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Methods to Find the Solution for the Intuitionistic Fuzzy Assignment Problem with Ranking of Intuitionistic Fuzzy Numbers

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ABSTRACT: In this paper, Intuitionistic Fuzzy Assignment Problem with the Trapezoidal Intuitionistic fuzzy numbers is introduced. Methods are proposed to find the solution of Intuitionistic Fuzzy Assignment Problem. Ranking method based on the magnitude of membership function and non-membership function of a Intuitionistic Fuzzy Number is utilized to order the Intuitionistic fuzzy numbers. Numerical example is provided to illustrate the methods.

KEYWORDS:Intuitionistic Fuzzy numbers, Trapezoidal Intuitionistic Fuzzy Numbers, Ranking of Intuitionistic fuzzy numbers, Intuitionistic fuzzy Hungarian method, Intuitionistic fuzzy Approximation Method.

I. INTRODUCTION

The idea of intuitionistic fuzzy set (IFS) introduced by Atanassov (1986) is the generalization of Zadeh's (1965) fuzzy set. An IFS is characterized by membership degree as well as non-membership degree. Since its introduction, the IFS theory has been studied and applied in different areas including decision making. In recent past, ranking intuitionistic fuzzy numbers (IFNs) draws the attention of several researchers. Nehi (2010) ranked IFNs based on characteristic values of membership and non – membership functions of IFN. Ranking of trapezoidal IFNs based on value and ambiguity indices were given by De and Das (2012), Rezvani (2012) and many more approaches were subsequently developed. Bellman and Zadeh (1970) introduced the concept of decision making in fuzzy environment. The concept of optimization in intuitionistic fuzzy environment was given by Angelov(1997). In this paper, Intuitionistic Fuzzy Hungarian Method is developed to find the optimal assignment of Intuitionistic Fuzzy Assignment Problems with Trapezoidal Intuitionistic fuzzy numbers. Ranking method based on magnitude of membership function and non-membership function of a Intuitionistic Fuzzy Number proposed by Sagaya &Henry (2013) is utilized to order the Intuitionistic fuzzy numbers. Numerical example is provided to illustrate the methods.

II. PRELIMINARIES

In this section, some basic concepts related to IFS are discussed. (Atanassov (1986, 1989), Nehi (2010))

Let *X* be a classical set of objects called the universal set, where an element of *X* is denoted by *x*. **Definition1:** An IFS *A* in *X* is given by $A = \{(x, \mu_A(x), \nu_A(x))/x \in X\}$ where the functions $\mu_A, \nu_A : X \to [0,1]$ are functions such that $0 \le \mu(x) + \nu(x) \le 1 \forall x \in X$. For each *x* the numbers $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of the element $x \in X$ to the set, which is a subset of *X*, respectively. **Definition2:** An IFS $A = \{(x, \mu_A(x), \nu_A(x))/x \in X\}$ is called IF- normal, if there exists at least two points $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1, \nu_A(x_1) = 1$.



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Definition3: An IFS $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ of the real line is called IF- convex if $\forall x_1, x_2 \in R, \forall \lambda \in [0,1],$ $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \ge \mu_A(x_1) \land \mu_A(x_2)$ and

 $v_A(\lambda x_1 + (1 - \lambda)x_2) \ge v_A(x_1) \land v_A(x_2)$. Thus A is IF-convex if its membership function is fuzzy convex and its non membership function is fuzzy concave.

Definition4: An IFS $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ of the real line is called an Intuitionistic Fuzzy Number (IFN) if (a) A is IF-normal, (b) A is IF-convex, (c) μ_A is upper semi continuous and ν_A is lower semi continuous, (d) A = $\{x \in X / \nu_A(x) < 1\}$ is bounded.

