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Determination of Solution of Optimal Power Flow Problem Using Sensitivity Analysis

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ABSTRACT: In this paper we propose an approach to solving the disturbed Optimal Power Flow problem(OPF).The methodology consists the use of a sensitivity analysis to estimate new solutions after some disturbances occurred in the starting from optimal solution obtained by an OPF program. These disturbances can be load variations at one or more buses in the system. The sensitivity analysis technique is based on second-order information and in Karush-Kuhn-Tucker (KKT)conditions. Obtaining the solution after disturbances occur in system is straightforward and does not require initial and correction parameters as other methods like penalty and barrier used in conventional OPF programs. The numerical results presented highlighted the potential of this methodology for the solution of the disturbance OPF problem.

KEYWORDS: sensitivity analysis; optimal power flow problem; disturbances; Karush-Kuhn-Tucker conditions.

I. INTRODUCTION

An Electric Power System (EPS) operates on a permanent basis, but due to fluctuations in demands a disturbance is introduced. If the control variables, after a disturbance, remain constant, this will result a new operation point in the network buses with variable voltages and phase angles. These changes may take one or more variables state out of its operational limits. Regain an operating point would be necessary to solve the new disturbed problem or control exercise on the system, working in the control variables. If this setting is successful, then the dependent variables will return to the operating state, within its boundaries. From fundamental importance in a control system, the full understanding of how variations in the system affect their status. Such an understanding can be obtained from the sensitivity analysis without the need to process or reprocess several cases Optimal Power Flow (OPF). Not only the sensitivity of a variable can be obtained for any variable, but the sensitivity of any controlled variables can be obtained with respect to all the control variables and disturbances of the system. The sensitivity analysis is required in many areas of research in an EPS. Sensitivity analysis application has been used with great emphasis being widely applied in reactive power planning [9,10].

II. RELATED WORK

The authors Belhadj. (1996) [3] and Aumuller (2002) [1] used the sensitivity in voltage instability study and Kishore the study of voltage instability. Park & Lee (1997) [12] presented a study of sensitivity to power system restoration. The sensitivity analysis proposed by Fiacco (1976) [6] was used in resolving the OPF problem by T. S. Dillon (1980) [5], where the objective was to minimize the cost of generation. Gribik. (1990) [8], based on the theorem Fiacco (1976) [6], but worked with relaxed inequality constraints, resulting in an incomplete sensitivity matrix. In this paper we will use a sensitivity analysis to determine the new point operation to the OPF problem occurred after a disturbance in the system. The objective function is to minimize active power losses in transmission, this function is non-linear, non-convex and non-separable. This sensitivity study also permit analysis of the system easy and direct way, i.e. checking the new network performance (target function) and sensitivity of a given variable with respect to variations in demand. This paper is organized as follows: first in section 2 will be presented to OPF problem formulation and in section 3 the development of sensitivity methodology used; then in section 4, the application of the methodology will be presented to OPF problem, and numerical tests performed to evaluate the potential of the sensitivity methodology; and in section 5 will be presented the conclusions.

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III. OPTIMAL POWER FLOW PROBLEM FORMULATIONO

The OPF problem can be considered a non-linear programming problem (NLP), and is represented mathematically by:

$$min\, f(x)$$
$$s.a\, g_i(x) = 0, \quad i = 1, \ldots, m \qquad (1)$$
$$h_i(x) \leq 0, \quad j = 1, \ldots, r$$
$$x^{min} \leq x \leq x^{max}$$

xmin and xmaxrepresenting the vectors lower and upper bounds respectively; f (x) is the loss of active transmitted power; g (x) = 0 represents the group of balance equations of power flow; h (x) ≤ 0 is the set of functional restraints, ie limits of active and reactive power in transmission lines and transformers, limits of reactive power injection into control buses and active power injection into the slack bus. This is a typical non-linear problem. The solution of the problem (1) can be obtained by any OPF program. We can mention some of them: [see 2, 4, 7]. In this work used Baptista, E.C. (2004) [2].

