

# Supremum and Infimum of Set of Real Numbers

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## I. INTRODUCTION

Let  $S$  denote a set (a collection of objects). The notation  $x \in S$  means that the object  $x$  is in the set  $S$ , and we write  $x \notin S$  to indicate that  $x$  is not in  $S$ . We assume there exists a nonempty set  $R$  of objects, called real numbers, which satisfy the ten axioms listed below. The axioms fall in a natural way into three groups which we refer to as the field axioms, the order axioms, and the completeness axiom (also called the least-upper-bound axiom or the axiom of continuity)[10]. Along with the  $R$  of real numbers we assume the existence of two operations, called addition and multiplication, such that for every pair of real numbers  $x$  and  $y$  the sum  $x+y$  and the product  $xy$  are real numbers uniquely determined by  $x$  and  $y$  satisfying the following axioms[9]. (In the axioms that appear below,  $x, y, z$  represent arbitrary real numbers unless something is said to the contrary.). In this section, we define axioms[6].

### The field axioms

**Axiom 1.**  $x+y = y+x, xy = yx$  (commutative laws)**Axiom 2.**  $x+(y+z) = (x+y)+z, x(yz) = (xy)z$  (associative laws)**Axiom 3.**  $x(y+z) = xy +xz$  (distributive law).**Axiom 4.** Given any two real numbers  $x$  and  $y$ , there exists a real number  $z$  such that  $x+z=y$ . This  $z$  is denoted by  $y-x$ ; the number  $x-x$  is denoted by  $0$ . (It can be proved that  $0$  is independent of  $x$ .) We write  $-x$  for  $0-x$  and call  $-x$  the negative number of  $x$ [1].**Axiom 5.** There exists at least one real number  $x \neq 0$ , then there exists a real number  $z$  such that  $xz=y$ . This  $z$  is denoted by  $y/x$ ; the number  $x/x$  is denoted by  $1$  and can be shown to be independent of  $x$ [2]. We write  $x^{-1}$  for  $1/x$  if  $x \neq 0$  call  $x^{-1}$  the reciprocal of  $x$ .

### The order axioms

We also assume the existence of a relation  $<$  which establishes an ordering among the real numbers and which satisfies the following axioms[3]:

**Axiom 6.** Exactly one of the relations  $x=y, x < y, x > y$  holds.**Note.**  $x > y$  means the same as  $y < x$ .**Axiom 7.** If  $x < y$ , then for every  $z$  we have  $x+z < y+z$ .**Axiom 8.** If  $x > 0$  and  $y > 0$ , then  $xy > 0$ .**Axiom 9.** If  $x > y$  and  $y > z$ , then  $x > z$ .**Note.** A real number  $x$  is called positive if  $x > 0$ , and negative if  $x < 0$ . We denote by  $R^+$  the set of all positive real numbers, and by  $R^-$  the set of all negative numbers[4].**Definition 1.1.** Let  $S$  be a set of real numbers. If there is a real number  $b$  such that  $x \leq b$  for every  $x$  in  $S$ , then  $b$  is called an upper bound for  $S$  and we say that  $S$  is bounded above by  $b$ .

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We say an upper bound every number greater than  $b$  will also be an upper upper bound. If an upper bound  $b$  is also a member of  $S$ , then  $b$  is called the largest member or the maximum element of  $S$ . There can be at most one such  $b$ . If it exists, we write  $b = \max S$ .

**Definition 1.2.** Let  $S$  be a set of real numbers bounded above. a real number  $b$  is called a least upper bound or supremum for  $S$  if it has the following two properties:

- a)  $b$  is an upper bound for  $S$ .
- b) No number less than  $b$  is an upper bound for  $S$ .

**The completeness Axiom 10.** Every nonempty set  $S$  of real numbers [5] which is bounded above has a supremum; that is, there is a real number  $b$  such that  $b = \sup S$ .

**Comparison property.** Given [6] nonempty subsets  $S$  and  $T$  of  $\mathbf{R}$  such that  $s \leq t$  for every  $s$  in  $S$  and  $t$  in  $T$ . If  $T$  has a supremum then  $S$  has a supremum and  $\sup S \leq \sup T$ .

## II. MAIN RESULTS

In this section, we prove some theorems on infimum and supremum of set of real numbers.

**Theorem 2.1.** Show that the sup and the inf [7] of a set are uniquely determined whenever they exist.

*Proof.* Given a nonempty set  $S (\subseteq \mathbf{R})$ , and assume  $\sup S = a$  and  $\sup S = b$ , we show  $a = b$  as follows. Suppose that  $a > b$ , and thus choose

$$\varepsilon = \frac{a-b}{2}, \text{ then there exists a } x \in S \text{ such that}$$

$$b < \frac{a+b}{2} = a - \varepsilon < x < a.$$

which implies that

$$b < x$$

which contradicts to  $b = \sup S$ . Similarly for  $a < b$ . Hence,  $a = b$ .

**Theorem 2.2.** Find the sup and inf of each of the following sets of real number [8]s:

(a) All numbers of the form  $2^{-p} + 3^{-q} + 5^{-r}$ , where  $p, q,$  and  $r$  take on all positive integer values.

*Proof.* Define  $S = \{2^{-p} + 3^{-q} + 5^{-r} : p, q, r \in \mathbf{N}\}$ . Then it is clear that  $\sup S = \frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ , and  $\inf S = 0$ .

(b)  $S = \{x : 3x^2 - 10x + 3 < 0\}$

*Proof.* Since  $3x^2 - 10x + 3 = (x-3)(3x-1)$ , we know that  $S = (\frac{1}{3}, 3)$ .

Hence,  $\sup S = 3$  and  $\inf S = \frac{1}{3}$ .

(c)  $S = \{x : (x-a)(x-b)(x-c)(x-d) < 0\}$ , where  $a < b < c < d$ .

*Proof.* It is clear that  $S = (a, b) \cup (c, d)$ . Hence,  $\sup S = d$  and  $\inf S = a$ .

**Theorem 2.3.** Prove the comparison property for suprema [9].

*Proof.* Since  $s \leq t$  for every  $s \in S$  and  $t \in T$ , fixed  $t_0 \in T$ , then  $s \leq t_0$  for all  $s \in S$ . Hence, by Axiom 10, we know that  $\sup S$  exists. In addition, it is clear  $\sup S \leq \sup T$  [1].

**Remark 2.3.** There is a useful result, we write it as a reference. Let  $S$  and  $T$  be two nonempty subsets of  $\mathbf{R}$ . If  $S \subseteq T$  and  $\sup T$  exists, then  $\sup S$  exists and  $\sup S \leq \sup T$

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