

(An ISO 3297: 2007 Certified Organization) Vol. 5, Issue 4, April 2016

# High Gain Observer Design for Nonlinear Web Winding System

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**ABSTRACT**: State estimation has become an important area of research in the industrial field. This is because there are many applications that demand the knowledge of many of the state variables, if not all of them.

The purpose of this paper is to introduce a nonlinear high gain observer in order to estimate the whole process state variables. Whenever some construction conditions hold, it is possible to obtain estimates that converge asymptotically to the actual values. As a particular application, we consider the Web Winding System for a reversing mill. This WWS is widely recognized as a difficult problem for the purpose of control. Finally, computer simulations are developed for showing the performance of the proposed nonlinear observer.

**KEYWORDS**: web winding system (wws), high gain observer, nonlinear system, systems uniformly observable, estimated parameters.

### I. INTRODUCTION

The resolution of many industrial problems requires disposal at all times state measures of the studied system. So the quality of the solution, for example a control problem, depends at least in part on the accuracy of the measured state. In the case of a cold rolling system, there must be measures of electromagnetic quantities, mechanical quantities and thermal quantities. There are physical sensors that directly provide these measures, only they are expensive and fragile. To overcome this difficulty, we use estimation algorithms online non-measurable variables. These algorithms, called observers or software sensors, based on the knowledge of the system model and the measures provided by physical sensors available. The observer's synthesis for nonlinear systems is usually much more difficult than for linear systems. For systems admitting an exact linearization with variable change or may be observability canonical form, a type of Luenberger observer might suit [1-3]. The web winding system dynamics (WWS) is described by its strongly nonlinear behaviour. In all cases of winding or unwinding a web material, there is the flatness problem[4-5].

In the present work, we propose a nonlinear observer applied to the nonlinear affine control system. The paper is organized as follows. The web winding model is described in Section 2, with a brief description of the system. We detail the proposed observer and prove its convergence in Section 3. In Section 4 the observability analysis of the wws are treated and evaluated using simulation of the web winding model. The last section concludes the paper.

### II. PLANT MODEL

### A. WWS Description

The web system is very important in a rolling mill, because its parameters determine the strip quality. Among its parameters, we quote:

• The entry and exit of the traction forces.



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• The entry and exit velocities ensured by the winders motors, and the work rolls velocity (Fig.1).

• The pressure force or the variations between the work rolls and their parallelism.

The variation of the exit strip flatness evolves because of the thermal dilation of the cylinders [6-7], but also due to the elasticity forces [8]. To avoid this phenomenon, the traction forces are applied to limit the elasticity of the rolled material. Programmable automats, which are called AGC (Automatic Gauge Control system); ensure the thickness control [9]. Their goal is to maintain the strip thickness uniform in spite of the changing of the acting factors. We start with the development of a mathematical model describing the dynamic behavior of the system.



Fig. 1. Interactions between the components of the cold rolling system.

#### B. Global Model

Let us consider the nonlinear model of the wws [10-11] defined by the state representation (1) which can be put in the general form of the nonlinear affine control system [4]:

$$\begin{cases} \dot{x} = f(x, u) = f(x) + g(x)u \\ y = h(x) \end{cases}$$
(1)  
with:  

$$x = [V_1 \quad T_1 \quad V_2 T_3 \quad V_3]^T = ; y = [T_1 \quad V_2 \quad T_3]^T \text{et } u = [U_1 \quad U_2 \quad U_3]^T \\
= \begin{bmatrix} \frac{gr_1^2}{J_1} x_2 - \frac{1}{\tau_{em1}} x_1 \\ -\frac{1}{L} x_2 x_3 - \frac{ES}{L} x_1 + \frac{ES}{L} x_3 \\ -\frac{gr_2^2}{J_2} x_2 + \frac{gr_2^2}{J_2} x_4 - \frac{1}{\tau_{em2}} x_3 \\ -\frac{1}{L} x_3 x_4 - \frac{ES}{L} x_3 + \frac{ES}{L} x_5 \\ -\frac{gr_3^2}{J_3} x_4 - \frac{1}{\tau_{em3}} x_5 \end{bmatrix} h(x) = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix}$$
(2)  

$$g(x) = [g_1, g_2, g_3] = \begin{bmatrix} \frac{r_1}{\tau_{em1} k_1} & 0 & 0 \\ 0 & \frac{r_2}{\tau_{em2} k_2} & 0 \\ 0 & 0 & \frac{r_3}{\tau_{em3} k_3} \end{bmatrix}$$
(3)

These parameters characterize the system: it is multivariable, strongly coupled, nonlinear and timevarying. The model (1) is composed of three subsystems: The first one is of the state vector  $[V_1,T_1]^T$ , controlled by the tension  $U_1$ , the second has as a state vector  $[V_2]$ , is controlled by  $U_2$  and the third subsystem has as a state vector  $[T_2, V_3]^T$ , is controlled by the tension  $U_3$ .



