

Numerical Solution of Imbibition Phenomenon in a Slightly Dipping Porous Medium with Magnetic Fluid

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ABSTRACT: In this paper, we have discussed imbibition phenomenon in double phase flow through slightly dipping porous media. The mathematical formulation leads to non linear partial differential equation governing the phenomenon in a homogeneous medium with magnetic fluid. The mathematical solution has been obtained by finite element method with appropriate initial and boundary condition. Finite element method is a numerical method for finding an approximation solution of differential equation in finite region or domain. We obtained graphical representation of the solution using a Matlab coding.

KEYWORDS: Slightly dipping porous media, fluid flow, magnetic fluid, finite element method.

I. INTRODUCTION

It is well known that when a porous medium filled with some fluid is brought into contact with another fluid which preferentially wets the medium, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. Such a phenomena arising due to difference in wetting abilities is called counter-current imbibition. These phenomena have been formally discussed by Brownscombe and Dyes [4], Enright [5], Braham and Richarson [6], Rijik [7], Rijik et al [9], for homogeneous porous medium.

II. STATEMENT OF THE PROBLEM

We consider the finite cylindrical piece of oil saturated porous matrix of length $L (=1)$ is inclined at small angle θ with the horizontal. The present problem of linear counter current imbibition is furnished, when the three sides of the matrix are bounded by impermeable surfaces and the open end is exposed to liquid involving magnetic fluid (as in figure).

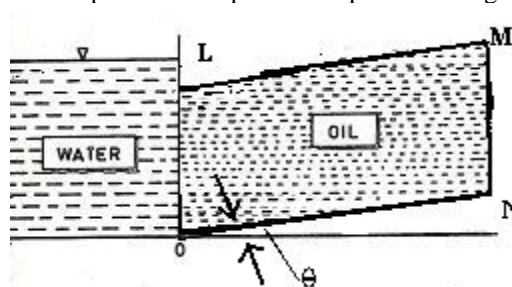


Figure 1.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

III. MATHEMATICAL FORMULATION OF THE PROBLEM

The equation of filtration velocity of injected liquid (w), becomes

$$V_w = -\left(\frac{K_w}{\delta_w}\right)K\left[\frac{\partial P_w}{\partial x} + \gamma H \frac{\partial H}{\partial x} + \rho_w g \sin \theta\right] \quad \text{..... (1)}$$

Where, $\gamma = \mu_o \lambda + \frac{16\lambda\mu_o\lambda^2 r^3}{9(l+2)^3}$ and equation of velocity of native liquid (V_o) is

$$V_o = -\left(\frac{K_o}{\delta_o}\right)K\left[\frac{\partial P_o}{\partial x} + \rho_o g \sin \theta\right] \quad \text{..... (2)}$$

Where, the phase densities ρ_w and ρ_o are regarded as constant, δ_w and δ_o are assumed to be invariant of magnetic field, g is the acceleration due to gravity and θ is the inclination. At the imbibition face, we have, $V_o = -V_w$ (3)

Also, $P_c = P_o - P_w$ (4) by the definition of capillary pressure .

On simplifying equation (1) and equation (2) by using equation (3) and (4), we obtain

$$\frac{\partial P_w}{\partial x} = -\left[\frac{-K_w \delta_o \left[\gamma H \frac{\partial H}{\partial x} + \rho_w g \sin \theta\right] - K_o \delta_w \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin \theta\right]}{(K_o / \delta_o + K_w / \delta_w)}\right] \left(\frac{\partial P_c}{\partial x}\right)$$

Substituting above equation into equation (1), we obtain

$$V_w = \frac{K K_o K_w}{(K_o / \delta_o + K_w / \delta_w)} \left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x} + (\rho_o - \rho_w) g \sin \theta\right]$$

