

# Comparative Study on Eigen Values and Polarisation in 2D and 3D Hydrogen Atoms

C.Radhika, R..Suganya

Department of Physics, Bharath Institute of Higher Education and Research, Chennai, India

Department of Physics, Bharath Institute of Higher Education and Research, Chennai, India

**ABSTRACT:** The theoretical study of the 2D hydrogen atom problems has been solved and the energy Eigen values are obtained and compared with the 3D values. Hydrogen atom is an example of Central force problem. It is a two particle system. The Polarisation of 3D Hydrogen atom can be derived using Dalgarno and Lewis Method and this was extended to 2D Hydrogen atom.

**KEY WORDS :** Eigen Value, Energy, Hydrogen Atom, polarization.

## I.INTRODUCTION

Hydrogen atom is an example of Central force problem. An Hydrogen atom is made up of two interacting particles, ie charge and nucleus (1,4,5). The interaction is done by coulomb attraction. The Hydrogen atom is a two particle system consisting of an atomic nucleus of charge of an electron and nucleus. The Polarisation of 3D Hydrogen atom can be derived using Dalgarno and Lewis Method and this was extended to 2D Hydrogen atom.

## ENERGY EIGEN VALUE

The Energy Eigen value of different atomic state was defined by the principal quantum number 'n'. The Energy Eigen value of different atomic states is discrete.[7,8]

$$\text{Let } \lambda = \frac{2\mu Z e^2}{\alpha \hbar^2}$$

$$\text{Also } \alpha^2 = \frac{8\mu E}{\hbar^2}$$

Substituting the value of  $\alpha$ ,

$$\lambda^2 = \frac{\mu e^4 Z^2}{2\hbar^2 E}$$

$\lambda^2$  is the principal quantum number 'n'.

Therefore

$$E = \frac{\mu e^4 Z^2}{2\hbar^2 \lambda^2}$$

Replacing  $\lambda$  by 'n',

$$E_n = \frac{\mu e^4 Z^2}{2\hbar^2 n^2}$$

This is the expression for the Energies of different atomic states.

So in general,

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 9, September 2015

$$E_n = \frac{E_0}{n^2} \dots\dots\dots (1)$$

For 2D hydrogen atom , the main quantum number starts from Zero. Hence the energy of 2D hydrogen atom is [9]

$$E_n = \frac{E_0}{(n+\frac{1}{2})^2} \dots\dots\dots (2)$$

When n = 0, 
$$E_0 = \frac{E_0}{(0+\frac{1}{2})^2} = 4 E_0$$

## II.POLARIZABILITY USING DALGARNO LEWIS METHOD

### Polarisation of 3D hydrogen atom

The second order Stark effect can be interpreted in terms of an induced dipole moment (1,2). The induced dipole moment is proportional to the applied electric field and the same direction and the ratio  $\alpha$  of dipole moment to field strength is called polarisability(3,4,7). It is easily seen that these conditions hold exactly for a charged isotropic harmonic oscillator and the energy change in this case is  $-1/2\alpha E^2$ . [10,11]

$$W_2 = -1/2\alpha E^2.$$

Also 
$$W_2 = -9/4E^2 a_0^3.$$

Comparing we get , 
$$\alpha = 9/2a_0^3.$$

For a 3D hydrogen atom in the ground state, the polarisability can be found out using Dalgarno Lewis method as follows.

The second order energy correction  $W_2$  is given by,

$$W_2 = \int_n^1 \frac{|(m|H|n)|^2}{E_m - E_n}$$

The ground state wave function of 2D hydrogen atom is [12]

$$\Psi_{00} = \sqrt{\frac{1}{\pi a_0^3}} e^{-r/a_0}$$

$$\psi_1^{(1)} = \frac{-rC}{\sqrt{\pi}} \left(1 + \frac{r}{2}\right) e^{-r} \cos\theta$$

The second order energy correction is

$$W_2 = \int \psi_{00}^* H' \psi_1^{(1)} d\tau$$

Substituting we get ,  $W_2 = -9/4 c^2$

$W_2 = (-9/4) E^2 a_0^3$  ,  $a_0$  is the Bohr radius.

$$W_2 = -1/2\alpha E^2. = (-9/4) E^2 a_0^3$$

Therefore polarisability  $\alpha_{3D} = 9/2 a_0^3$

## III.RESULT AND DISCUSSION

The Energy Eigen value of different atomic states has been found by substituting different values of 'n' and it was compared for 2 and 3 dimensions. The results are tabulated below

# International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 4, Issue 9, September 2015

### Comparison of eigen values in 2D and 3D hydrogen atoms.

3D	2D
$E_n = E_0/n^2, n=1,2,\dots$	$E_n = E_0/(n+1/2)^2, n= 0,1,2,\dots$
$E_{gr\ n=1} = E_0/1 = E_0$	$E_{gr\ n=0} = E_0/(1/4) = 4E_0$
$E_{n=2} = E_0/4$	$E_{n=1} = E_0/(9/4) = 4E_0/9$
$E_{n=3} = E_0/9$	$E_{n=2} = E_0/(25/4) = 4E_0/25$

Hence in two dimensional hydrogen atom the main quantum number starts from zero and so it has a zero point energy whereas in 3D the main quantum number starts from one and it has no zero point energy. Also in 3D the binding energy of excited ground state is  $E_0$  but in 2D it is  $4E_0$ . So the large binding energy in 2D can be understood by decreasing the width of quantum well structure.

The polarisability of 2D hydrogen atom is compared with the 3D hydrogen atom. We get the ratio of polarisability to be  $\alpha_{2D} / \alpha_{3D} = 1.3 * 10^{-2}$  which agrees well with other works.

Though there has not been any work reported on the measurement of polarisability of any low dimensional system, there have been some estimates of donor polarisability in a quantum well of GaAs / Ga<sub>1-x</sub>Al<sub>x</sub>As system. These estimates were made using variation methods. Our results are compared with these works in the following table.

### Polarisability estimates of a 2D hydrogenic system

	Exact	+	@	#
$\alpha_{2D} / \alpha_{3D}$	$1.3 * 10^{-2}$	$.09 * 10^{-2}$	$0.2 * 10^{-2}$	$0.32 * 10^{-2}$

Exact : Present Work

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### IV.ACKNOWLEDGEMENT

I am very thankful to honorable chairman Pro-chancellor, Vice Chancellor, Registrar, Dean R&D and Dean Engineering of Bharath University for their continuous support and encouragement given to carry out this research work.

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