IV. SENSETIVITY TECHNIQUE

The sensitivity analysis technique is based on the theorem proposed by Fiacco [6]. The theorem uses the sensitivity analysis of the first order applied to thesecond order of local solution. This technique can be used to estimate the new solution of NLP problem occurred after disturbances.The sensitivity analysis technique presented in this section considers the disturbances in equality constraints, when there a variation in the equality constraints (active and reactive power in the load bus). The mathematical formulation used sensitivity analysis technique is presented below.

The problem (1) disturbances are introduced $\varepsilon_i, i = 1, \ldots, m$, the equality constraints

$$min\, f(x)$$
$$s.a\, g_i(x) + \varepsilon_i = 0, \quad i = 1, \ldots, m \qquad (2)$$
$$h_i(x) \leq 0, \quad j = 1, \ldots, r$$
$$x^{min} \leq x \leq x^{max}$$

Where, $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_m)^T$ is the disturbance vectors. It is associated with the following Lagrangian function of problem (2):

$$L(x, \mu, \lambda, \varepsilon) = f(x) + \sum_{i=1}^{m} \lambda_i [g_i(x) + \varepsilon_i] + \sum_{j=1}^{r} \mu_j h_j(x) \qquad (3)$$

Where λ is the vector of Lagrange multipliers associated with the equality constraints, and μ is the vector of Lagrange multipliers associated with the inequality constraints.

To apply the sensitivity analysis technique, you must first have the optimum solution to the problem, (x^*, λ^*, μ^*), without disturbance ie, the solution to the problem (2) with ε = 0. In this paper, this solution is obtained by OPF program. The sensitivity analysis technique considers the gradient of the Lagrangian function, the conditions for further clearances and disturbed equality constraints, that is:

$$\nabla_x L(x, \mu, \lambda, \varepsilon) = 0$$
$$\mu_j [h_j(x)] = 0 \quad j = 1, \ldots \ldots, r \quad (4)$$
$$g_i(x) + \varepsilon_i = 0 \quad i = 1, \ldots \ldots, m$$

where, μ ≥ 0 and λ unrestricted. The gradient of the Lagrangian function (3) is represented by:

$$\nabla_x L(x, \mu, \lambda, \varepsilon) = \nabla_x f(x) + \sum_{i=1}^{m} \lambda_i \nabla_x g_i(x) + \sum_{j=1}^{r} \mu_j h_j(x) \qquad (5)$$

The roots of nonlinear system (4) are determined linearizing the system at optimum (x^*, λ^*, μ^*), which results the following linear system:

$$\nabla_x L(x^*, \lambda^*, \mu^*, \varepsilon) + \nabla^2_{xx} L(x^*, \lambda^*, \mu^*, \varepsilon)\Delta x + \nabla^2_{x\mu} L(x^*, \lambda^*, \mu^*, \varepsilon)\Delta\mu + \nabla^2_{x\lambda} L(x^*, \lambda^*, \mu^*, \varepsilon)\Delta\lambda = 0$$
$$\mu_j^* h_j(x^*) + \mu_j^* \nabla_x h_j(x^*)\Delta x + h_j(x^*)\Delta\mu = 0, \; j = 1, \ldots, r \qquad (6)$$
$$[g_i(x^*) + \varepsilon_i] + \nabla_x g_i(x^*)\Delta x = 0, \quad i = 1, \ldots, m$$

Rewriting (6) and eliminating invalid terms are:

$$\nabla_{xx} L(x^*, \lambda^*, \mu^*, \varepsilon)\Delta x + \nabla_x h_j L(x^*)\Delta\mu + \nabla_x g_i(x^*)\Delta\lambda = 0$$
$$\mu_j^* \nabla_x h_j(x^*)\Delta x + h_j(x^*)\Delta\mu = 0 \qquad (7)$$

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$\varepsilon_i + \nabla_x g_i(x^*)\Delta x = 0$