(5)

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#### **III. OBSERVER SYNTHESIS**

### A. Observers

Consider the following dynamic systems:

$$(\Sigma):\begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases} \qquad (O):\begin{cases} \dot{z} = \hat{f}(z, u, y) \\ \hat{x} = H(z, u, y) \end{cases}$$
(4)

where  $z \in M$  a variety differentiable such as  $H:M \rightarrow \vartheta$  is a submersion,  $\vartheta$  is open  $\Re^n$  constituting state space of  $\Sigma$ ; u and y are respectively the  $\Sigma$  inputs and outputs.

The most used observer structure is:

$$(O) \begin{cases} \hat{x} = f(\hat{x}, u) - G(g)(h(\hat{x}) - y) \\ \dot{g} = \varphi(\hat{x}, u, y, g) \end{cases}$$

G(g) is called observer gain and  $G(g)(h(\hat{x})-y)$  is the correction term. In the following, we will recall some existing results on the synthesis of observers.

#### B. Nonlinear MIMO affine in control systems uniformly observable

The equation of the observer in the original coordinates is written as follows:

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u - \theta \left(\frac{\partial \Phi(\hat{x})}{\partial x}\right)^{-1} \Delta_{\theta}^{-1} S^{-1} C^{T} \left(h(\hat{x}) - y\right)$$

This observer is a copy of the model, plus a corrective term which is explicitly given. Moreover, its setting is made through the choice of a single parameter, ie  $\theta$ .

We present an observable form characterizing class of uniformly observable of nonlinear multi-output systems. Next, we will present a particular class of uniformly observable of nonlinear multi-output systems that can presented in a canonical form. This canonical form, also known as the first canonical form was proposed by [12]. This form can be interpreted as a generalization of the canonical form characterizing the uniformly observable single-output systems [13-15].

#### C. Canonical form of an observer

We consider systems that can be put in the following form:

$$\begin{cases} \dot{x} = Ax + \varphi(u, x) \\ y = Cx = x^{1} \end{cases}$$
(6)

The exit  $y \in \Re^P$ , the input  $u \in \Re^m$  and  $\varphi(u,x)$  a triangular structure with respect to x, i.e :

$$\varphi(u, x) = \begin{pmatrix} \varphi^{1}(u, x^{1}) \\ \varphi^{2}(u, x^{1}, x^{2}) \\ \vdots \\ \varphi^{q}(u, x) \end{pmatrix}$$

The matrix A of  $n \times n$  dimension is the anti-offset matrix block:

$$A = \begin{bmatrix} 0_{p} & I_{p} & 0_{p} & & 0_{p} \\ \vdots & & I_{p} & & 0_{p} \\ 0_{p} & & & \ddots & \vdots \\ 0_{p} & & & 0_{p} & I_{p} \\ 0_{p} & & & 0_{p} & 0_{p} \end{bmatrix}$$
(7)

The observation matrix *C* has the following specific structure:



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$C = \begin{bmatrix} I_p & 0_p & \cdots & 0_p \end{bmatrix}$	(8)
A high gain observer for the system (6) is the following dynamical system:	
$\dot{\hat{x}} = A\hat{x} + \varphi(u, \hat{x}) - \theta \Delta_{\theta}^{-1} S^{-1} C^T (C\hat{x} - y)$	(9)
Where:	
• $\theta > 0$ is the observer adjustment parameter.	
• S is the unique solution of the algebraic Lyapunov equation as follows:	
$S + A^T S + S A = C^T C$	(10)
• $\Delta_{\theta}$ is the following diagonal matrix:	(10)
$\Delta_{\theta} = diag\left(I_{p}, \frac{1}{\theta}I_{p}, \dots, \frac{1}{\theta^{q-1}}I_{p}\right)$	(11)

The solution *S* of equation (10) and the vector  $S^{-1}C^{T}$  can be expressed as follows:

$$S(i, j) = (-1)^{(i+j)} C_{i+j-2}^{j-1} I_p \ 1 \text{ for } i, j \le n$$
(12)

$$S^{-1}C^{T} = \begin{pmatrix} C_{q}^{1}I_{p} \\ C_{q}^{2}I_{p} \\ \vdots \\ C_{q}^{q}I_{p} \end{pmatrix}$$
(13)

Where  $I_P$  is the identity matrix  $P \times P$ .