Since, $\frac{K_o K_w}{K_o \delta_w + K_w \delta_o} \approx \frac{K_o}{\delta_o}$, Above equation reduces to the form

$$V_w = \frac{K K_o}{\delta_o} \left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x} + (\rho_o - \rho_w) g \sin \theta\right] \quad \text{..... (5)}$$

The equations of continuity of the two phases,

$$P\left(\frac{\partial S_w}{\partial t}\right) + \frac{\partial}{\partial x} \left(\frac{K K_o}{\delta_o} \left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x} + (\rho_o - \rho_w) g \sin \theta\right]\right) = 0$$

For definiteness of the mathematical analysis, we assume standard relationship between phase saturation and relative permeability, above equation becomes,

$$P\left(\frac{\partial S_w}{\partial t}\right) + \frac{K}{\delta_o} \frac{\partial}{\partial x} \left((1 - \alpha S_w) \left[-\beta_o \frac{\partial S_w}{\partial x} - \gamma H \frac{\partial H}{\partial x} + (\rho_o - \rho_w) g \sin \theta\right]\right) = 0 \quad \text{..... (6)}$$

This is the desired non linear differential equation of motion which describes the linear counter current imbibition phenomena in a homogeneous porous medium with effect of magnetic fluid.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Considering the magnetic fluid H in the x -direction only, we may write, $H = \frac{\Lambda}{x^n}$ where Λ is a constant parameter and n is an integer. Using the value of H for $n = -1$ in equation (6), we get,

$$C_1 \left(\frac{\partial S_w}{\partial t} \right) - C_2 \frac{\partial}{\partial x} \left((1 - \alpha S_w) \frac{\partial S_w}{\partial x} \right) - C_3 \frac{\partial}{\partial x} ((1 - \alpha S_w)x) + C_4 \frac{\partial}{\partial x} ((1 - \alpha S_w)) = 0 \quad \dots\dots\dots (7)$$

where $C_1 = P$, $C_2 = \frac{K\beta_o}{\delta_o}$, $C_3 = \frac{\gamma K \Lambda^2}{\delta_o}$, $C_4 = \frac{K}{\delta_o} (\rho_o - \rho_w) g \sin \theta$

A set of suitable boundary conditions,

$$S_w(x, 0) = 0 \quad \text{for all } x > 0; \quad \dots\dots\dots (8)$$

$$S_w(0, t) = S_{w0}; \quad S_w(L, t) = S_{w1} \quad \text{for all } t \geq 0 \quad \dots\dots\dots (9)$$

Equation (7) is reduced to dimensionless form by setting,

$$x^* = x/L, \quad t^* = \frac{t}{L^2(C_1/C_2)}, \quad (1 - \alpha S_w(x, t)) = S_w^*(x^*, t^*)$$

So that $\frac{\partial S_w}{\partial t} - \frac{\partial}{\partial x} \left(S_w \frac{\partial S_w}{\partial x} \right) - C_5 \frac{\partial}{\partial x} (S_w x) - C_6 \frac{\partial S_w}{\partial x} = 0 \quad \dots\dots\dots (10)$

Where $C_5 = \frac{-C_3 L^2 \alpha}{C_2}$, $C_6 = \frac{C_4 L \alpha}{C_2}$,

The initial and boundary conditions (8) & (9) now becomes,

$$S_w(x, 0) = 1 \quad \forall \quad x > 0 \quad \dots\dots\dots (11)$$

$$S_w(0, t) = 1 - \alpha S_{w0}; \quad S_w(L, t) = 1 - \alpha S_{w1} \quad \text{for all } t \geq 0 \quad \dots\dots\dots (12)$$

Equation (10) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in homogeneous medium with effect of magnetic fluid. A Matlab Code is prepared and executed with $C_5 = 4.046 \times 10^{-11}$, $C_6 = 0.001 \times 1.11$; $h = 1/15$, $k = 0.002223$ for 225 time levels, $S_{w0} = 0.5$ and $S_{w1} = 0$. Curves indicating the behavior of Saturation of injected fluid with respect to various time period.