The set of equations (7) can be represented in matrix form by:

$$\begin{bmatrix} x(\varepsilon) - x^* \\ \mu(\varepsilon) - \mu^* \\ \lambda(\varepsilon) - \lambda^* \end{bmatrix} = - \begin{bmatrix} \nabla^2 L^* & H^* & G^* \\ M^*(H^*)^T & D_H^* & 0 \\ (G^*)^T & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \varepsilon \end{bmatrix} \quad (8)$$

Wherein

$$\begin{bmatrix} x(\varepsilon) - x^* \\ \mu(\varepsilon) - \mu\mu^* \\ \lambda(\varepsilon) - \lambda^* \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta\mu \\ \Delta\lambda \end{bmatrix};$$

$\nabla^2 L^* = \nabla_{xx}^2 L(x^*, \lambda^*, \mu^*, \varepsilon).$

They are:

$$\nabla^2 L^* = \nabla_{xx}^2 f(x^*) + \sum_{i=1}^{m} \lambda_i \nabla_{xx}^2 g_i(x^*) + \sum_{j=1}^{r} \mu_j h_j(x^*); M^* = \begin{bmatrix} \mu_1^* & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mu_r^* \end{bmatrix};$$

$$D_H^* = \begin{bmatrix} h_1(x^*) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_r(x^*) \end{bmatrix}; G^* = \left[\nabla_{g_1}(x^*), \ldots., \nabla_{g_m}(x^*)\right]; H^* = \left[\nabla_{h_1}(x^*), \ldots., \nabla_{h_r}(x^*)\right]$$

And $\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_m \end{bmatrix}$

The matrix system (8) is used to obtain the new state of the network when a disturbance ε in equality constraints are imposed.

4.1 Algorithm

A proposed algorithm to solve the problem of OPF consists of the following steps:

Having the optimal solution of OPF problem undisturbed (ε = 0), impose disturbances in the problem (ε ≠ 0).

Using (8) to find the new state of the network.

Check the condition of KKT. If you are satisfied within a given tolerance the problem is solved, but to get the optimal solution apply an OPF program

V. TESTENG AND RESULTS

In order to prove the efficiency of the proposed approach, we will give an application example illustrate OPF problem. The computational implementation was held in FORTRAN language.

5.1 didactic Example

To illustrate the technique described, we will present two tests. The first with a disturbance of 2% and the second with 10% in the equality constraint of the problem (9). After of completed disturbances problem, we will estimate the new solutions. Consider following disturbed problem:

$$min x_1^2 + x_2^2$$
$$s.a x_1 + 2x_2 = 1 + \varepsilon_1 \quad (9)$$
$$x_1^2 - x_2 \geq 0$$

As described in the algorithm, section 3, we solve the problem without disturbance to find the initial optimum spot, '*'. After we found the optimum solution applies the technique of sensitivity analysis in order to estimate the new optimum spot, "o" when ε1 ≠ 0. To validate technique, compare the estimated point with the optimum point after a disturbance.

In problem (9), where all the constraints are active, we can associate the following Lagrangian function:

$L = x_1^2 + x_2^2 + \lambda_1(x_1 + 2x_2 - 1 - \varepsilon_1) + \mu_1(x_1^2 - x_2)$

The solution of problem (9) can be easily found. Applying optimality conditions, we have:

$\nabla L(x_1^2, x_2^2, \lambda_1, \mu_1) = 0$

Solving the problem for ε1 = 0, we have the solution for the base case shown in Table 1 and visualized in Figure 1. The circles correspond to the contour function goal, the parabola and straight to the constraint of the problem, as can be seen in Figure 1.

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Table 1 - Optimal solution of the problem for ε1 = 0.

