

On The Transcendental Equation with Six Unknowns

$$\sqrt[2]{x^2 + 3y^2} + \sqrt[4]{X^3 + Y^3} = z^2 + w^2$$

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ABSTRACT: The transcendental equation with six unknowns represented by the Diophantine equation $\sqrt[2]{x^2 + 3y^2} + \sqrt[4]{X^3 + Y^3} = z^2 + w^2$ is analysed for its patterns of non-zero distinct integral solutions.

KEYWORDS: Transcendental equation, integral solutions, surd equation. M.Sc 2000 Mathematics subject classification: 11D99.

Notations:

$t_{m,n}$ – Polygonal numbers of rank n with sides m.

ky_n – Kynea number of rank n.

J_n – Jacobsthal number of rank n.

j_n – Jacobsthal-Lucas number of rank n.

I. INTRODUCTION

Diophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems are algebraic equations [1,2,3]. It seems that much work has not been done to obtain integral solutions of transcendental equations. In this context one may refer [5-18]. This communication analyses a transcendental equation with six unknowns given by $\sqrt[2]{x^2 + 3y^2} + \sqrt[4]{X^3 + Y^3} = z^2 + w^2$ for its integer solutions.

II. METHOD OF ANALYSIS

The Transcendental equation with six unknowns under consideration is given by

$$\sqrt[2]{x^2 + 3y^2} + \sqrt[4]{X^3 + Y^3} = z^2 + w^2 \tag{1}$$

Introduction of the transformations,

$$\left. \begin{aligned} x &= 3u^2 - v^2, y = 2uv, \\ X &= u(u^3 + v^3), Y = v(u^3 + v^3) \end{aligned} \right\} \tag{2}$$

in (1) leads to

$$u^2(u + 3) + v^2(1 + v) = z^2 + w^2 \tag{3}$$

To solve (3), it is seen that, $u+3$ is a perfect square when $u = k^2 + 2k - 2$ and $1+v$ is a perfect square when $v = k^2 + 2k$.

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Thus, the values of z and w satisfying (3) may be taken as

$$z = z(k) = (k^2 + 2k - 2)(k + 1) \tag{4}$$

$$w = w(k) = (k^2 + 2k)(k + 1) \tag{5}$$

Substituting the above values of u and v in (2), the corresponding values of x , y , X and Y are obtained as

$$x = x(k) = 3(k^2 + 2k - 2)^2 - (k^2 + 2k)^2 \tag{6}$$

$$y = y(k) = 2(k^2 + 2k)(k^2 + 2k - 2) \tag{7}$$

$$X = X(k) = (k^2 + 2k - 2)[(k^2 + 2k - 2)^3 + (k^2 + 2k)^3] \tag{8}$$

$$Y = Y(k) = (k^2 + 2k)[(k^2 + 2k - 2)^3 + (k^2 + 2k)^3] \tag{9}$$

Therefore, (4) to (9) represent the non-zero distinct integer solutions to (1).

Properties:

1. $4(Y - X)$ is written as the sum of two squares.
2. $(4k^2 + 4k - 2) \frac{Y(2k)}{X(2k)} = 8\alpha_{3,k}$
3. $\frac{yX}{Y} + 4 = 2(ky_n)^2$.
4. $(x - y) = 24J_{2n} + 24J_n + 20$.
5. Each of the following expressions is a Nasty Number [4].
 - (i). $3(x - y + 20)$
 - (ii). $6(2x + 4)$
 - (iii). $\frac{3yY}{X}$
 - (iv). $6 \left[1 + (k^2 + 2k - 2) \frac{Y}{X} \right]$
 - (v). $\frac{3yX}{Y}$.

However, (3) is also solved through applying a linear transformation as illustrated below:

Introducing the linear transformations

$$v = u + 2, z = r + s, w = r - s, (r \neq s \neq 0) \tag{10}$$

in (3), it becomes

$$u^3 + 5u^2 + 8u + 6 = r^2 + s^2 \tag{11}$$

After performing a few numerical calculations, it is observed that (11) is satisfied by

$$\left. \begin{aligned} r &= k^3 + 3k^2 + k - 1 \\ s &= k + 1 \\ u &= k^2 + 2k - 2 \end{aligned} \right\} \tag{12}$$

In view of (10), we have

$$v = k^2 + 2k$$

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$$\left. \begin{aligned} z &= k^3 + 3k^2 + 2k \\ w &= k^3 + 3k^2 - 2 \end{aligned} \right\} \quad (13)$$

Substituting the above values of u and v in (2), the corresponding values of x, y, X and Y are given by

$$\left. \begin{aligned} x &= x(k) = 2[k^4 + 4k^3 - 2k^2 - 12k + 6] \\ y &= y(k) = 2[k^4 + 4k^3 + 2k^2 - 4k] \\ X &= X(k) = [k^2 + 2k - 2][(k^2 + 2k - 2)^3 + (k^2 + 2k)^3] \\ Y &= Y(k) = [k^2 + 2k][(k^2 + 2k - 2)^3 + (k^2 + 2k)^3] \end{aligned} \right\} \quad (14)$$

Thus, (13) & (14) represent the corresponding non-zero distinct integer solutions to (1). The values of r, s and u satisfying (11) are presented in Table 1 below:

Table 1:

S. No	r	s	u
1	1	7	2
2	3	5	-1
3	2	-2	3
4	8	3	11
5	7	9	-8
6	$k[\alpha + k + 2]$	$k + 1$	$\alpha - 1$

In the above table $\alpha = k^2 + 2k - 1$.

Substituting the above values of r, s, and u in (10) and employing (2), the corresponding distinct integer solutions to (1) are presented in Table 2 below:

Table 2:

S. No	v	z	w	x	y	X	Y
1	4	8	-6	-4	16	144	288
2	1	8	-2	2	-2	0	0
3	5	0	4	2	30	456	760
4	13	11	5	194	286	38808	45864
5	-6	16	-2	156	96	5824	4368
6	$\alpha + 1$	$k[\alpha + k + 3]$	$k[\alpha + k + 1] - 2$	$2\alpha^2 - 4\alpha + 1$	$2\alpha^2 - 1$	$(\alpha - 1)^4 + (\alpha^2 - 1)(\alpha + 1)^2$	$(\alpha + 1)^4 + (\alpha^2 - 1)(\alpha - 1)^2$

III. CONCLUSION

In this paper, we have presented infinitely many non-zero distinct integer solutions to the transcendental equation with six unknowns given by $\sqrt[2]{x^2 + 3y^2} + \sqrt[4]{X^3 + Y^3} = z^2 + w^2$. As these equations are rich in variety, one may search for other forms of transcendental equations with six or more unknowns for their integer solutions.

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