A is a Trapezoidal Intuitionistic Fuzzy Number (TIFN) with parameters $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le$ **Definition 5:** $b_3 \leq a_4 \leq b_4$ and denoted by

 $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ In this case,

$$\mu_{A}(x) = \begin{cases} 0 & if \quad x \le a_{1} \\ \frac{x-a_{1}}{a_{2}-a_{1}} & if \quad a_{1} < x < a_{2} \\ 1 & if \quad a_{2} \le x \le a_{3} \\ \frac{x-a_{4}}{a_{3}-a_{4}} & if \quad a_{3} < x < a_{4} \end{cases} \text{ and } \nu_{A}(x) = \begin{cases} 1 & if \quad x \le b_{1} \\ \frac{x-b_{1}}{b_{2}-b_{1}} & if \quad b_{1} < x < b_{2} \\ 0 & if \quad b_{2} \le x \le b_{3} \\ \frac{x-b_{4}}{b_{3}-b_{4}} & if \quad b_{3} < x < b_{4} \\ 1 & if \quad b_{4} \le x \end{cases}$$

If in a TIFN A, we let $b_2 = b_3$ (and hence $a_2 = a_3$) then it will be a Triangular Intuitionistic Fuzzy Number(TrIFN) with parameters $b_1 \le a_1 \le b_2$ $(a_2 = a_3 = b_3) \le a_4 \le b_4$ and denoted by $A = (b_1, a_1, b_2, a_4, b_4)$.

Arithmetic Operations of TIFNs

The Arithmetic Operations between two TIFNs defined on universal set of real numbers R, are reviewed as in Atanassov (1986, 1989), Nehi (2010), AmitKumar (2013)

Atanassov (1986, 1989), Nehi (2010), AmitKumar (2013) (i) Let $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $B = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$ be two TIFNs, then $A \oplus B = (b_1 + b'_1, a_1 + a'_1, b_2 + b'_2, a_2 + a'_2, a_3 + a'_3, b_3 + b'_3, a_4 + a'_4, b_4 + b'_4)$ (ii) Let $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $B = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$ be two TIFNs, then $A \oplus B = (b_1 - b'_4, a_1 - a'_4, b_2 - b'_3, a_2 - a'_3, a_3 - a'_2, b_3 - b'_2, a_4 - a'_1, b_4 - b'_1)$ (iii) Let $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ be a TIFN and r be a real number then $rA = \begin{cases} (rb_1, ra_1, rb_2, ra_2, ra_3, rb_3, ra_4, rb_4) \text{ if } r > 0 \\ (rb_4, ra_4, rb_3, ra_3, ra_2, rb_2, ra_1, rb_1) \text{ if } r < 0 \end{cases}$ (iv)Let $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $B = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$ be two non-negative TIFNs, then $A \otimes B = (b, b'_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $B = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$ be two non-negative TIFNs, then $A \otimes B = (b_1 b'_1, a_1 a'_1, b_2 b'_2, a_2 a'_2, a_3 a'_3, b_3 b'_3, a_4 a'_4, b_4 b'_4)$

III. RANKING OF INTUITIONISTIC FUZZY NUMBERS

In this paper, based on the magnitude of a fuzzy number introduced in Abbasbandy & Hajjari(2009), an ordering method for Intuitionistic fuzzy numbers proposed by Sagaya & Henry (2013) is used to order the Intuitionistic fuzzy numbers and it is as follows.

For an arbitrary IFN $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$, the magnitude of membership and non-membership function for IFN A denoted by $Mag(A_{\mu})$, $Mag(A_{\gamma})$ are defined respectively, as

$$\operatorname{Mag}(A_{\mu}) = \frac{1}{2} \left(\int_{0}^{1} (f_{A}^{-1}(r) + g_{A}^{-1}(r) + a_{2} + a_{3}) f(r) dr \right)$$
 and

$$\operatorname{Mag}(A_{\gamma}) = \frac{1}{2} \left(\int_{0}^{1} (h_{A}^{-1}(r) + k_{A}^{-1}(r) + b_{2} + b_{3}) f(r) dr \right)$$
 where the function f(r) is a regular reducing

function on [0,1] with f(0)=0, f(1)=1 and $\int_0^1 f(r) dr = \frac{1}{2}$. Obviously function f(r) can be considered as a weighting function. In actual applications, function f(r) can be chosen according to the actual situation. In this chapter we use f(r)= r. In particular, let $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ be a TIFN.