$\varepsilon_1 = 0$	x_1	x_2	λ_1	μ_1	$f(x)$
'*'	0,5000	O,2500	-0,5000	-0,5000	0,3125

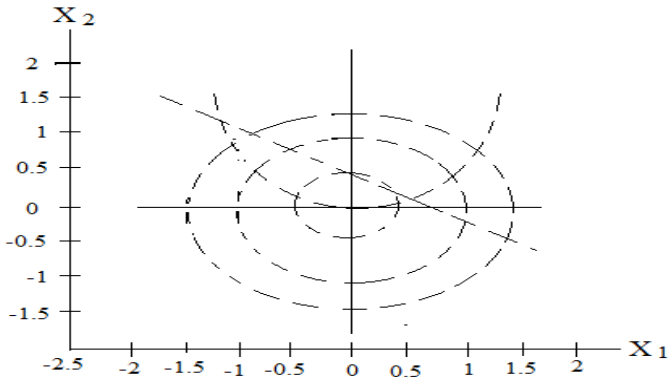

Figure 1 - Graphic representation of the optimal solution of the problem for ε1 = 0.

We apply a disturbance of ε1 =0.02 and ε1 = 0.10 corresponds to the variation of 2% and 10%respectively, in the resources of the equality constraint. For each case we compare the solution obtained by the sensitivity technique with a type of Lagrangian method. The Tables 2 and 3 show the points obtained via sensitivity analysis "o", and the exact spot,"*" , for ε1 =0.02 and ε1 = 0.10, respectively, which can be viewed in Figures 2 and 3 respectively. The solid line in the figure corresponds to the position of the new disturbed equality constraint.

Applying this example shows that the sensitivity analysis technique can be applied in NLP problems with good results. Obtaining new solutions by sensitivity analysis unlike the NLP methods solving problem, it is not iterative and not depends on the choice of initial parameters and correction, which makes their effective application. The dual variables are also estimated, thus providing more information of the problem, as can be seen in columns 4 and 5 of the Tables 2 and 3 represented by the variables λ1 and μ1.

Table 2 - post-disturbance Solutions ε1 = 0.02.

$\varepsilon_1 = 0.02$	x_1	x_2	λ_1	μ_1	$f(x)$
"O"	0.4933	0.2433	-0.4933	-0.5000	0.3025
'*'	0.4933	0.2433	-0.4933	-0.5000	0.3025

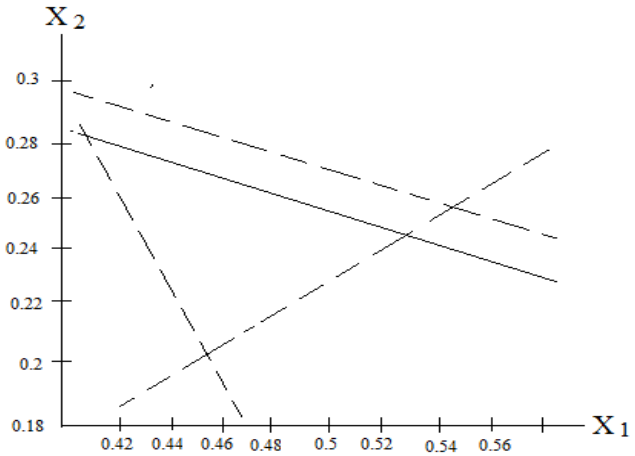

Figure 2 - Graphical representation of the solution to ε1 = 0.02

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Table 3 - post-disturbance Solutions ε1 = 0.10.

$\varepsilon_1 = 0.02$	x_1	x_2	λ_1	μ_1	$f(x)$
"O"	0.4659	0.2171	-0.4666	-0.4992	0.2649
'*'	0.4667	0.2267	-0.4667	-0.5000	0.2647