IV. FINITE ELEMENT METHOD

For the problem under consideration, the length variable x varies between 0 and L (figure 2(a)). This domain is divided in to set of linear elements (figure 2(b)).

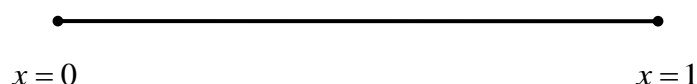


Figure 2(a).

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

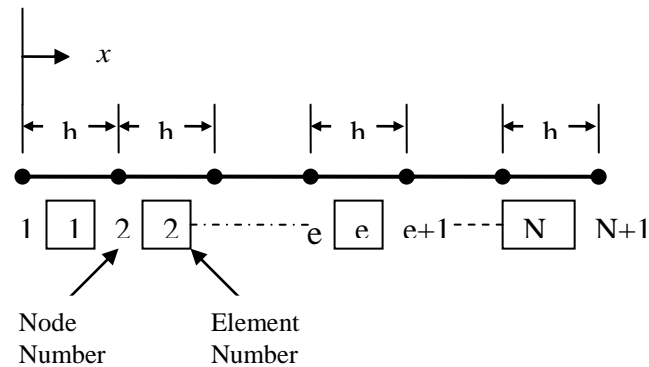


Figure 2(b).

Now, the variational form of given PDE is

$$J(S_w) = \frac{1}{2} \int_R \left[S_w \left(\frac{\partial S_w}{\partial x} \right)^2 + 2S_w \frac{\partial S_w}{\partial t} + (C_5 x + C_6) S_w \frac{\partial S_w}{\partial x} \right] dx \quad \dots\dots\dots (13)$$

Choose an arbitrary linear element $R^{(e)} = [S_1^{(e)}, S_2^{(e)}]$ & obtain interpolation function for $R^{(e)}$ using Lagrange interpolation Method such as

$$S^{(e)}(x) = \sum_{j=1}^2 N_j(x) S_j^{(e)} = N^{(e)} \phi^{(e)} = \phi^{(e)T} N^{(e)T} \quad \dots\dots\dots (14)$$

where $N^{(e)} = [N_1 \ N_2]$ & $\phi^{(e)} = [S_1 \ S_2]^T$

and N_1, N_2 are shape function for linear element.

Now, apply Variational Method to $R^{(e)}$, therefore equation (13) becomes

$$J(S^{(e)}) = \frac{1}{2} \int_{R^{(e)}} \left[S^{(e)} \left(\frac{\partial S^{(e)}}{\partial x} \right)^2 + 2S^{(e)} \frac{\partial S^{(e)}}{\partial t} + (C_5 x + C_6) S^{(e)} \frac{\partial S^{(e)}}{\partial x} \right] dx \quad \dots\dots\dots (15)$$

$$\text{as, } S^{(e)}(x) = N^{(e)} \phi^{(e)} = \phi^{(e)T} N^{(e)T}$$

$$\therefore \frac{\partial S^{(e)}}{\partial x} = \frac{\partial N^{(e)}}{\partial x} \phi^{(e)} = \phi^{(e)T} \frac{\partial N^{(e)T}}{\partial x} \text{ and } \left(\frac{\partial S^{(e)}}{\partial x} \right)^2 = \phi^{(e)T} \frac{\partial N^{(e)T}}{\partial x} \frac{\partial N^{(e)}}{\partial x} \phi^{(e)}$$

\therefore Equation 15 becomes

$$J(S^{(e)}) = \frac{1}{2} \int_{R^{(e)}} \phi^{(e)T} \left[N^{(e)} \phi^{(e)} \left(\frac{\partial N^{(e)T}}{\partial x} \frac{\partial N^{(e)}}{\partial x} \right) \phi^{(e)} + 2 \left(N^{(e)T} N^{(e)} \right) \frac{\partial \phi^{(e)}}{\partial t} + (C_5 x + C_6) N^{(e)T} \frac{\partial N^{(e)}}{\partial x} \phi^{(e)} \right] dx \quad \dots\dots\dots (16)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