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Then $Mag(A_{\mu}) = = \frac{1}{12} (a_1 + 5a_2 + 5a_3 + a_4)$ and $Mag(A_{\gamma}) = \frac{1}{6} (b_1 + 2b_2 + 2b_3 + b_4)$ And the algorithm of ordering is as follows

Algorithm

As a ranking method, two TIFNs A and B are compared using the following steps :

Step1 : Compute $Mag(A_{\mu})$ and $Mag(B_{\mu})$. Then compare them. If they are equal then go to the step 2. Otherwise rank *A* and *B* according to the relative position of $Mag(A_{\mu})$ and $Mag(B_{\mu})$.

Step2 : Compute $Mag(A_{\gamma})$ and $Mag(B_{\gamma})$. Then compare them. If they are equal then *A* and *B* are equal. Otherwise rank *A* and *B* according to the relative position of

-Mag(A_{ν}) and -Mag(B_{ν}).

Therefore for any two TIFNs A and B, we define the ranking of A and B by the Mag(.)as follows:

(i) $\operatorname{Mag}(A_{\mu}) > \operatorname{Mag}(B_{\mu})$ if and only if $A \geq B$,

(ii) $Mag(A_{\mu}) < Mag(B_{\mu})$ if and only if $A \prec B$,

(iii) $Mag(A_{\mu}) = Mag(B_{\mu})$ and $Mag(A_{\nu}) = Mag(B_{\nu})$ if and only if $A \sim B$,

(iv) $\operatorname{Mag}(A_{\mu}) = \operatorname{Mag}(B_{\mu})$ and $\operatorname{Mag}(A_{\gamma}) > \operatorname{Mag}(B_{\gamma})$ if and only if $A \succ B$.

(v) $\operatorname{Mag}(A_{\mu}) = \operatorname{Mag}(B_{\mu})$ and $\operatorname{-Mag}(A_{\nu}) < \operatorname{-Mag}(B_{\nu})$ if and only if $A \prec B$.

Then the order \geq and \leq is formulated as $A \geq B$ if and only if $A \geq B$ or $A \sim B$, $A \leq B$ if and only if $A \prec B$ or $A \sim B$.

IV. THE INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM

The Intuitionistic Fuzzy Assignment Problem (IFAP) can be stated in the form of nxn fuzzy cost table $[\tilde{c}_{ij}]$ of real numbers as given in the following table :

	Jobs					
	1	2		j		n
1	č ₁₁	õ ₁₂		€ _{1j}		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
2	č ₂₁	č ₂₂		\tilde{c}_{2j}		\tilde{c}_{2n}
2	•	•	•	•	•	•
	•	•	•	•	•	•
	•	•	•	•	•	•
i	€ _{i1}	€i2		с _{іј}		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
Persons	•	•	•	•	•	•
	•	•	•	•	•	•
n	€ _{nl}	\tilde{c}_{n2}		$\tilde{\mathbf{c}}_{\mathbf{nj}}$		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

The costs or time \tilde{c}_{ij} are Intuitionistic trapezoidal fuzzy numbers. $\tilde{c}_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, c_{ij}^{(5)}, c_{ij}^{(6)}, c_{ij}^{(7)}, c_{ij}^{(8)}]$ is the cost of assigning the jth job to the ith person. The objective is to minimize the total cost of assigning all the jobs to the available persons (one job to one person).

V. METHODS TO SOLVE IFAP

In this section, Intuitionistic Fuzzy Hungarian Method and Intuitionistic Fuzzy Approximation Method are proposed to find the optimal assignment of the IFAP.



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Intuitionistic Fuzzy Hungarian Method

An efficient method based on the concept of Intuitionistic fuzzy opportunity costs is the Intuitionistic Fuzzy Hungarian method. This method of solving IFAP consists of the following steps :

Step1: Determine the Intuitionistic fuzzy cost matrix from the given problem. Then test whether it is balanced or not.

(i) If it is a balanced one (i.e., the number of persons are equal to the number of jobs) then go to step 3.

(ii) If it is an unbalanced one (i.e., the number of persons are not equal to the number of jobs) then go to step 2.

Step2: Add dummy rows or dummy columns, so that the Intuitionistic fuzzy cost matrix becomes a Intuitionistic fuzzy square matrix. The Intuitionistic fuzzy cost entries of dummy rows/columns are always Intuitionistic fuzzy zero.

