \frac{\partial N^{(e)}}{\partial x} \right) \varphi^{(e)} + 2 \left(N^{(e)T} N^{(e)} \right) \frac{\partial \varphi^{(e)}}{\partial t} + C_0 x N^{(e)T} \frac{\partial N^{(e)}}{\partial x} \varphi^{(e)} + \frac{\alpha}{c_1 \sqrt{c_2 T - x^2}} N^{(e)T} \right] dx \quad \dots\dots\dots (16)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

For minimization, first differentiate equation (16) with respect to $\phi^{(e)}$

$$\frac{\partial J^{(e)}}{\partial \phi^{(e)}} = \int_{R^{(e)}} \left[N^{(e)} \phi^{(e)} \left(\frac{\partial N^{(e)T}}{\partial x} \frac{\partial N^{(e)}}{\partial x} \right) \phi^{(e)} + 2 \left(N^{(e)T} N^{(e)} \right) \frac{\partial \phi^{(e)}}{\partial t} + C_0 x N^{(e)T} \frac{\partial N^{(e)}}{\partial x} \phi^{(e)} + \frac{\alpha}{c_1 \sqrt{c_2 T - x^2}} N^{(e)T} \right] dx$$

Now, $\frac{\partial J^{(e)}}{\partial \phi^{(e)}} = 0$. Therefore The Element equation is,

$$A^{(e)} \frac{\partial \phi^{(e)}}{\partial T} + B^{(e)}(\phi^{(e)}) \phi^{(e)} + C^{(e)} \phi^{(e)} + F^{(e)} = 0 \quad \dots\dots\dots (17) \text{ where,}$$

$$A^{(e)} = \int_{S_1^{(e)}}^{S_2^{(e)}} \left(N^{(e)T} N^{(e)} \right) dx, \quad B^{(e)}(\phi^{(e)}) = \int_{S_1^{(e)}}^{S_2^{(e)}} N^{(e)} \phi^{(e)} \left(\frac{\partial N^{(e)T}}{\partial x} \frac{\partial N^{(e)}}{\partial x} \right) dx$$

$$C^{(e)} = C_0 \int_{S_1^{(e)}}^{S_2^{(e)}} x \left(N^{(e)T} \frac{\partial N^{(e)}}{\partial x} \right) dx, \quad F^{(e)} = \int_{S_1^{(e)}}^{S_2^{(e)}} \frac{\alpha}{c_1 \sqrt{c_2 T - x^2}} N^{(e)T} dx$$

By Gauss Legendre Quadrature Method, Interpolation function becomes $N_1(z) = \frac{1}{2}(1-z); N_2(z) = \frac{1}{2}(1+z)$.

Thus, element matrix transform to,

$$A^{(e)} = \int_{-1}^1 \left(N^{(e)T} N^{(e)} \right) J dz \approx \sum_{I=1}^r A^{(e)}(z_I) W_I$$

$$B^{(e)}(\phi^{(e)}) = \int_{-1}^1 N^{(e)} \phi^{(e)} \left(\frac{1}{J} \frac{\partial N^{(e)T}}{\partial X} \frac{1}{J} \frac{\partial N^{(e)}}{\partial X} \right) J dz \approx \sum_{I=1}^r B^{(e)}(z_I) W_I$$

$$C^{(e)} = C_0 \int_{-1}^1 \left(\frac{h}{2}(1+z_I) \right) \left(N^{(e)T} \frac{1}{J} \frac{\partial N^{(e)}}{\partial z} \right) J dz \approx \sum_{I=1}^r C^{(e)}(z_I) W_I$$

$$F^{(e)} = \frac{\alpha}{c_1} \int_{-1}^1 \left(\sqrt{c_2 T - \left(\frac{h}{2}(1+z_I) \right)^2} \right) \left(N^{(e)T} \right) J dz \approx \sum_{I=1}^r F^{(e)}(z_I) W_I$$