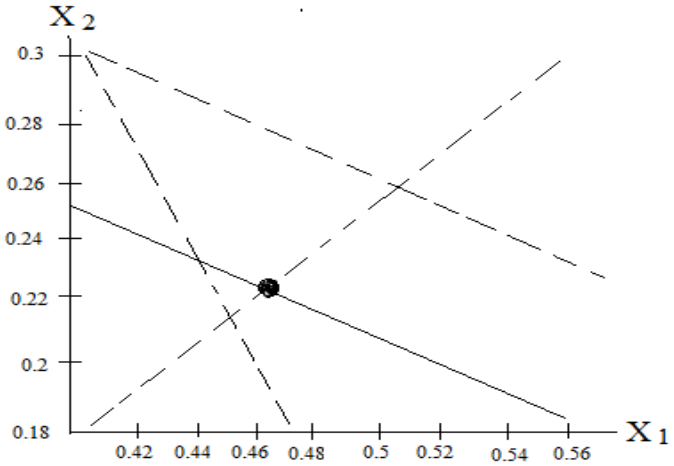

Figure 3 - Graphical representation of the solution to ε1 = 0.10.

5.2 Application in the optimal power flow problem

Two studies will be analyzed using sensitivity analysis technique to OPF problem related to the systems: IEEE 30, IEEE 300. The data from IEEE systems are in place http://www.ee.washington.edu/research/pstca/. The OPF problem of objective function used was active power transmission loss, which is a non-linear function, non-convex and not severable, which after Monticelli (1992)[11] makes the problem more difficult to resolve.

5.2.1System IEEE 30 bus

IEEE 30 bus system composed by: one generation bus (slack), five control bus of fixed reactive and active power generation, 24 load bus, 37 transmission lines, 4 transformers with variables tap. The OPF problem associated with IEEE 30 bar system consists of 53 equality constraints, 5 constraints channeled of reactive power, four tap channeled restrictions, and 30 voltage channeled restrictions.

In this system we performed a 5% load increase on all the carriers. THE Table 4 shows the variable voltage magnitude, drive in V, drive and phase angle θ in degree, the system buses prior to the disturbance and after a disturbance estimated by sensitivity analysis

Table 4 - Solution IEEE system 30 bars

Bus k	Base Case		After Disturbance	
	Vk(p.u)	θk(degree)	Vk(p.u)	θk(degree)
1	1,1000	0.0000	1,1000	0,0000
2	1.0819	-4.9276	1,0802	-5,0554
3	1.0639	-6.9875	1,0622	-7,21608
4	1.0553	-8.6113	1,0533	-8,89574
5	1.0569	-13.0530	1,0447	-13,3225
6	1.0505	-10.2456	1,0484	-10,5744
7	1.0416	-11.8931	1,0389	-12,224
8	1.0513	-10.9565	1,0486	-11,285
9	1.0530	-13.3326	1,0564	-13,8379

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10	1.0457	-14.9722	1,0461	-15,5671
11	1.0716	-13.3326	1,0864	-13,8379
12	1.0504	-14.2300	1,0498	14,7858
13	1.0744	-14.2300	1,0756	-14,7858
14	1.0361	-15.0821	1,0349	-15,6799
15	10328	-15.2079	1,0315	-15,8121
16	1.0403	-14.8483	1,0397	-15,4387
17	1.0388	-15.1422	1,0387	-15,7452
18	1.0252	-15.8253	1,0239	-16,4646
19	1.0238	-16.0158	1,0226	-16,6676
20	1.0284	-15.8064	1,0276	-16,4433
21	1.0332	-15.4058	1,0328	-16,0207
22	1.0336	-15.3839	1,0331	-16,0075
23	1.0236	-15.5693	1,0218	16,1866
24	1.0204	-15.7609	1,0185	-16,3805
25	1.0193	-15.3809	1,0164	-15,9728
26	1.0015	-15.9485	0,9977	-16,5725
27	1.0271	-14.8096	1,0241	-15,3665
28	1.0474	-10.8395	1,0446	-11,19
29	1.0071	-15.9880	1,0030	-16,6119
30	0.9949	-16.9345	0,9902	-17,6141
losses	16.27 MW		17,22 MW	

When analyzing the results in Table 4, we observe that the magnitudes of voltage remained within its boundaries of 0.95 to 1.10 p.u. The biggest error in the inequality equations occurred in buses 6 and 3 for the active and reactive powers respectively, which they were in the order of 10-4pu. The active power transmission losses increased from 16.27 MW to 17.22 MW. The active power generation in slack bus had an increase of 7.8 MW after a disturbance. It is important to note that the voltage at the Bus 1 was at the upper limit, and even after a disturbance in system the limit was satisfied. This was due to the Lagrange multiplier associated with the voltage restrictions at the active bus in the solution of the base case.

