

A Coloring of Special Kind of Graphs

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ABSTRACT: In this paper we give a schematic way to color special kind of Graphs. We also make a deep survey on graph coloring to all kinds of special graphs in Practice

KEYWORDS: coloring, domination, dominator coloring.

I. INTRODUCTION

The **chromatic number** is the minimum number of colors needed in a proper coloring of a graph. Graph coloring is used as a model for a vast number of practical problems involving allocation of scarce resources (e.g., scheduling problems), and has played a key role in the development of graph theory and, more generally, discrete mathematics and combinatorial optimization[7]. Graph k -colorability is NP-complete in the general case, although the problem is solvable in polynomial time for many classes.

A **dominating set** S is a subset of the vertices in a graph such that every vertex in the graph either belongs to S or has a neighbor in S . The domination number is the order of a minimum dominating set[5].

We start with notation and more formal problem definitions.

Let $G = (V(G), E(G))$ be a graph with $n = |V(G)|$ and $m = |E(G)|$. For any vertex $v \in V(G)$, the open neighborhood of v is the set $N(v) = \{u \mid uv \in E(G)\}$ and the closed neighborhood is the set $N[v] = N(v) \cup \{v\}$. Similarly, for any set $S \subseteq V(G)$, $N(S) = \bigcup_{v \in S} N(v) - S$ and $N[S] = N(S) \cup S$. A set S is a dominating set if $N[S] = V(G)$ [4]. The minimum cardinality of a dominating set of G is denoted by $\gamma(G)$. The distance, $d(u, v)$, between two vertices u and v in G is the smallest number of edges on a path between u and v in G . The eccentricity, $e(v)$, of a vertex v is the largest distance from v to any vertex of G . The radius, $\text{rad}(G)$, of G is the smallest eccentricity in G . The diameter, $\text{diam}(G)$, of G is the largest eccentricity in G [6].

A graph coloring is a mapping $f : V(G) \rightarrow C$, where C is a set of colors (frequently $C \subseteq \mathbb{Z}^+$). A coloring f is proper if, for all $x, y \in V(G)$, $x \in N(y)$ implies $f(x) \neq f(y)$.

We begin our study with an example of finding the dominator chromatic number for the graph in Figure 1 in order to show an approach in finding the dominator chromatic number of a graph[3].

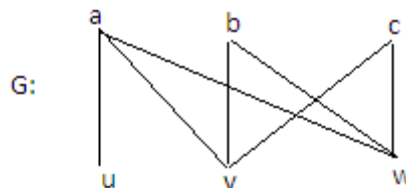


Figure 1: An example

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Example 1: The bipartite graph G in Figure 1 has $\chi_d(G) = 3$.

Proof:

Since G is a bipartite graph, at least two colors are needed to obtain a proper coloring of the graph, namely vertices $a, b,$ and c should receive one color, and vertices $u, v,$ and w should receive a different one. Note that this coloring is unique up to isomorphism[2]. However then, the vertex u does not dominate a color class since it is not the only one of its color class (so it cannot dominate its own class), and also it is not adjacent to all vertices in the other color class (thus not dominating the other color class either). Therefore, at least 3 colors are needed for the graph, and so $\chi_d(G) \geq 3$.

Now, define a coloring where the vertices $a, b,$ and c receive color 1, vertex u receives color 2, and vertices v and w receive color 3. Since each of the vertices $a, b,$ and c dominate the color class 3, the vertex u dominates the color class 2, and each of the vertices v and w dominate the color class 1, it follows that [3] colors will suffice, and so $\chi_d(G) \leq 3$. Therefore $\chi_d(G) = 3$.

Note that in the above graph every color class is dominated by some vertex[1]. This does not always happen, as we will see below in paths or multistars for example.

We now proceed with classes of graphs[6,8,3].

Proposition A: The dominator chromatic number for some classes of graphs.

- (1) The star $K_{1,n}$ has $\chi_d(K_{1,n}) = 2$.
- (2) The path P_n of order $n \geq 3$ has $\chi_d(P_n) = 2 + \lceil n/3 \rceil$ otherwise.
- (3) The complete graph K_n has and $\chi_d(K_n) = n$.

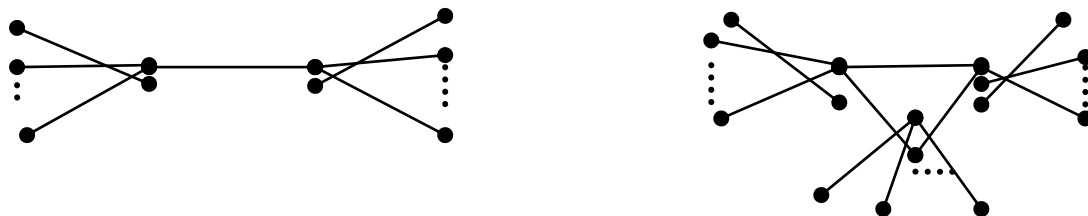


Figure 2: A 2-star graph $K_2(a_1, a_2)$ and a 3-star graph $K_3(a_1, a_2, a_3)$

We now investigate other classes of graphs that have not been studied. The star can be generalized to the multistar graph $K_m(a_1, a_2, \dots, a_m)$, which is formed by joining $a_i \geq 1$ ($1 \leq i \leq m$) end-vertices to each vertex x_i of a complete graph K_m , with $V(K_m) = \{x_1, x_2, \dots, x_m\}$. A 2-star and a 3-star graphs are shown in Figure 2. We present the dominator chromatic number of a few other classes for completeness.

Theorem 1.1. For every graph, $\gamma(G) \leq \gamma_{cpl}(G) \leq 2\gamma(G)$.

Proof. For any coloring π with associated partition S , any S -dominating set S is a dominating set, and $\gamma(G; S) \geq \gamma(G)$. Trivially then $\gamma(G) \leq \gamma_{cpl}(G)$. Assume that S is a coupled-partition of $V(G)$ with associated coloring π and $\gamma_{cpl}(G) = \gamma(G; S)$. Let D be a minimum dominating set, so $\gamma(G) = |D|$, and let $S = \{v \in V(G) : \exists w \in D \text{ with } \pi(v) = \pi(w)\}$. Then $\gamma_{cpl}(G) = \gamma(G; S) \leq |S| \leq 2|D| = 2\gamma(G)$.

Note that, in general, $\gamma(G; S) \leq \gamma(G) \cdot \max\{|V_i| : V_i \in S\}$.

Let $G_{2n} = K_n \times P_2$ be the graph on $2n \geq 6$ vertices obtained from two disjoint copies of complete graph K_n by adding a matching between them. We have $\gamma(G_{2n}) = 2$, $\gamma_{cpl}(G_{2n}) = 2 = \gamma(G_{2n})$ if n is odd, and $\gamma_{cpl}(G_{2n}) = 4 = 2\gamma(G_{2n})$ if n is even.



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