For $A^{(e)}$, $C^{(e)}$ and $F^{(e)}$, degree of polynomial $p=2$ then $r=2$ and For $B^{(e)}$, $p=1$ then $r=1$. z_1 and W_1 are corresponding gauss points and gauss weights. Then, the element matrix becomes,

$$A^{(e)} = \frac{h^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad B^{(e)}(\phi^{(e)}) = \frac{1}{2h^{(e)}} \begin{bmatrix} S_1 + S_2 & -S_1 - S_2 \\ -S_1 - S_2 & S_1 + S_2 \end{bmatrix}, \quad C^{(e)} = \frac{C_0 h^{(e)}}{6} \begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix}, \quad F^{(e)} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \text{ where,}$$

$$F_1 = \frac{\alpha}{c_1} \sum_{I=1}^r \left(\sqrt{c_2 T - \left(\frac{h}{2}(1+z_I) \right)^2} \right) \left(\frac{h}{2}(1-z_I) \right) W_I,$$

..... (18)

$$F_2 = \frac{\alpha}{c_1} \sum_{I=1}^r \left(\sqrt{c_2 T - \left(\frac{h}{2}(1+z_I) \right)^2} \right) \left(\frac{h}{2}(1+z_I) \right) W_I$$

Assembling of elements: For a uniform mesh of N elements, by equation (17) and equation (18), the assembled equation becomes,

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

$$A \frac{\partial \phi}{\partial T} - B(\phi)\phi + C \phi + F1 = 0 \quad \dots\dots\dots(19)$$

where, $A = \frac{h}{6} \begin{bmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & (2+2) & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & (2+2) & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & (2+2) & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{bmatrix}$

$$B(\phi) = \frac{1}{2h} \begin{bmatrix} S_1+S_2 & -S_1-S_2 & 0 & 0 & \dots & 0 & 0 \\ -S_1-S_2 & S_1+2S_2+S_3 & -S_2-S_3 & 0 & \dots & 0 & 0 \\ 0 & -S_2-S_3 & S_2+2S_3+S_4 & -S_3-S_4 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & S_{14}+2S_{15}+S_{16} & -S_{15}-S_{16} \\ 0 & 0 & 0 & 0 & \dots & -S_{15}-S_{16} & S_{15}+S_{16} \end{bmatrix}$$

$$C = \frac{C_0 h}{6} \begin{bmatrix} -1 & -2 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & -2 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 & -2 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{bmatrix}$$

$$F1 = \begin{bmatrix} F_1^1 \\ F_2^1 + F_1^2 \\ F_2^2 + F_1^3 \\ \vdots \\ \vdots \\ F_2^{N-1} + F_1^N \\ F_2^N \end{bmatrix}$$

$$\phi = [S_1 \quad S_2 \quad \dots \quad \dots \quad S_{16}]^T \quad \dots\dots\dots (20)$$

Equation (19) represents the assembled equation.

Time approximation: We have obtained the finite element equation in the global form, which represent a system of simultaneous ordinary differential equation. We introduced δ family of approximations which approximates weighted average of a dependent variable of two consecutive time steps by linear interpolation of the values of the variable at the

two time steps such as, $S_j = \delta S_j^{n+1} + (1-\delta)S_j^n \quad \dots\dots\dots (21)$. The time derivatives $\dot{\theta}_j$ are replaced by Forward finite

difference formula such as $\dot{S}_j = \frac{S_j^{n+1} - S_j^n}{k} \quad \dots\dots\dots (22)$.

In view of (21) and (22), equation (19) written as,
 $[A + \delta k(B(\phi^{(n+1)})) + C] \phi^{(n+1)} = [A - (1-\delta)k(B(\phi^{(n)})) + C] \phi^{(n)} + F1^{(n)} \quad \dots\dots\dots (23)$

where $\delta = 1/2$ and $n = 0, 1, 2, \dots$

For a uniform mesh of N elements, by equation (20), the global equation (23) takes the form,

$$[K(\phi^{(n+1)})] \phi^{(n+1)} = [F_1(\phi^{(n)})] \phi^{(n)} + F1^{(n)} = F(\phi^{(n)}) + F1^{(n)} = F \quad \dots\dots\dots (24)$$