For minimization, first differentiate equation (16) with respect to $\phi^{(e)}$

$$\frac{\partial J^{(e)}}{\partial \phi^{(e)}} = \int_{R^{(e)}} \left[N^{(e)} \phi^{(e)} \left(\frac{\partial N^{(e)T}}{\partial x} \frac{\partial N^{(e)}}{\partial x} \right) \phi^{(e)} + 2 \left(N^{(e)T} N^{(e)} \right) \frac{\partial \phi^{(e)}}{\partial t} + (C_5 x + C_6) N^{(e)T} \frac{\partial N^{(e)}}{\partial x} \phi^{(e)} \right] dx$$

Now, $\frac{\partial J^{(e)}}{\partial \phi^{(e)}} = 0 \quad \therefore$ The Element equation is

$$A^{(e)} \frac{\partial \phi^{(e)}}{\partial T} + B^{(e)}(\phi^{(e)}) \phi^{(e)} + C^{(e)} \phi^{(e)} = 0 \quad \dots\dots\dots (17)$$

where $A^{(e)} = \int_{S_1^{(e)}}^{S_2^{(e)}} \left(N^{(e)T} N^{(e)} \right) dx$; $B^{(e)}(\phi^{(e)}) = \int_{S_1^{(e)}}^{S_2^{(e)}} N^{(e)} \phi^{(e)} \left(\frac{\partial N^{(e)T}}{\partial x} \frac{\partial N^{(e)}}{\partial x} \right) dx$;

$$C^{(e)} = \int_{S_1^{(e)}}^{S_2^{(e)}} (C_5 x + C_6) \left(N^{(e)T} \frac{\partial N^{(e)}}{\partial x} \right) dx$$

By Gauss Legendre Quadrature Method, we evaluate these integral. Thus, element matrix transform to

$$A^{(e)} = \int_{-1}^1 \left(N^{(e)T} N^{(e)} \right) J dz \approx \sum_{I=1}^r A^{(e)}(z_I) W_I$$

$$B^{(e)}(\phi^{(e)}) = \int_{-1}^1 N^{(e)} \phi^{(e)} \left(\frac{1}{J} \frac{\partial N^{(e)T}}{\partial X} \frac{1}{J} \frac{\partial N^{(e)}}{\partial X} \right) J dz \approx \sum_{I=1}^r B^{(e)}(z_I) W_I$$

$$C^{(e)} = \int_{-1}^1 \left(C_5 \left(\frac{h}{2} (1 + z_I) \right) + C_6 \right) \left(N^{(e)T} \frac{1}{J} \frac{\partial N^{(e)}}{\partial z} \right) J dz \approx \sum_{I=1}^r C^{(e)}(z_I) W_I$$

For $A^{(e)}$ and $C^{(e)}$, degree of polynomial $p=2$ then $r=2$ and For $B^{(e)}$, $p=1$ then $r=1$. z_I and W_I are corresponding gauss points and gauss weights with respect to 'r'. Then, the element matrix becomes,

$$A^{(e)} = \frac{h^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad B^{(e)}(\phi^{(e)}) = \frac{1}{2h^{(e)}} \begin{bmatrix} S_1 + S_2 & -S_1 - S_2 \\ -S_1 - S_2 & S_1 + S_2 \end{bmatrix},$$

$$C^{(e)} = \frac{1}{2} \begin{bmatrix} -\frac{C_5 h^{(e)}}{3} - C_6 & -\frac{2C_5 h^{(e)}}{3} - C_6 \\ \frac{C_5 h^{(e)}}{3} + C_6 & \frac{2C_5 h^{(e)}}{3} + C_6 \end{bmatrix}, \quad \dots\dots\dots (18)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Assembling of elements: For a uniform mesh of N elements, by the equation (17) and equation (18), the assembled equation becomes , $A \frac{\partial \phi}{\partial T} - B(\phi)\phi + C \phi + D \phi = 0$ (19)

