

# **Higher Order Reduction Using Simple Routh Approximation Method (SRAM) Based on Affine Arithmetic (AA)**

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**ABSTRACT:** This paper summarizes the reduction of higher order or a large scale continuous time system is introduced. The proposed method is Simple Routh Approximation Method (SRAM) of order reduction based on affine arithmetic (AA). SRAM is one of higher reduction for complex system plants without changing the characteristics of the plants. The proposed method generates stable low order or reduced order models for the given original stable high order systems. Affine Arithmetic based higher order reduction by SRAM is similar to standard interval arithmetic, but AA to some extent that it automatically keeps track of rounding and truncation errors for each computed value. However, by taking into account correlations between operands and sub-formulas, Affine Arithmetic is able to provide much tighter bounds.

**KEYWORDS:** Model Reduction, SRAM, Shebshev principles, Affine Arithmetic, Interval Arithmetic.

## **I. INTRODUCTION**

Almost all Engineering physical systems require high order mathematical models for their representation. Because, in physical design the order of the system is extremely high [1]. There are many disadvantages associated with high order systems in the like, designing, memory requirement application of available methods for analysing the stability. Therefore for realization, computation, control, and other purposes, a high-order system is represented by a low order without changing the significant features of the original system. The generated low order model from the original systems describes the behaviour of the original system closely.

Analysis and design of a high order control system is both tedious and costly, it is highly desirable to replace such a higher order system by a system of lower order. Many techniques have been developed to approximate the higher systems by low order models in frequency domain [1, 3].

Model reduction has turn a tool in analysis and simulation of high-order dynamic systems, circuit simulation, control design and numerous other fields dealing with complex physical systems. Order reduction is used to accelerate the simulation of such large scale systems [2].

The approximation of the high order systems by low order models is an active area of research in many engineering applications especially in control system analysis and design. Many researchers have attracted the attention on interval systems and to study the stability and the transient analysis of interval systems.

The analysis and design problems in engineering and science involve uncertainty to varying degree. The uncertainty may be specified in a different ways, such as probabilistic, convex or fuzzy descriptions. However in many systems the coefficients are constant but uncertain within a finite range. Such systems are classified as interval systems [4, 5].

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In this paper, Routh Approximation Method for model order reduction of affine arithmetic based is proposed. The coefficients of the reduced order both denominator and numerator polynomials are obtained by using Routh approximants, avoiding the need to find the time moments of the original system.

## II. INTERVAL ARITHMETIC (IA)

Interval arithmetic is also known as interval analysis, a real quantity  $x$  is represented by an interval  $[x] = [x_{\min}, x_{\max}]$  of floating-point numbers [11]. Interval arithmetic is simple tool to solve the range of problems. Those intervals are added, subtracted, multiplied, etc., in such a way that each computed interval  $[x]$  is guaranteed to contain the corresponding ideal quantity  $x$  [9].

## III. ADVANTAGE OF INTERVAL ARITHMETIC (IA)

The advantage of interval arithmetic is to model and analysis potential uncertainties in the quantities by holding internal computation errors such as rounding errors [10].

## IV. DISADVANTAGE OF INTERVAL ARITHMETIC (IA)

Unfortunately, Interval arithmetic often yields an interval that is much wider than the exact range of the computed function. Note that the interval arithmetic subtraction routine cannot assume that the two given intervals denote the same ideal quantity, since they could also denote two independent quantities that happen to have the same range [11]

## V. AFFINE ARITHMETIC (AA)

Affine arithmetic is proposed to overcome the error explosion problem of interval arithmetic. Besides finding a range for each ideal quantity, affine arithmetic also keeps track of correlations between those quantities. The approximation error occurred in each affine arithmetic operation depend on the size of the input intervals. Therefore, if the input intervals are small enough, each operation will provide a fairly tight estimate of the exact range of the corresponding quantity. Affine arithmetic is potentially useful in every numeric problem where one needs guaranteed enclosures to smooth functions [9-10].