Where,

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

$$K(\varphi^{(n+1)}) = \begin{bmatrix} \frac{h}{3} + \delta k \left(\frac{1}{2h} (\sum_{i=1}^{(n+1)} (-1)^{i-1} \sum_{j=2}^{(n+1)} C_{ij}^{\varphi}) - \frac{C_{ij}^{\varphi}}{6} \right) & \dots & \dots & 0 & \dots & \dots & 0 & \dots & \dots & 0 \\ \frac{h}{6} + \delta k \left(\frac{1}{2h} (-1)^{n+1} \sum_{j=3}^{(n+1)} C_{ij}^{\varphi} \right) & 0 & \dots & 0 & \dots & \dots & 0 & \dots & \dots & 0 \\ \frac{2h}{3} + \delta k \left(\frac{1}{2h} (\sum_{i=1}^{(n+1)} \sum_{j=2}^{(n+1)} + \sum_{j=3}^{(n+1)} C_{ij}^{\varphi}) - \frac{C_{ij}^{\varphi}}{6} \right) & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{h}{6} + \delta k \left(\frac{1}{2h} (-1)^{n+1} \sum_{j=3}^{(n+1)} C_{ij}^{\varphi} \right) & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & \dots & \dots & 0 & \dots & \dots & 0 \\ 0 & \dots & \dots & 0 & \dots & \dots & 0 & \dots & \dots & 0 \end{bmatrix}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

$$\phi^{(n)} = \begin{bmatrix} S_1^{(n)} \\ S_2^{(n)} \\ S_3^{(n)} \\ \vdots \\ \vdots \\ S_{15}^{(n)} \\ S_{16}^{(n)} \end{bmatrix}, \quad F1^{(n)} = \begin{bmatrix} F1^{(1)} \\ F1^{(2)} \\ F1^{(3)} \\ \vdots \\ \vdots \\ F1^{(15)} \\ F1^{(16)} \end{bmatrix},$$

F=

$$\begin{bmatrix} \frac{h}{3} - (1-\delta)k \left(\frac{1}{2h}(S_1^{(n)} + S_2^{(n)}) + \frac{C_0 h}{6} \right) S_1^{(n)} + \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h}(-S_1^{(n)} - S_2^{(n)}) + \frac{C_0 h}{3} \right) S_2^{(n)} + F1^{(1)} \\ \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h}(-S_1^{(n)} - S_2^{(n)}) - \frac{C_0 h}{6} \right) S_1^{(n)} + \frac{2h}{3} - (1-\delta)k \left(\frac{1}{2h}(S_1^{(n)} + 2S_2^{(n)} + S_3^{(n)}) - \frac{C_0 h}{6} \right) S_2^{(n)} + \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h}(-S_2^{(n)} - S_3^{(n)}) + \frac{C_0 h}{3} \right) S_3^{(n)} + F1^{(2)} \\ \vdots \\ \vdots \\ \frac{2h}{3} - (1-\delta)k \left(\frac{1}{2h}(S_{14}^{(n)} + 2S_{15}^{(n)} + S_{16}^{(n)}) - \frac{C_0 h}{6} \right) S_{15}^{(n)} + \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h}(-S_{15}^{(n)} - S_{16}^{(n)}) + \frac{C_0 h}{3} \right) S_{16}^{(n)} + F1^{(15)} \\ \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h}(-S_{15}^{(n)} - S_{16}^{(n)}) - \frac{C_0 h}{6} \right) S_{15}^{(n)} + \frac{h}{3} - (1-\delta)k \left(\frac{1}{2h}(S_{15}^{(n)} + S_{16}^{(n)}) - \frac{C_0 h}{3} \right) S_{16}^{(n)} + F1^{(16)} \end{bmatrix}$$

where K is called global stiffness matrix and F is called global generalized force vector and N+1 is total number of global nodes .

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Imposing boundary conditions: we now apply the boundary condition (12) to the global equation (24) of the problem and simplifying, we get