#### IV. APPLICATION TO THE WEB WINDING SYSTEM (WWS)

### A. Observability analysis of the wws

Consider the following model of the wws:

$$\begin{cases} \dot{x}_{1} = \frac{gr_{1}^{2}}{J_{1}} x_{2} - \frac{1}{\tau_{em1}} x_{1} + \frac{r_{1}}{\tau_{em1}.k_{1}} U_{1} \\ \dot{x}_{2} = -\frac{1}{L} x_{2} x_{3} - \frac{ES}{L} x_{1} + \frac{ES}{L} x_{3} \\ \dot{x}_{3} = -\frac{gr_{2}^{2}}{J_{2}} x_{2} + \frac{gr_{2}^{2}}{J_{2}} x_{4} - \frac{1}{\tau_{em2}} x_{3} + \frac{r_{2}}{\tau_{em2}.k_{2}} U_{2} \\ \dot{x}_{4} = -\frac{1}{L} x_{3} x_{4} - \frac{ES}{L} x_{3} + \frac{ES}{L} x_{5} \\ \dot{x}_{5} = -\frac{gr_{3}^{2}}{J_{3}} x_{4} - \frac{1}{\tau_{em3}} x_{5} + \frac{r_{3}}{\tau_{em3}.k_{3}} U_{3} \end{cases}$$
(14)

The observability matrix for the wws model becomes:



(15)

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	0	1	0	0	0 ]
	$-\frac{ES}{L}$	$-\frac{x_3}{L}$	$-\frac{x_2}{L} + \frac{ES}{L}$	0	0
0 =	0	0	1	0	0
	0	0	0	1	0
	0	0	$-\frac{x_4}{L}-\frac{ES}{L}$	$-\frac{x_3}{L}$	$\frac{ES}{L}$

The WWS observability result is (locally) observable in the sense of rank. Under the local observability criterion, it follows that the model (14) is locally observed in the sense of rank if the *O* matrix is non-singular. For this purpose, it simply checks that the determinant of this matrix is non-zero:

$$\det(O) = \left(\frac{ES}{L}\right)^2 > 0 \tag{16}$$

The determinant is always positive which allows us to deduce that the WWS is locally observable.

#### B. Observer synthesis for the WWS

We proceed to the change of coordinates  $z = \Phi(x)$  as follows:

• First output :

$$z_1^{1} = y_1 = x_2$$
  

$$\dot{z}_1^{1} = -\frac{1}{L} y_1 \cdot y_2 - \frac{ES}{L} x_1 + \frac{ES}{L} y_2$$
  

$$= -\frac{ES}{L} x_1 + \frac{1}{L} (ES - y_1) y_2 = z_2^{1} + \varphi_{11}(y, u)$$

With:

$$\begin{cases} z_2^1 = -\frac{ES}{L} x_1 \\ \varphi_{11}(y,u) = \frac{1}{L} (ES - y_1) y_2 \\ \dot{z}_1^2 = -\frac{ES}{L} \dot{x}_1 = -\frac{ES}{L} \left( \frac{gr_1^2}{J_1} x_2 - \frac{1}{\tau_{em1}} x_1 + \frac{r_1}{\tau_{em1}.k_1} U_1 \right) = \varphi_{12}(z_2^1, y, u) \end{cases}$$

• Second output :

$$z_{1}^{2} = y_{2} = x_{3}$$

$$\dot{z}_{1}^{2} = \dot{x}_{3} = -\frac{gr_{2}^{2}}{J_{2}}x_{2} + \frac{gr_{2}^{2}}{J_{2}}x_{4} - \frac{1}{\tau_{em2}}x_{3} + \frac{r_{2}}{\tau_{em2}.k_{2}}.U_{2} = \varphi_{21}(y, u)$$

• Third output :

 $z_1^3 = y_3 = x_4$ 



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$$\dot{z}_{1}^{3} = \dot{x}_{4} = -\frac{1}{L}y_{2}y_{3} - \frac{ES}{L}y_{2} + \frac{ES}{L}x_{5} = \frac{ES}{L}x_{5} - \frac{1}{L}(ES - y_{3})y_{2}$$
$$= z_{2}^{3} + \varphi_{31}(y, u)$$

With:

$$\begin{cases} z_2^3 = \frac{ES}{L} x_5 \\ \varphi_{31}(y, u) = -\frac{1}{L} (ES - y_3) y_2 \\ \dot{z}_2^3 = \frac{ES}{L} \dot{x}_5 = \frac{ES}{L} \left( -\frac{gr_3^2}{J_3} x_4 - \frac{1}{\tau_{em3}} x_5 + \frac{r_3}{\tau_{em3}} . U_3 \right) \\ = \varphi_{32}(z_2^3, y, u) \end{cases}$$