In affine arithmetic, each input or computed quantity  $x$  is represented by an affine form  $[x]$ , which is a first degree polynomial

$$x = x_0 + x_1 \varepsilon_1 + \dots + x_n \varepsilon_n, \quad -1 \leq \varepsilon_i \leq 1$$

Where  $x_0, x_1, \dots, x_n$  are known floating-point numbers, and  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$  are symbolic variable

## VI. ADVANTAGE OF AFFINE ARITHMETIC

Like interval arithmetic, affine arithmetic keeps track automatically of the round-off and truncation errors affecting each computed quantity. In addition, affine arithmetic keeps track of correlations between those quantities. Affine arithmetic is able to provide much tighter intervals than interval arithmetic. Affine arithmetic is also computation model that permits reasoning over the ranges and maintains linear dependencies between affine quantities in order to reduce some of the over approximation [10].

## VII. PROPOSED METHOD

A simple way for reducing higher order reduction using RAM affine arithmetic based is explained. The coefficients of the higher reduced order obtained by using Routh approximation, without finding the time moments of the original system. In this method only Alpha table is required to be formulated [6,7].

The higher order continuous system transfer function is given as follows:

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$$G ( S ) = \frac{B_0 s^n + B_1 s^{n-1} + \dots + B_{n-1} s^1 + B_n}{A_0 s^n + A_1 s^{n-1} + \dots + A_{n-1} s^1 + A_n}$$

Alpha table is formulated as follows:

Alpha ( $\alpha$ ) table

From the transfer function which can evaluate the  $\alpha$ -parameters ( $\alpha_1, \alpha_2, \dots, \alpha_n$ ) by formulating the Routh-type array as follows [6, 7]:

$$\alpha_1 = \frac{A_0}{A_1} \langle \begin{matrix} A_0 & A_1 & A_2 & A_3 & A_4 & \dots & A_N \\ A_1 & A_2 & A_3 & A_4 & \dots & A_N \end{matrix} \rangle$$

$$\alpha_2 = \frac{C_0}{C_1} \langle \begin{matrix} C_0 & C_1 & C_2 & C_3 & C_4 & \dots & C_N \\ C_1 & C_2 & C_3 & C_4 & \dots & C_N \end{matrix} \rangle$$

$$\alpha_3 = \frac{D_1}{D_2} \langle \begin{matrix} D_1 & D_2 & D_3 & D_4 & D_5 & \dots & D_{N-1} \\ D_2 & D_3 & D_4 & D_5 & \dots & D_{N-1} \end{matrix} \rangle$$

$$\alpha_4 = \frac{E_1}{E_2} \langle \begin{matrix} E_1 & E_2 & E_3 & E_4 & E_5 & \dots & E_{N-2} \\ E_2 & E_3 & E_4 & E_5 & \dots & E_{N-2} \end{matrix} \rangle$$

$$\alpha_5 = \frac{F_1}{F_2} \langle \begin{matrix} F_1 & F_2 & F_3 & F_4 & \dots & F_{N-3} \\ F_2 & F_3 & F_4 & \dots & F_{N-3} \end{matrix} \rangle$$

$$\alpha_6 = \frac{G_1}{G_2} \langle \begin{matrix} G_1 & G_2 & G_3 & \dots & G_{N-4} \\ G_2 & G_3 & G_4 & \dots & G_{N-4} \end{matrix} \rangle$$