5.2.2System IEEE 30 bus

IEEE 300 bus system composed by: one generation bus (slack), 68 control bus of fixed reactive and active power generation, 231 load bus, 411 transmission lines, 107 transformers with variables tap. The OPF problem associated with IEEE 300 bar system consists of 530 equality constraints, 68 constraints channeled of reactive power, 107 tap channeled restrictions, and 300 voltage channeled restrictions.

This system performed a 5% load increase on load buses 49, 51, 52 and 55 maintaining constant power factor. Using a sensitivity analysis technique was estimated the solution after a disturbance. In this test, we analyzed the sensitivity increase ratio of load buses to the injection reactive power of the generation bus. After applying the technique presented here, we observed that injections reactive power remained within its boundaries. The biggest errors in equal equations occurred in the load bar 526. The error for active power was lower than 9 × 10-4pu and error for reactive power, less than 2 × 10-4pu. Losses in the system increased from 408.31 MW to 410.06 MW after increase in load. The slack bus had an increase of 9.7 MW for his generation after a disturbance. Table 5 shows the variation of injection reactive power with the load increased on the system.

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Table 5 - Analysis of the sensitivity of the reactive power injected into the system in relation to 5% filler increase the load buses 49, 51, 52 and 55.

Bus k	Reactive before disturbance (MVAR)	Reactive after disturbance (MVAR)	injection Reactive Variation (MVAR)
7094	72.8	85.8	13.0
8	-59.9	-59.2	0.7
10	56.6	57.7	1.1
20	102.9	101.6	-1.3
63	52.0	49.0	-3.0
76	143.3	143.7	0.4
84	120.1	120.2	0.1
91	81.5	81.0	-0.5
92	81.8	81,6	-0.2
98	-60.4	-59.8	0.6
108	3.2	3.9	0.7
119	981.6	987.5	5.9
124	98.2	97.7	-0.5
125	191.5	191	-0.5
138	192.4	192.5	0.1
141	72.3	72.4	0.1
143	267.9	276.1	8.2
146	17.7	18.2	0.5
147	-44.5	-44.2	0.3
149	24.0	24.1	0.1
152	-48.9	-48.8	0.1
153	-15.1	-14.9	0.2
156	-6.2	-5.9	0.3
170	78.2	79,4	1.2
171	447.6	448.5	0.9
176	34.1	34.6	0.5
177	39.9	40.8	0.9
185	19.1	18.9	-0.2
186	274.6	272.3	-2.3
187	215.9	214.3	-1.6
190	145.1	143.8	-1.3
191	696.0	698.7	2.7
198	66.2	66.2	0.0
rather buses	7049, 119 e 143		

This test shows the sensitivity of the injected reactive power into the system with respect to a load increased in a given group of buses. It is possible to using the technique to determining the sensitivity of any variable with respect to the variation in a determined load.

VI. CONCLUSION

This paper presents a methodology for solving the problem of disturbed OPF which can be extended to other NLP problems. The methodology consists, firstly, obtaining an initial optimal solution for the base case by the OPF and the

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use of sensitivity analysis technique to estimate new solutions after disturbances occurredin demand. With applying this technique it was possible to obtain new operating points for the system without the need to reprocess the OPF program. The main contribution of the method compared to the OPF programs are: correction, such as barrier and penalties used in the conventional OPF programs. The sensitivity analysis technique can also be used to estimate solutions where OPF algorithms cannot successful, for example, in some cases studies of expansion of the EPS.

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