$$\text{where, } A = \frac{h}{6} \begin{bmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & (2+2) & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & (2+2) & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & (2+2) & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{bmatrix}$$

$$B(\phi) = \frac{1}{2h} \begin{bmatrix} S_1 + S_2 & -S_1 - S_2 & 0 & 0 & \dots & 0 & 0 \\ -S_1 - S_2 & S_1 + 2S_2 + S_3 & -S_2 - S_3 & 0 & \dots & 0 & 0 \\ 0 & -S_2 - S_3 & S_2 + 2S_3 + S_4 & -S_3 - S_4 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & S_{14} + 2S_{15} + S_{16} & -S_{15} - S_{16} \\ 0 & 0 & 0 & 0 & \dots & -S_{15} - S_{16} & S_{15} + S_{16} \end{bmatrix}$$

$$C = \frac{1}{2} \begin{bmatrix} -\frac{C_5 h}{3} - C_6 & -\frac{2C_5 h}{3} - C_6 & 0 & 0 & \dots & 0 & 0 \\ \frac{C_5 h}{3} + C_6 & \frac{C_5 h}{3} & -\frac{2C_5 h}{3} - C_6 & 0 & \dots & 0 & 0 \\ 0 & \frac{C_5 h}{3} + C_6 & \frac{C_5 h}{3} & -\frac{2C_5 h}{3} - C_6 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & \frac{C_5 h}{3} & -\frac{2C_5 h}{3} - C_6 \\ 0 & 0 & 0 & 0 & \dots & \frac{C_5 h}{3} + C_6 & \frac{2C_5 h}{3} + C_6 \end{bmatrix}, \quad \phi = [S_1 \ S_2 \ \dots \ S_{11}]^T$$

..... (20)

Equation (19) represents the assembled equation.

Time approximation: We have obtained the finite element equation in the global form, which represent a system of simultaneous ordinary differential equation. We introduced δ family of approximations which approximates weighted average of a dependent variable of two consecutive time steps by linear interpolation of the values of the variable at the

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

two time steps such as, $S_j = \delta S_j^{n+1} + (1-\delta)S_j^n$ (21). The time derivatives $\dot{\theta}_j$ are replaced by Forward

finite difference formula such as $\dot{S}_j = \frac{S_j^{n+1} - S_j^n}{k}$ (22).

In view of (21) and (22), equation (19) written as,

$$[A + \delta k(B(\phi^{(n+1)}) + C)]\phi^{(n+1)} = [A - (1-\delta)k(B(\phi^{(n)}) + C)]\phi^{(n)} \text{ ----- (23) where } \delta = 1/2 \text{ and } n = 0, 1, 2, \dots$$

For a uniform mesh of N elements, by equation (20), the global equation (23) takes the form,

$$\left[K(\phi^{(n+1)}) \right] \phi^{(n+1)} = \left[F_1(\phi^{(n)}) \right] \phi^{(n)} = F(\phi^{(n)}) \text{ (24) where,}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