Table: a alpha table formulation

<p>Alpha values</p> $\alpha_1 = \frac{A_0}{A_1}$ $\alpha_2 = \frac{C_1}{C_2}$ $\alpha_3 = \frac{D_1}{D_2}$ $\alpha_4 = \frac{E_1}{E_2}$ $\alpha_5 = \frac{F_1}{F_2}$ $\alpha_6 = \frac{G_1}{G_2}$	<p>For i odd</p> $C_i = A_i, \quad i = 1, 2, \dots, n$ $D_1 = C_1$ $D_i = A_{i+1}, \quad i = 3, 5, \dots, n$ $E_1 = D_2$ $E_i = A_{i+2}, \quad i = 3, 5, \dots, n$ $F_1 = E_2$ $F_i = A_{i+3}, \quad i = 3, 5, \dots, n$ $G_1 = F_2$ $G_i = A_{i+4}, \quad i = 3, 5, \dots, n$ <p>For i even</p> $C_i = A_i - (A_0 A_{i+1} / A_1), \quad i = 1, 2, \dots, n$ $D_i = C_i - (C_1 C_{i+2} / C_2), \quad i = 1, 2, \dots, n$ $E_i = D_{i+1} - (D_1 D_{i+2} / D_2), \quad i = 1, 2, \dots, n$
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	$F_i = E_{i+1} - (E_1 E_{i+2} / E_2), \quad i = 1, 2, \dots, n$ $G_i = F_{i+1} - (F_1 F_{i+2} / F_2), \quad i = 1, 2, \dots, n$
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Generally equation for reduced order is given the follows:

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{\alpha \sum_{j=A}^{k-1} B_j s^j}{s^k + \alpha \sum_{j=0}^{k-1} A_j s^j}$$

Asymptotically stable system with closed-loop transfer function reduced order for different order is given by:

## VIII. RESULTS AND DISCUSSION

Using Simple Routh approximation method of large scale reduction based on affine arithmetic algorithm. The above formula gives reduced order with same characteristics of higher order system.

### Example 1

Let us consider the fourth order stable system.

$$G(s) = \frac{[28, 29]s^3 + [496, 497]s^2 + [1800, 1801]s + [2400, 2401]}{[2, 3]s^4 + [36, 37]s^3 + [204, 205]s^2 + [360, 361]s + [240, 241]}$$

Calculated values of alpha using SRAM that is given in alpha tables based on AA.

$$\begin{aligned} \alpha_1 &= [0.6648, 0.6694] \\ \alpha_2 &= [1.9889, 2.0132] \\ \alpha_3 &= [5.4189, 6.0334] \\ \alpha_4 &= [9.2117, 16.5056] \end{aligned}$$

Reduced order transfer functions using calculated alpha values depend the given equations by AA.

$$\begin{aligned} R_1 &= \frac{[6.648, 6.67]}{[1, 1]s + [0.6648, 0.6695]} \\ R_2 &= \frac{[9.944, 10.05]s + [13.25, 13.4]}{[1, 1]s^2 + [1.989, 2.014]s + [1.325, 1.345]} \\ R_3 &= \frac{[14.99, 16.57]s^2 + [54.37, 60.09]s + [72.49, 80.11]}{[1, 1]s^3 + [6.166, 6.834]s^2 + [10.87, 12.048]s + [7.248, 8.041]} \end{aligned}$$

Calculated values of alpha using SRAM which is given in alpha tables based on Interval Arithmetic.

$$\begin{aligned} \alpha_1 &= [0.6648, 0.6694] \\ \alpha_2 &= [1.9882, 2.0142] \end{aligned}$$

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$$\alpha_3 = [ 5 . 4274 \quad , 5 . 6633 \quad ]$$

$$\alpha_4 = [ 15 . 9858 \quad , 16 . 5118 \quad ]$$

Based on calculated values of alpha, we calculate reduced transfer functions by using above equations for all orders.

$$R_1 = \frac{[ 6 . 621 \quad , 6 . 697 \quad ]}{[1,1]s + [ 0 . 6621 \quad , 0 . 6722 \quad ]}$$

$$R_2 = \frac{[ 9 . 872 \quad , 10 . 12 \quad ]s + [ 13 . 16 \quad , 13 . 49 \quad ]}{[1,1]s^2 + [ 1 . 974 \quad , 2 . 028 \quad ]s + [ 1 . 316 \quad , 1 . 354 \quad ]}$$

$$R_3 = \frac{[ 14 . 76 \quad , 15 . 81 \quad ]s^2 + [ 53 . 58 \quad , 57 . 35 \quad ]s + [ 71 . 44 \quad , 76 . 39 \quad ]}{[1,1]s^3 + [ 6 . 073 \quad , 6 . 52 \quad ]s^2 + [ 10 . 72 \quad , 11 . 49 \quad ]s + [ 7 . 144 \quad , 7 . 668 \quad ]}$$