Step3: Select the row minimum in each row by the ranking method and then subtract it from each row entry of that row. **Step4:** Select the column minimum from each column by using the ranking method in the resulting Intuitionistic fuzzy assignment problem after using step3 and then subtract the column minimum from each column entry of that column. Each column and row now has at least one Intuitionistic fuzzy element with rank zero.

Step 5: In the modified Intuitionistic fuzzy assignment table obtained in step4, search for Intuitionistic fuzzy optimal assignment as follows

a) Examine the rows successively until a row with a single Intuitionistic fuzzy element with rank zero is found. Assign this Intuitionistic fuzzy element with rank zero and cross off all other Intuitionistic fuzzy element with rank zero in its column. Continue this for all the rows.

b) Repeat the procedure for each column of reduced Intuitionistic fuzzy assignment table.

c) If a row and / or column has two or more Intuitionistic fuzzy element with rank zero then assign arbitrary any one of that and cross off all other Intuitionistic fuzzy element with rank zero of that row/column.

d) Repeat a) through c) above successively until the chain of assigning or cross ends.

Step 6: If the number of assignments is equal to n, the order of the Intuitionistic fuzzy cost matrix, Intuitionistic fuzzy optimal solution is reached. If the number of assignments is less than n, the order of the Intuitionistic fuzzy cost matrix, go to the step 7.

Step 7: Draw the minimum number of horizontal and / or vertical lines to cover

all the Intuitionistic fuzzy zeros of the reduced Intuitionistic fuzzy assignment matrix. This can be done by using the following:

- i) Mark rows that do not have any assigned Intuitionistic fuzzy zero.
- ii) Mark columns that have Intuitionistic fuzzy zeros in the marked rows.
- iii) Mark rows that do have assigned Intuitionistic fuzzy zeros in the marked columns.
- iv) Repeat ii) and iii) above until the chain of marking is completed.
- v) Draw lines through all the unmarked rows and marked columns.
- This gives the desired minimum number of lines.

Step 8: Develop the new revised reduced Intuitionistic fuzzy cost matrix as follows

- i) Find the smallest entry of the reduced Intuitionistic fuzzy cost matrix not covered by any of the lines.
- ii) Subtract this entry from all the uncovered entries and add the same to all the entries lying at the intersection of any two lines.

Step9: Repeat step 6 to step 8 until Intuitionistic fuzzy optimal solution to the given Intuitionistic fuzzy assignment problem is attained.

Intuitionistic Fuzzy Approximation Method

The proposed steps of Intuitionistic Fuzzy Approximation method to find a optimal assignment for IFAP is as follows.

Step 1: Test whether the given IFAP is balanced or not.

(i) If it is a balanced one (i.e., the number of persons are equal to the number of jobs) then go to step 3.

(ii) If it is an unbalanced one (i.e., the number of persons are not equal to the number of jobs) then go to step 2.

Step2: Add dummy rows or dummy columns, so that the Intuitionistic fuzzy cost matrix becomes a Intuitionistic fuzzy square matrix. The Intuitionistic fuzzy cost entries of dummy rows/columns are always Intuitionistic fuzzy zero.



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Step3: Determine the minimum and the next minimum Intuitionistic fuzzy costs in each row and column of IFAP using the ranking method.

Step4: Determine the Intuitionistic fuzzy difference between the minimum and next minimum Intuitionistic fuzzy costs in each row and column and display them alongside the Intuitionistic fuzzy assignment matrix against the respective rows. Similarly compute the differences for each column.

Step5: Identify the row or column with the largest difference among all the rows and columns. If a tie occurs use arbitrary tie-breaking choice. Let the greatest difference correspond to ith row and let \tilde{c}_{ij} be the smallest Intuitionistic fuzzy cost in the ith row. Assign that Intuitionistic fuzzy entry and cross off the ith row and jth column.

Step6: Recompute the column and row Intuitionistic fuzzy differences for the reduced IFAP and go to step5. Repeat the procedure until each row and column has exactly one Intuitionistic fuzzy assignment.

VI. NUMERICAL EXAMPLE

Consider a Intuitionistic fuzzy assignment problem with rows representing three persons P_1 , P_2 , P_3 and columns representing the three jobs J_1 , J_2 , J_3 . The following is the cost matrix $[\tilde{C}_{ij}]$ whose elements are Intuitionistic trapezoidal fuzzy numbers.