$$K(\phi^{(n+1)}) = \begin{bmatrix} \frac{h}{3} + \phi \left(\frac{1}{24} (s_1^{(n+1)} + s_2^{(n+1)}) - \frac{C_5 h}{6} + \frac{C_6}{2} \right) & \frac{h}{6} + \phi \left(\frac{1}{24} (-s_1^{(n+1)} - s_2^{(n+1)}) - \frac{C_5 h}{6} + \frac{C_6}{2} \right) & 0 & \dots & 0 \\ \frac{h}{6} + \phi \left(\frac{1}{24} (-s_1^{(n+1)} - s_2^{(n+1)}) + \frac{C_5 h}{6} + \frac{C_6}{2} \right) & \frac{2h}{3} + \phi \left(\frac{1}{24} (s_1^{(n+1)} + s_2^{(n+1)}) - s_3^{(n+1)} - \frac{C_5 h}{6} + \frac{C_6}{2} \right) & 0 & \dots & 0 \\ 0 & \frac{h}{6} + \phi \left(\frac{1}{24} (-s_1^{(n+1)} - s_2^{(n+1)}) - 2s_3^{(n+1)} + s_4^{(n+1)} + \frac{C_5 h}{6} + \frac{C_6}{2} \right) & \frac{2h}{3} + \phi \left(\frac{1}{24} (s_1^{(n+1)} + s_2^{(n+1)}) - 2s_3^{(n+1)} - s_4^{(n+1)} + \frac{C_5 h}{6} + \frac{C_6}{2} \right) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \frac{h}{6} + \phi \left(\frac{1}{24} (-s_1^{(n+1)} - s_2^{(n+1)}) - s_5^{(n+1)} - \frac{C_5 h}{6} + \frac{C_6}{2} \right) \\ 0 & 0 & 0 & \dots & \frac{h}{6} + \phi \left(\frac{1}{24} (-s_1^{(n+1)} - s_2^{(n+1)}) - s_5^{(n+1)} - \frac{C_5 h}{6} + \frac{C_6}{2} \right) \end{bmatrix}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

$$F_1(\phi^{(n)}) = \begin{bmatrix} \frac{h}{3} - (1-\delta)k \left(\frac{1}{2h} (s_1^{(n)} + s_2^{(n)}) - \frac{C_5 h}{6} - \frac{C_6}{2} \right) & \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-s_1^{(n)} - s_2^{(n)}) - \frac{C_5 h}{3} - \frac{C_6}{2} \right) & 0 & \dots & 0 & \dots & 0 \\ \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-s_1^{(n)} - s_2^{(n)}) + \frac{C_5 h}{6} + \frac{C_6}{2} \right) & \frac{2h}{3} - (1-\delta)k \left(\frac{1}{2h} (s_1^{(n)} + 2s_2^{(n)} + s_3^{(n)}) + \frac{C_5 h}{6} \right) & \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-s_1^{(n)} - s_2^{(n)}) - \frac{C_5 h}{3} - \frac{C_6}{2} \right) & 0 & \dots & 0 & 0 \\ 0 & \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-s_2^{(n)} - s_3^{(n)}) + \frac{C_5 h}{6} + \frac{C_6}{2} \right) & \frac{2h}{3} - (1-\delta)k \left(\frac{1}{2h} (s_2^{(n)} + 2s_3^{(n)} + s_4^{(n)}) + \frac{C_5 h}{6} \right) & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & \frac{2h}{3} - (1-\delta)k \left(\frac{1}{2h} (s_1^{(n)} + 2s_5^{(n)} + s_6^{(n)}) + \frac{C_5 h}{6} \right) & \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-s_1^{(n)} - s_6^{(n)}) - \frac{C_5 h}{3} - \frac{C_6}{2} \right) \\ 0 & 0 & 0 & \dots & 0 & \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-s_1^{(n)} - s_6^{(n)}) + \frac{C_5 h}{6} + \frac{C_6}{2} \right) & \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (s_1^{(n)} + s_6^{(n)}) + \frac{C_5 h}{3} + \frac{C_6}{2} \right) \end{bmatrix}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

$$\phi^{(n)} = \begin{bmatrix} S_1^{(n)} \\ S_2^{(n)} \\ S_3^{(n)} \\ \vdots \\ \vdots \\ S_{15}^{(n)} \\ S_{16}^{(n)} \end{bmatrix}$$