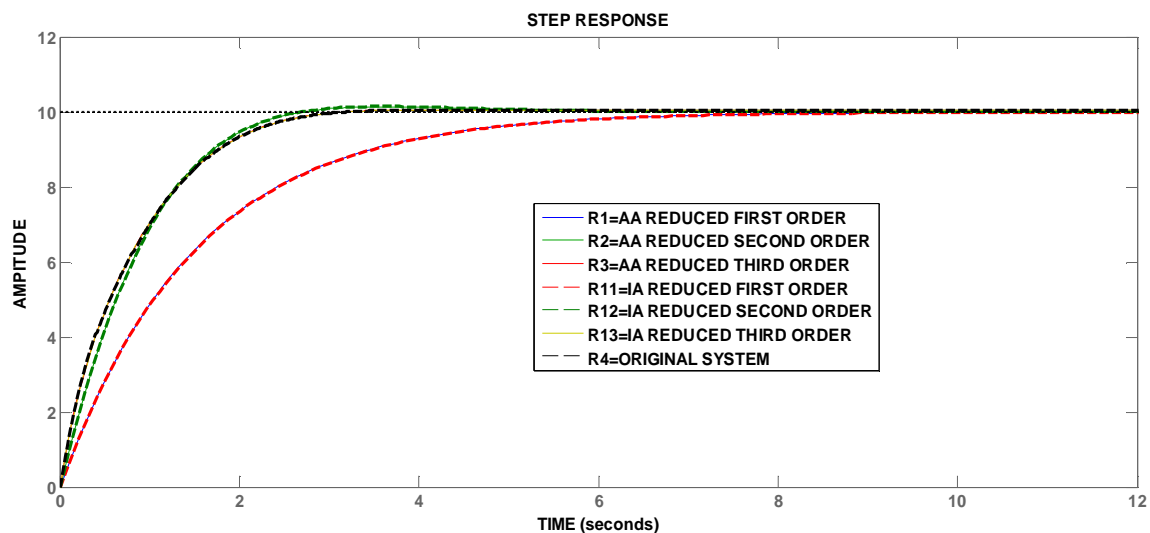


Figure: a Reduced orders and original system forth order lower bound.

The above figure (a) is lower bound order model of the original system and reduced order models. From the figure, we can understand that transient response is similar for reduced order models and original system. Steady state response of original system and reduced order models are same.

The reduced order model transfer functions of different upper bound orders and original system are given by step response as indicated:

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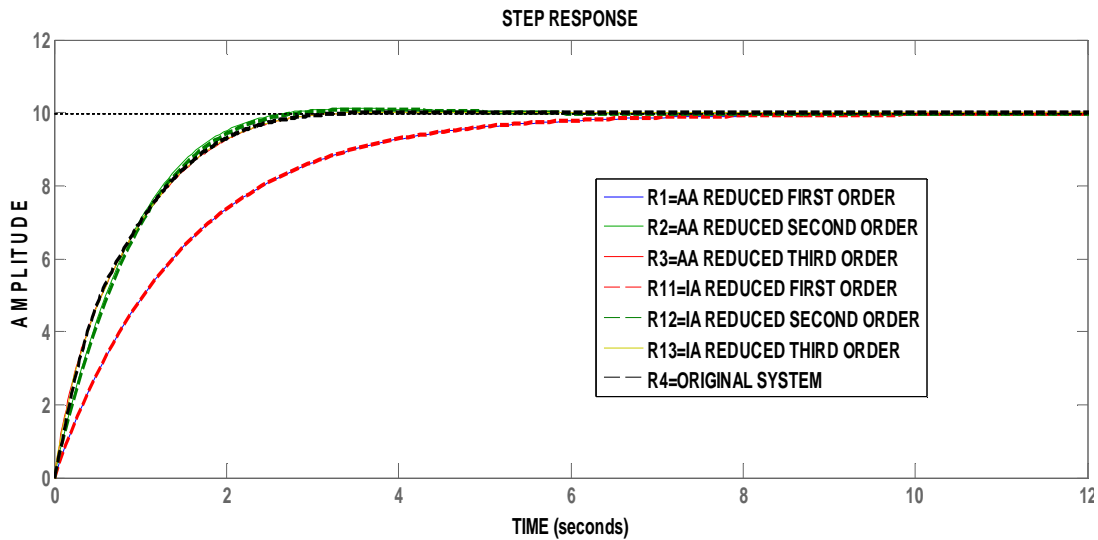


