

Optimal Solution of Transportation Problem Based on Revised Distribution Method

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ABSTRACT: Transportation problem is a special type of linear programming problem. The objective of this paper is to find an optimal solution for the transportation problem which having objective function is to be maximized using new approach Revised Distribution Method, and Vogel's Approximation Method. This new approach Revised Distribution Method is based on allocating units to the cells in the transportation matrix starting with minimum requirement or availability to the cell with minimum cost in the transportation matrix and then try to find an optimum solution to the given transportation problem .the proposed method is easy to apply both type of balanced and unbalanced transportation problem with maximize or minimize objective function.

KEYWORDS: Transportation problem. Linear programming problem, Vogel's Method, REDI Method,

I. INTRODUCTION

The problem of transportation or distribution arises due to shipment of goods to the destination of their requirement from various sources of the origin .The transportation model or distribution model is also a part of linear programming as it also inherits the objective of minimization of cost or maximization of profit. In the case of large scale production an industry may have production centers at various locations where from the goods are sent to the warehouses or godowns further send these goods to the distribution centers for the supply to customers. In transportation model the procedure of sending goods from one destination to another is pre- determined along with their respective cost of transportation .the transportation model mainly deals that how the cost of transportation can be minimized or the revenue of transportation can be maximized by satisfying the requirement of various destination within the known constraints of different sources of supply. The transportation model was first presented by FL Hitchcock in 1941. It was further developed by TC Koopmans (1949) and GB Dantzing (1951) towards the formulation and solution of linear programming problem. It is why the transportation model is regarded as a specific type of linear programming problem which analyse the transportation of certain homogeneous goods or services from their different sources of origins to their different destination of requirements.

II.MATHEMATICAL FORMULATION OF TRANSPORTATION PROBLEM

Suppose that there are m sources and n destination. let a_i be the number of supply units available at source i ($i = 1, 2, \dots, m$) and let b_j be the number of demands units required at destination j ($j= 1, 2, \dots, n$) let c_{ij} represent the unit transportation cost for transporting the units from sources i to destination j . the objective is to determine the number of units to be transported from source i to destination j so that the total transportation cost is minimum .if x_{ij} is the number of units shipped from source i to destination j , then the equivalent lpp model will be

$$\text{Minimize (Total cost)} \quad Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to the constraint

$$\sum_{j=1}^n x_{ij} = a_i \quad i= 1, 2 \dots n \text{ (supply constraints)}$$

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$$\sum_{i=1}^m x_{ij} = b_j \quad j= 1, 2, \dots, m \text{ (demand constraints)}$$

and $x_{ij} \geq 0$ for all i and j

Existence of feasible solution a necessary and sufficient condition for a feasible solution to the transportation problem is

$$\text{Total supply} = \text{Total Demand}$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

The representation of constraints equation i.e. supply constraints and demand constraints in the matrix form is very useful in simplification to find the optimal solution that minimize the total transportation cost (table 1)

Origin	Destination s					Supply
	1	2	3	j	n	
1	C_{11} x_{11}	C_{12} x_{12}	C_{13} x_{13}	C_{1j} x_{1j}	C_{1n} x_{1n}	a_1
2	C_{21} x_{21}	C_{22} x_{22}	C_{23} x_{23}	C_{2j} x_{2j}	C_{2n} x_{2n}	a_2
3	C_{31} x_{31}	C_{32} x_{32}	C_{33} x_{33}	C_{3j} x_{3j}	C_{3n} x_{3n}	a_3
i	C_{i1} x_{i1}	C_{i2} x_{i2}	C_{i3} x_{i3}	C_{ij} x_{ij}	C_{in} x_{in}	a_i
m	C_{m1} x_{m1}	C_{m2} x_{m2}	C_{m3} x_{m3}	C_{mj} x_{mj}	C_{mn} x_{mn}	a_m
Demand	b_1	b_2	b_3	b_j	b_n	

Table 1

III. TERMINOLOGY USED IN TRANSPORTATION MODEL

Feasible Solution- Non negative valves of X_{ij} where $i = 1, 2, \dots, m$ and $j= 1, 2, \dots, n$ which satisfy the constraints of supply and demand is called feasible solution

Basic Feasible Solution- if the no of positive allocation are $(m+n -1)$ where $m =$ number of rows and $n =$ number of column.