$$F(\phi^{(n)}) =$$

$$\begin{bmatrix} \frac{h}{3} - (1-\delta)k \left(\frac{1}{2h} (S_1^{(n)} + S_2^{(n)}) - \frac{C_5 h}{6} - \frac{C_6}{2} \right) S_1^{(n)} + \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-S_1^{(n)} - S_2^{(n)}) - \frac{C_5 h}{3} - \frac{C_6}{2} \right) S_2^{(n)} \\ \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-S_1^{(n)} - S_2^{(n)}) + \frac{C_5 h}{6} + \frac{C_6}{2} \right) S_1^{(n)} + \frac{2h}{3} - (1-\delta)k \left(\frac{1}{2h} (S_1^{(n)} + 2S_2^{(n)} + S_3^{(n)}) + \frac{C_5 h}{6} \right) S_2^{(n)} + \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-S_2^{(n)} - S_3^{(n)}) - \frac{C_5 h}{3} - \frac{C_6}{2} \right) S_3^{(n)} \\ \vdots \\ \frac{2h}{3} - (1-\delta)k \left(\frac{1}{2h} (S_{14}^{(n)} + 2S_{15}^{(n)} + S_{16}^{(n)}) + \frac{C_5 h}{6} \right) S_{15}^{(n)} + \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-S_{15}^{(n)} - S_{16}^{(n)}) - \frac{C_5 h}{3} - \frac{C_6}{2} \right) S_{16}^{(n)} \\ \frac{h}{6} - (1-\delta)k \left(\frac{1}{2h} (-S_{15}^{(n)} - S_{16}^{(n)}) + \frac{C_5 h}{6} + \frac{C_6}{2} \right) S_{15}^{(n)} + \frac{h}{3} - (1-\delta)k \left(\frac{1}{2h} (S_{15}^{(n)} + S_{16}^{(n)}) + \frac{C_5 h}{3} + \frac{C_6}{2} \right) S_{16}^{(n)} \end{bmatrix}$$

where K is called global stiffness matrix and F is called global generalized force vector and N+1 is total number of global nodes .

Imposing boundary conditions: We now apply the boundary condition (12) to the global equation (24) of the problem and simplifying, we get

$$[K(\phi^{(n+1)})]\phi^{(n+1)} = F \dots\dots\dots (25)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Where,

$$K(\phi^{(n+1)}) = \begin{pmatrix} 1 & 0 & \dots & 0 & \dots & 0 \\ \frac{A}{6} + \phi \left(\frac{1}{20} (\phi^{(n+1)} - \phi^{(n)}) + \frac{C_2 A}{6} \right) & \frac{2A}{3} + \phi \left(\frac{1}{20} (\phi^{(n+1)} - \phi^{(n)}) + \frac{C_2 A}{6} \right) & \dots & \frac{A}{6} + \phi \left(\frac{1}{20} (\phi^{(n+1)} - \phi^{(n)}) + \frac{C_2 A}{6} \right) & \dots & 0 \\ 0 & \frac{A}{6} + \phi \left(\frac{1}{20} (\phi^{(n+1)} - \phi^{(n)}) + \frac{C_2 A}{6} \right) & \dots & \frac{2A}{3} + \phi \left(\frac{1}{20} (\phi^{(n+1)} - \phi^{(n)}) + \frac{C_2 A}{6} \right) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \frac{A}{6} + \phi \left(\frac{1}{20} (\phi^{(n+1)} - \phi^{(n)}) + \frac{C_2 A}{6} \right) & \dots & \frac{2A}{3} + \phi \left(\frac{1}{20} (\phi^{(n+1)} - \phi^{(n)}) + \frac{C_2 A}{6} \right) & \dots & 0 \\ 0 & \dots & \dots & \dots & \dots & 1 \end{pmatrix}$$

Vol. 4, Issue 11, November 2015

10995

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Thus, equation (25) is the resulting system of nonlinear algebraic equation.