Figure: b Reduced orders and original system forth order upper bound.

The upper bound reduced order models and original system have the same properties with lower bound. The steady state response and transient response are same for all orders. The reduced order models have all characteristics of higher order systems. The higher order and lower order systems are same with same with all characteristics, but different only by the amplitude.

## Example 2

Let us consider six order stable system given by folowing transfer function:

$$G(s) = \frac{[2,3]s^5 + [70,71]s^4 + [762,763]s^3 + [3610,3611]s^2 + [7700,7701]s + [6000,6001]}{[1,1]s^6 + [41,42]s^5 + [571,572]s^4 + [3491,3492]s^3 + [10060,10061]s^2 + [13100,13101]s + [6000,6001]}$$

Original formulated values of alpha table using AA.

$$\alpha_1 = [ 0 . 4580 \quad , 0 . 4581 \quad ]$$

$$\alpha_2 = [ 1 . 5481 \quad , 1 . 5485 \quad ]$$

$$\alpha_3 = [ 3 . 2075 \quad , 3 . 2121 \quad ]$$

$$\alpha_4 = [ 5 . 8501 \quad , 5 . 9782 \quad ]$$

$$\alpha_5 = [ 12 . 0794 \quad , 15 . 3945 \quad ]$$

$$\alpha_6 = [ 10 . 1248 \quad , 36 . 0193 \quad ]$$

Based on above calculated values of alpha, reduced transfer functions order model using Simple Routh approximation method based on affine arithmetic given as follows for all orders.

$$R_1 = \frac{[ 0 . 458 \quad , 0 . 4581 \quad ]}{[ 1, 1 ]s + [ 0 . 458 \quad , 0 . 4581 \quad ]}$$

$$R_2 = \frac{[ 0 . 9099 \quad , 0 . 9102 \quad ]s + [ 0 . 709 \quad , 0 . 7094 \quad ]}{[ 1, 1 ]s^2 + [ 1 . 548 \quad , 1 . 549 \quad ]s + [ 0 . 709 \quad , 0 . 7094 \quad ]}$$

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$$R_3 = \frac{[1.22, 1.222]s^2 + [2.919, 2.924]s + [2.274, 2.278]}{[1,1]s^3 + [3.666, 4.974]s^2 + [4.966, 4.974]s + [2.274, 2.278]}$$

$$R_4 = \frac{[1.509, 1.528]s^3 + [8.157, 8.347]s^2 + [17.82, 18.28]s + [13.88, 14.24]}{[1,1]s^4 + [7.653, 7.799]s^3 + [23.09, 23.65]s^2 + [30.31, 31.09]s + [13.88, 14.24]}$$

$$R_5 = \frac{[1.77, 2.132]s^4 + [21, 28.68]s^3 + [100, 142.9]s^2 + [213.6, 307.9]s + [166.4, 240]}{[1,1]s^5 + [15.65, 20.15]s^4 + [96.69, 136.4]s^3 + [279, 400.8]s^2 + [363.3, 523.9]s + [166.4, 240]}$$

Calculated values of alpha using SRAM based on IA using alpha tables.