Optimal Solution- a feasible solution is said to be optimal solution if it minimize total transportation cost

Balanced Transportation Problem - a transportation problem in which the total supply from all sources is equal to the total demand in all the destinations.

Unbalanced Transportation Problem- problem which are not balance are called unbalanced.

Matrix Terminology- in the matrix the square are called cells and form columns vertically and rows horizontally.

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None degenerate basic feasible solution- A basic feasible solution to a $m \times n$ transportation problem is said to be non degenerate if

1. The total number of non negative allocation is exactly $m+n-1$ and
2. These $m+n-1$ allocation are in independent position

Degenerate Basic Feasible Solution- if the no. of allocation in basic feasible solution is less than $m+n-1$.

IV. A NEW APPROACH FOR SOLVING TRANSPORTATION PROBLEM

This section presents Revised Distribution Method to solve the Maximization type transportation problem which is different from the preceding method. Revised Distribution method is also easy to apply both type of balanced [1] and unbalanced transportation problem [2].

Step 1. Start with converting maximization problem into a minimization problem by subtracting all the elements from the highest element in the given transportation table. The modified transporting minimization problem can be solved in the usual manner.

Step 2. Start with the minimum value in the supply column and demand row. If tie occurs, then select the demand or supply value with least cost [7].

Step 3. Compare the figure of available supply in the row and demand in the column and allocate the units equal to capacity or demand whichever is less.

Step 4. If the demand in the column is satisfied, move to the next minimum value in the demand row and supply column.

Step 5. Repeat steps 2 and 3 until capacity condition of all the sources demand conditions of all destination have been satisfied.

Example. the following transportation problem to maximize the profit

Sources	Destination				Supply
	D ₁	D ₂	D ₃	D ₄	
S ₁	15	51	42	33	23
S ₂	80	42	26	81	44
S ₃	90	40	66	60	33
Demand	23	31	16	30	

Table 1. Transportation problem to maximize the profit

Since the given problem is to maximize the profit we convert this into minimization type by subtracting all the elements (profit values) from the highest profit value of 90. Hence we have the following loss matrix (table 2)

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Sources	Destination				Supply
	D ₁	D ₂	D ₃	D ₄	
S ₁	75	39	48	57	23
S ₂	10	48	64	9	44
S ₃	0	50	24	30	33
Demand	23	31	16	30	100

Table 2. Transportation Matrix of the given balanced problem

In the table3 the minimum value of the demand row and supply column is 16.the cell contains smallest cost in the D₃ column is 24 ie cell (S₃, D₃). Hence allocate the entire 16 units to this cell i.e cell (S₃, D₃).

Sources	Destination				Supply
	D ₁	D ₂	D ₃	D ₄	
S ₁	75	39	48	57	23
S ₂	10	48	64	9	44
	0	50	16	30	33
S ₃			24		
Demand	23	31	16	30	100

Table 3 First allocation of the Transportation Matrix

In the table 4 the minimum value of the demand row and supply column is 17.the cell contains smallest cost in the D₁ column is 10 ie cell (S₃, D₁). Hence allocate the entire 17 units to this cell i.e cell (S₃, D₁).

Sources	Destination			Supply
	D ₁	D ₂	D ₄	
S ₁	75	39	57	23
S ₂	10	48	9	44
S ₃	17	0	30	17
Demand	23	31	30	84

Table 4 Second allocation of the Transportation Matrix

In the table5 the minimum value of the demand row and supply column is 6.the cell contains smallest cost in the D₁ column is 10 ie cell (S₂, D₁). Hence allocate the entire 6 units to this cell i.e cell (S₂, D₁).

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Sources	Destination			Supply
	D ₁	D ₂	D ₄	
S ₁	75	39	57	23
S ₂	6	10	48	9
Demand	6	31	30	67

Table 5 Third allocation of the Transportation Matrix

In the table 6 the minimum value of the demand row and supply column is 23. the cell contains smallest cost in the S₁ row is 39 i.e cell (S₁, D₂). Hence allocate the entire 23 units to this cell i.e cell (S₁, D₂).