	J1	J_2	J_3	2
P ₁	[1,2,4,5,7,8,10,12]	[2,3,5,6,9,10,12,14]	[1,3,5,6,9,10,11,13]	
\mathbf{P}_2	[3,4,6,7,9,11,13,15]	[1,2,4,6,10,12,14,16]	[2,3,5,7,10,12,14,15]	
P ₃	[2,4,6,8,10,11,12,14]	[2,3,4,5,8,9,10,11]	[3,4,5,7,11,12,13,14]	

The given problem is a balanced one.

Intuitionistic Fuzzy Hungarian Method

Using Step3 of the Intuitionistic fuzzy Hungarian Method, the following reduced Intuitionistic fuzzy assignment matrix is obtained.

Using Step 4 of the Intuitionistic fuzzy Hungarian Method, the following modified Intuitionistic fuzzy assignment matrix is obtained.

	J_1	\mathbf{J}_2	J_3	~
P ₁	[-22,-16,-8,-4,4,8,16,22]	[-25,-19,-11,-5,8,14,22,28]	[-25,-19,-11,-5,7,13,20,26]	
P ₂	[-24,-18,-10,-5,5,11,19,25]	[-30,-24,-16,-8,8,16,24,30]	[-28,-23,-15,-7,7,15,23,28]	
P ₃	[-20,-14,-7,-2,7,11,17,23]	[-24,-19,-13,-7,7,13,19,24]	[-22,-18,-12,-5,9,15,21,26]	
	L			

Now using Step 5 to Step 9 of the Intuitionistic fuzzy Hungarian Method, the following Intuitionistic fuzzy optimal assignment matrix is obtained.



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Therefore, the Intuitionistic fuzzy optimal assignment for the given Intuitionistic fuzzy assignment problem is

 $P_1 \longrightarrow J_1, P_2 \longrightarrow J_3, P_3 \longrightarrow J_2.$

The Intuitionistic fuzzy optimal total cost is calculated as $\tilde{c}_{11} + \tilde{c}_{23} + \tilde{c}_{32} = [1,2,4,5,7,8,10,12] \oplus [2,3,5,7,10,12,14,15] \oplus [2,3,4,5,8,9,10,11] = [5,8,13,17,25,29,34,38]$

Intuitionistic Fuzzy Approximation Method

Using Step3 and Step4 of the Intuitionistic fuzzy approximation Method, the following Intuitionistic fuzzy assignment matrix is obtained.

Now using the Step5 of the Intuitionistic fuzzy approximation Method, the following Intuitionistic fuzzy matrix is obtained.



Now using the Step6 of the generalized fuzzy approximation Method, the following Intuitionistic fuzzy optimal assignment matrix is obtained.



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	J_1	\mathbf{J}_2	J_3
	<pre>c</pre>		
P ₁	[1,2,4,5,7,8,10,12]	>[2,3,5,6,9,10,12,14]	[1,3,5,6,9,10,11,13]
P ₂	[3,4,6,7,9,11,13,15]	[1,2,4,6,10,12,14,16]	[2,3,5,7,10,12,14,15]
P ₃	[2,4,6,8,10,11,12,14]	[2,3,4,5,8,9,10,11]	[3,4,5,7,11,12,13,14]
	(\sim	

Therefore, the Intuitionistic fuzzy optimal assignment for the given Intuitionistic fuzzy assignment problem is

 $P_1 \longrightarrow J_1, P_2 \longrightarrow J_3, P_3 \longrightarrow J_2$

The Intuitionistic fuzzy optimal total cost is calculated as $[1,2,4,5,7,8,10,12] \oplus [2,3,5,7,10,12,14,15] \oplus [2,3,4,5,8,9,10,11] = [5,8,13,17,25,29,34,38]$

VII. CONCLUSION

Here, IFAP with trapezoidal intuitionistic fuzzy costs is considered. Ranking method based on magnitude of membership function and non-membership function of IFN is used to order the TIFNs. Intuitionistic Fuzzy Hungarian method and Intuitionistic Fuzzy Approximation method are proposed to find the optimal assignment of IFAP. Finally a numerical example is provided to discuss the methods and one can apply these approaches to solve any IFAP.

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