$$\alpha_1 = [0.4580, 0.4581]$$

$$\alpha_2 = [1.5481, 1.5485]$$

$$\alpha_3 = [3.2073, 3.2123]$$

$$\alpha_4 = [5.8484, 5.9807]$$

$$\alpha_5 = [12.2002, 15.5116]$$

$$\alpha_6 = [14.5193, 36.1516]$$

Reduced transfer functions of order models using Simple Routh approximation method based on interval arithmetic is given below.

$$R_{11} = \frac{[0.458, 0.4581]}{[1,1]s + [0.458, 0.4581]}$$

$$R_{12} = \frac{[0.9099, 0.9102]s + [0.709, 0.7094]}{[1,1]s^2 + [1.548, 1.549]s + [0.709, 0.7094]}$$

$$R_{13} = \frac{[1.22, 1.222]s^2 + [2.918, 2.924]s + [2.274, 2.279]}{[1,1]s^3 + [3.665, 3.67]s^2 + [4.965, 4.974]s + [2.274, 2.279]}$$

$$R_{14} = \frac{[1.509, 1.529]s^3 + [8.155, 8.351]s^2 + [17.81, 18.29]s + [13.88, 14.24]}{[1,1]s^4 + [7.651, 7.799]s^3 + [23.08, 23.66]s^2 + [30.3, 31.11]s + [13.88, 14.24]}$$

$$R_{15} = \frac{[1.798, 2.141]s^4 + [21.44, 28.91]s^3 + [102.4, 144.2]s^2 + [218.7, 310.8]s + [170.4, 242.2]}{[1,1]s^5 + [15.95, 20.67]s^4 + [98.92, 137.6]s^3 + [285.7, 404.5]s^2 + [372.1, 528.7]s + [170.4, 242.2]}$$

Reduced order figure for different order using interval arithmetic, affine arithmetic and original system by separate figures for both lower order and upper order.

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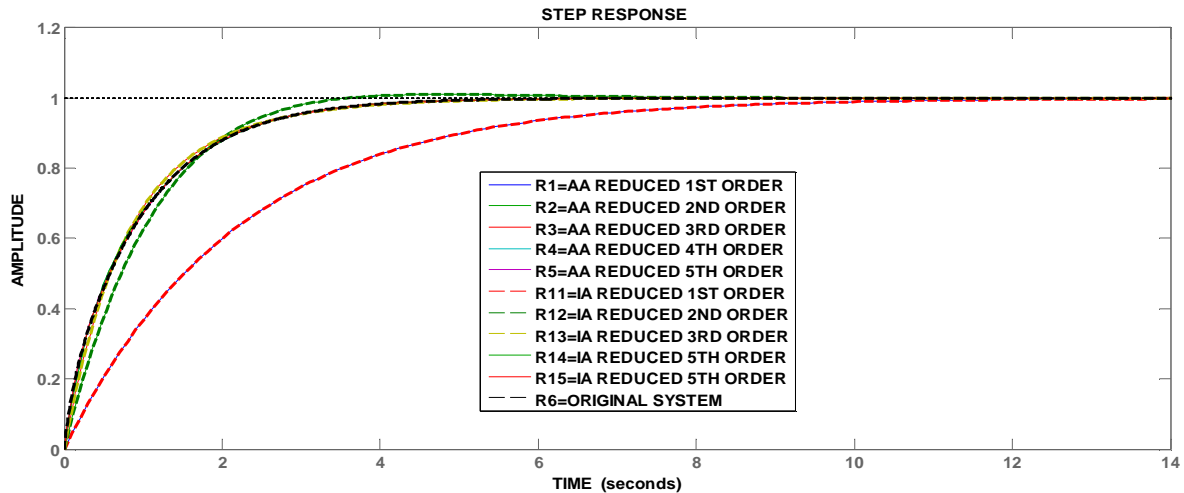


Figure: c Reduced orders and original system six order lower bound.

The above figure (c) is lower bound order model of the original system and reduced order models of six order system. From the figure, we can understand that transient response is similar for reduced order models and original system. Steady state response of original system and reduced order models are same.