Sources	Destination		Supply
	D ₂	D ₄	
S ₁	23	39	57
S ₂		48	9
Demand	31	30	61

Table 6 fourth allocation of the Transportation Matrix

In the table 7 the minimum value of the demand row and supply column is 8. the cell contains smallest cost in the D₂ column is 48 i.e cell (S₂, D₂). Hence allocate the entire 8 units to this cell i.e cell (S₂, D₂).

Sources	Destination		Supply
	D ₂	D ₄	
S ₂	8	48	9
Demand	8	30	38

Table 7 Fifth allocation of the Transportation Matrix

In the table 8 the remaining value of the demand row and supply column is 30. the cell contains smallest cost in the S₂ row is 9 i.e cell (S₂, D₄). Hence allocate the entire 30 units to this cell i.e cell (S₂, D₄).

Sources	Destination	Supply
	D ₄	
S ₂	30	9
Demand	30	30

Table 8 Sixth allocation of the Transportation Matrix

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Thus using REDI Method the basic feasible solution can be obtained as shown in table 9

Sources	Destination				Supply
	D ₁	D ₂	D ₃	D ₄	
S ₁		23	48	57	23
	75	39			
S ₂	6	8		30	44
	10	48	64	9	
S ₃	17		16	30	33
	0	50	24		
Demand	23	31	16	30	100

Table 9

Transportation cost is - $23 \times 51 + 6 \times 80 + 8 \times 42 + 81 \times 30 + 90 \times 17 + 16 \times 66$
 $= 1173 + 480 + 336 + 2430 + 1530 + 1056$
 $= 7005$

Basic Feasible solution by Vogel's Approximation Method (Table 10)

Sources	Destination				Supply
	D ₁	D ₂	D ₃	D ₄	
S ₁		23	48	57	23
	75	39			
S ₂	6	8		30	44
	10	48	64	9	
S ₃	17		16	30	33
	0	50	24		
Demand	23	31	16	30	100

Table 10

Transportation cost is - $23 \times 51 + 6 \times 80 + 8 \times 42 + 81 \times 30 + 90 \times 17 + 16 \times 66$
 $= 1173 + 480 + 336 + 2430 + 1530 + 1056$
 $= 7005$

Test for optimality

Once an initial solution is obtained the next step is to check its optimality in terms of feasibility of the solution and total minimum transportation cost. The modified distribution method is used to calculate opportunity cost associated with each unoccupied cell and then improving the current solution leading to an optimal solution. Hence using the MODI Method Table 11 shown the value of u_i and v_j containing the cost associated with the cells for which allocation have been made.

The opportunity cost for each of the occupied cell is determine by using the relation

$$d_{ij} = c_{ij} - (u_i + v_j)$$

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$$d_{11} = c_{11} - (u_1 + v_1) = 75 - (-9 + 10) = 74$$

$$d_{13} = c_{13} - (u_1 + v_3) = 48 - (-9 + 34) = 23$$

$$d_{14} = c_{14} - (u_1 + v_4) = 57 - (-9 + 9) = 57$$

$$d_{23} = c_{23} - (u_2 + v_3) = 64 - (0 + 34) = 30$$

$$d_{32} = c_{32} - (u_3 + v_2) = 50 - (-10 + 48) = 12$$

$$d_{34} = c_{34} - (u_3 + v_4) = 30 - (-10 + 9) = 31$$

Sources	Destination				Supply	u_i
	D ₁	D ₂	D ₃	D ₄		
S ₁	75	23 39	48	57	23	$u_1 = -9$
S ₂	6 10	8 48	64	30 9	44	$u_2 = 0$
S ₃	17 0	50	16 24	30	33	$u_3 = -10$
Demand	23	31	16	30	100	
v_j	$v_1 = 10$	$v_2 = 48$	$v_3 = 34$	$v_4 = 9$		

Table 11

According to the optimality criterion for cost minimizing transportation problem the solution by REDI method is optimal. Since the opportunity costs of the unoccupied cells are zero or positive and minimum total transportation cost of Rs. 7005.

V CONCLUSION

In this paper maximize type transportation problem is solve by REDI method , and Vogel’s methods and conclude that comparatively Vogel’s method, REDI method is simple and easy to understand . This method can be used for both type of balanced and unbalanced transportation problem which is having objective function of minimize or maximize.

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