The change of coordinates leads to:

$$z = \begin{pmatrix} z^{1} \\ z^{2} \\ z^{3} \end{pmatrix} = \begin{pmatrix} \Phi_{1}(x) \\ \Phi_{2}(x) \\ \Phi_{3}(x) \end{pmatrix} = \Phi(x)$$
(17)

Such as:

$$\begin{cases} z^{1} = \Phi_{1}(x) = \begin{pmatrix} z_{1}^{1} \\ z_{2}^{1} \end{pmatrix} = \begin{pmatrix} x_{2} \\ -\frac{ES}{L} x_{1} \end{pmatrix} \\ z^{2} = \Phi_{2}(x) = z_{1}^{2} = x_{3} \\ z^{3} = \Phi_{3}(x) = \begin{pmatrix} z_{1}^{3} \\ z_{2}^{3} \end{pmatrix} = \begin{pmatrix} x_{4} \\ \frac{ES}{L} x_{5} \end{pmatrix} \end{cases}$$

We then find the following overall system:

$$\begin{cases} \dot{z}^{1} = A_{1}z^{1} + \varphi_{1}(\tilde{z}^{1}, y, U_{1}) \\ \dot{z}^{2} = A_{2}z^{2} + \varphi_{2}(\tilde{z}^{2}, y, U_{2}) \\ \dot{z}^{3} = A_{3}z^{3} + \varphi_{3}(\tilde{z}^{3}, y, U_{3}) \end{cases}$$
$$\begin{cases} y_{1} = C_{1}.z^{1} \\ y_{1} = C_{2}.z^{2} \\ y_{3} = C_{3}.z^{3} \end{cases}$$

where :

(18)



(19)

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$$\begin{cases} \tilde{z}^{1} = z_{2}^{1} \\ \tilde{z}^{2} = 0 \\ \tilde{z}^{3} = z_{2}^{3} \end{cases}$$
  
and:  
$$\begin{cases} A_{1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = A_{3} \\ A_{2} = 0 \end{cases}$$
  
An observer from (18) is of the form:  
$$(\dot{z} = A\hat{z} + \alpha(\hat{z} \times U) + S^{-1}C^{T}(C\hat{z} = v))$$

 $\begin{cases} \hat{z} = A\hat{z} + \varphi(\hat{z}, y, U) + S^{-1}C^{T}(C\hat{z} - y) \\ \dot{S} = -\theta S - A^{T}S - SA + C^{T}C \end{cases}$ 

where:

$$\begin{cases} \hat{z}^{1} = A_{1}\hat{z}^{1} + \varphi_{1}\left(y, \hat{z}^{1}, U_{1}\right) - S_{\theta_{1}}^{-1}C_{1}^{T}\left(C_{1}\hat{z}^{1} - y_{1}\right) \\ \hat{z}^{2} = A_{2}\hat{z}^{2} + \varphi_{2}\left(y, \hat{z}^{2}, U_{2}\right) - S_{\theta_{2}}^{-1}C_{2}^{T}\left(C_{2}\hat{z}^{2} - y_{2}\right) \\ \hat{z}^{3} = A_{3}\hat{z}^{3} + \varphi_{3}\left(y, \hat{z}^{3}, U_{3}\right) - S_{\theta_{3}}^{-1}C_{3}^{T}\left(C_{3}\hat{z}^{3} - y_{3}\right) \end{cases}$$

The observer for the initial system:

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x}) \cdot u - \left[\frac{\partial \Phi}{\partial x}(\hat{x})\right] S_{\theta}^{-1} C^{T} \left(C\hat{x} - y\right)$$
<sup>(20)</sup>

#### V. DIGITAL RESULTS

The presentation of nonlinear observer are developed in this paper and illustrated through simulation. Figure 2 (a), (c) and (e) shows, respectively, the input, the work roll and the exit strip velocity. (b) and (d)are the upstream and downstream web tension.



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Fig.2. Comparison between the model and its estimated of each wws parameter through the proposed observer

It has noted that the good performance of the proposed observer was achieved despite the small initial gap between the model parameters and the estimated parameters. This proves that the local nature of the observer's stability will not be change in practice.

#### VI. CONCLUSION

In this paper, we have proposed a high-gain observer for the joint estimation of the whole state and all unknown entries for a class of nonlinear multi-output systems. The synthesis of this observer took place under certain sufficient conditions arising directly necessary and sufficient conditions established for linear systems.

Note that the proposed conditions can be relaxed if one allows to calculate the derivative with respect to time outputs and/or inputs known to express the unmeasured variables [18]. It can also release if the synthesis of an observer of the state without adopting any assumption about the unknown input is no longer set as [17] goal. In our work, these conditions were adopted to synthesize observers in the absence or non-assumption(s) on the unknown inputs, and using no derivative known input or output for the estimation of variables.



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