The reduced order model transfer functions of different upper bound orders and original system are given by step response as indicated:

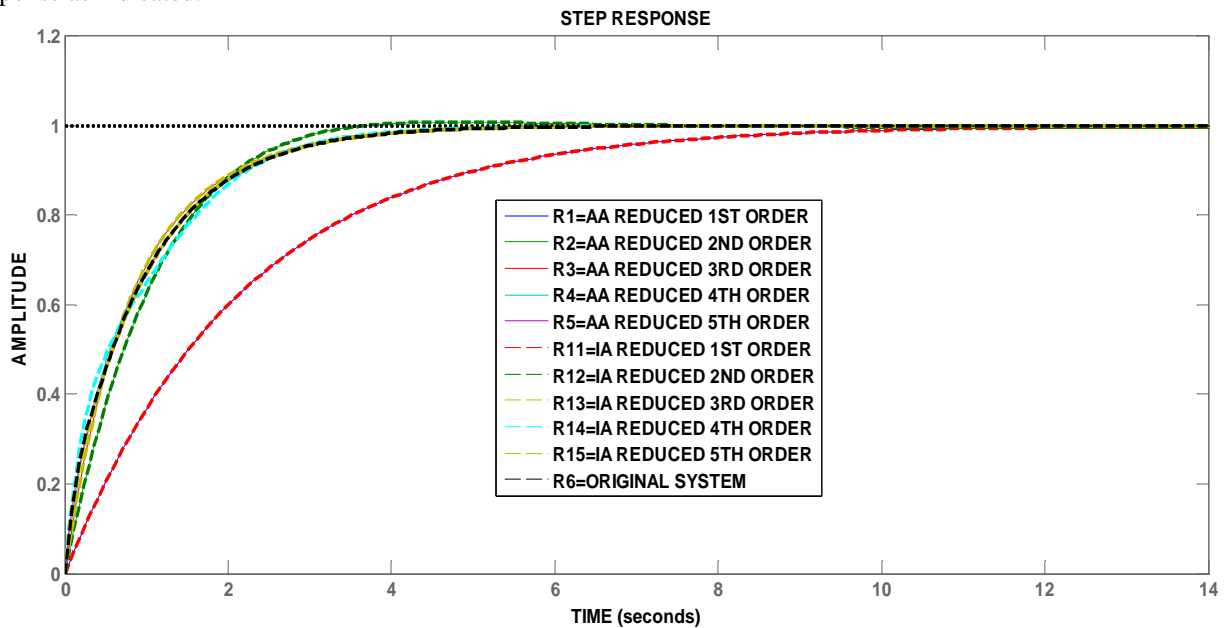


Figure: d Reduced orders and original system six order upper bound.

The upper bound reduced order models and original system have the same properties with lower bound. The steady state response and transient response are same for all orders. The reduced order models have all characteristics of higher order systems. Upper bound and lower bounds are only different by amplitude.



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## VIII. CONCLUSION

In practical application, the analysis, design and control of physical system is very difficult because of the order system is high. The disadvantages associated with high order systems like: designing, memory requirement and analysing the stability. The analysis and controller design of such higher order system is time consume and expensive. To solve these problems, Model reduction has become a tool in analysis and simulation of high-order variable systems. Model reduction represents a high-order system by a low-order model without changing the significant features of the original system. The generated low order model from the original systems describes the behaviour of the original system closely.

The advantage of interval arithmetic is to model and analysis potential uncertainties in the quantities and to hold internal computation errors such as rounding errors. However, Interval arithmetic often yields an interval that is much wider than the exact range of the computed function.

Like interval arithmetic, affine arithmetic keeps track automatically of the round-off and truncation errors affecting each computed quantity. Moreover, affine arithmetic keeps track of correlations between those quantities. Affine arithmetic is able to provide much tighter intervals than interval arithmetic. From the paper, we conclude that affine  $lkj+q$  arithmetic increase or decrease the interval depend n the application of system. Generally affine arithmetic more conservative as order system is increasing than interval arithmetic.

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