$$[K(\varphi^{(n+1)})]\varphi^{(n+1)} = F \dots\dots\dots (25)$$

Where,

$$K(\varphi^{(n+1)}) = \begin{bmatrix} 1 & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ \frac{h}{6} \omega \left(\frac{1}{2h} (-s^{(n+1)} - s^{(n+2)}) - \frac{C\varphi^h}{6} \right) & \frac{2h}{3} \omega \left(\frac{1}{2h} (s^{(n+1)} - s^{(n+2)}) + s^{(n+1)} - \frac{C\varphi^h}{6} \right) & \dots & \frac{h}{6} \omega \left(\frac{1}{2h} (-s^{(n+1)} - s^{(n+2)}) - \frac{C\varphi^h}{6} \right) & \dots & 0 & \dots & 0 \\ 0 & \frac{h}{6} \omega \left(\frac{1}{2h} (-s^{(n+1)} - s^{(n+2)}) - \frac{C\varphi^h}{6} \right) & \dots & \frac{2h}{3} \omega \left(\frac{1}{2h} (s^{(n+1)} - s^{(n+2)}) + s^{(n+1)} - \frac{C\varphi^h}{6} \right) & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \dots & \frac{2h}{3} \omega \left(\frac{1}{2h} (s^{(n+1)} - s^{(n+2)}) + s^{(n+1)} - \frac{C\varphi^h}{6} \right) & \dots & \frac{h}{6} \omega \left(\frac{1}{2h} (-s^{(n+1)} - s^{(n+2)}) - \frac{C\varphi^h}{6} \right) \\ 0 & 0 & \dots & 0 & \dots & 0 & \dots & 1 \end{bmatrix}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

V.GRAPHICAL REPRESENTATION AND CONCLUSION

A Matlab Code is prepared for 15 elements model and resulting equation (26) for $N = 15$ is solved by Gauss Elimination method.

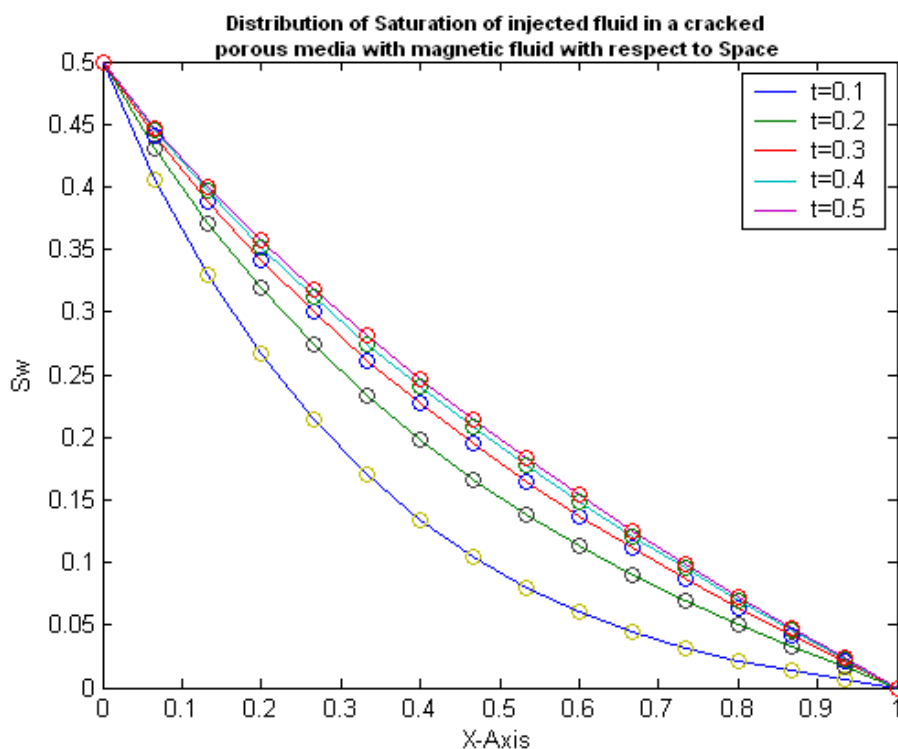


Figure 3.

In above graph, solution obtained with $C_0 = 4.046 \times 10^{-11}$, $c1(= C_2\sqrt{R}/D) = 0.3 \times 10^{11}$, $c2(= C_1/C_2R) = 90$, $\alpha = 1.11$, $L = 1, h = 1/15$, $k = 0.002223$ for 225 time levels. It is clear from graph that $S_w = S_0 (= 0.5)$ at layer $x = 0$ & there is no saturation of injected liquid at other end ($x = 1$) irrespective of time. At particular time, saturation of injected liquid involving magnetic fluid decrease with increase in value of x (or as we move ahead) and at $x = 1$, saturation is decreased to zero and as time increases, rate of increase of the saturation of injected liquid slows down at each layer

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