Solution of nonlinear algebraic equation: In the previous section, we obtained the assembled equation which is nonlinear. The assembled nonlinear equations after imposing boundary conditions are given by equation (25). We seek an approximate solution by the linearization which based on scheme

$$\left[K(\phi^{(n)}) \right] \phi^{(n+1)} = F \quad \dots\dots\dots (26)$$

where $\phi^{(n)}$ denotes the solution of the n iteration. Thus, the coefficient K_{ij} are evaluated using the solution $\phi^{(n)}$ from the previous iteration and the solution at the $(n+1)^{th}$ iteration can be obtained by solving equation (26) using Gauss Elimination Method. At the beginning of the iteration (i.e. $n=0$), we assume the solution $\phi^{(0)}$ from initial condition which requires to have $S_1^{(0)} = S_2^{(0)} = \dots\dots\dots = S_{N+1}^{(0)} = 1$.

V. GRAPHICAL REPRESENTATION AND CONCLUSION:

A Matlab Code is prepared for 15 elements model and resulting equation (26) for $N = 15$ is solved by Gauss Elimination method.

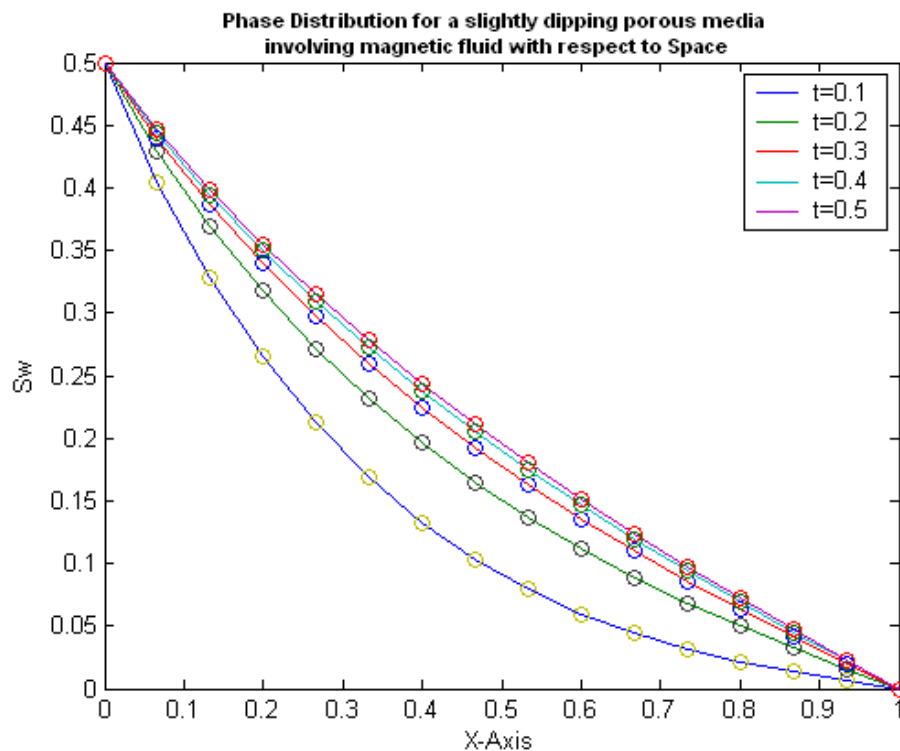


Figure 3.

In above graph, solution obtained with $C_5 = 4.046 \times 10^{-11}$, $C_6 = 0.001 \times 1.11$; $h = 1/15$, $k = 0.002223$ for 225 time levels. It is clear from graph that $S_w = S_0 (=0.5)$ at layer $x = 0$ & there is no saturation of injected liquid at other end ($x = 1$) irrespective of time. At particular time, saturation of injected liquid involving magnetic fluid decrease with increase in value of x (or as we move ahead) and at $x = 1$, saturation is decreased to zero and as time increases, rate of increase of the saturation of injected liquid decreases at each layer.

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(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

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