

Observations on Derived K-Fibonacci and Derived K- Lucas Sequences

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ABSTRACT: In this paper, we present some remarkable properties of derived k- Fibonacci and derived k- Lucas numbers. The identities are not considered earlier.

KEYWORDS: Derived k-Fibonacci sequence and derived k- Lucas sequence, Binet's formula. 2010 Mathematics

Subject Classification: 11B39, 11B83

I. INTRODUCTION

It is well known that the Fibonacci sequence is famous for its wonderful and amazing properties. Fibonacci composed a number text in which he did important work in number theory and the solution of algebraic equations. The equation of rabbit problem posed by Fibonacci is known as the first mathematical model for population growth. From the statement of rabbit problem, the famous Fibonacci numbers can be derived. This sequence of Fibonacci numbers is extremely fruitful and appears in different areas in mathematics and science.

The Fibonacci sequence, Lucas sequence, Pell sequence, Pell – Lucas sequence, Jacobsthal sequence and Jacobsthal – Lucas sequence are most prominent examples of recursive sequences.

The Fibonacci sequence [3] is defined by the recurrence relation $F_k = F_{k-1} + F_{k-2}, k \geq 2$ with $F_0 = 0, F_1 = 1$. The Lucas sequence [3] is defined by the recurrence relation $L_k = L_{k-1} + L_{k-2}, k \geq 2$ with $L_0 = 2, L_1 = 1$

The second order recurrence sequence has been generalized in two ways mainly, first by preserving the initial conditions and second by preserving the recurrence relation. In this context, one may refer [9].

D.Kalman and R.Mena [2] generalize the Fibonacci sequence by $F_n = aF_{n-1} + bF_{n-2}, n \geq 2$ with $F_0 = 0, F_1 = 1$

A.F.Horadam[1] defined generalized the Fibonacci sequence $\{H_n\}$ by $H_n = H_{n-1} + H_{n-2}, n \geq 3$ with $H_1 = p, H_2 = p + q$, where p and q are arbitrary integers.

B.Singh, O.Sikhwal and S.Bhatnagar [12], defined Fibonacci like sequence by recurrence relation $S_k = S_{k-1} + S_{k-2}, k \geq 3$ with $S_0 = 2, S_1 = 2$. The associated initial conditions S_0 and S_1 are the sum of the Fibonacci and Lucas sequences respectively. i.e, $S_0 = F_0 + L_0$ and $S_1 = F_1 + L_1$.

L.R.Natividad [11], Deriving a formula in solving Fibonacci like sequence. He found missing terms in Fibonacci like sequence and solved by standard formula.

V.K.Gupta, V.Y.Panwar and O.Sikhwal [7], defined generalized Fibonacci sequences and derived its identities connection formulae and other results. V.K.Gupta, V.Y.Panwar and N.Gupta [6], stated and derived identities for Fibonacci like sequence. Also, described and derived connection formulae and negation formulae for Fibonacci like sequence. B.Singh, V.K.Gupta and V.Y.Panwar [4], present many combination of higher powers of Fibonacci like sequence.

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The k-Fibonacci numbers defined by Falco 'n Plaza.A [5], depending only on one integer parameter k as follows: For any positive real number k, the k-Fibonacci sequence is defined recurrently by $F_{k,n} = kF_{k,n-1} + F_{k,n-2}, n \geq 2$ with $F_{k,0} = 0, F_{k,1} = 1$

In [10], A.D.Godase and M.B.Dhakne have presented some properties of k-Fibonacci and k-Lucas numbers by using matrices.

In [8], Yashwant K.Panwar, G.P.Rathore and Richa chawla have established some interesting properties of k-Fibonacci like numbers.

In this paper, we introduce the derived k-Fibonacci and derived k-Lucas sequences. We present some properties of derived k-Fibonacci and derived k-Lucas numbers. Also, we illustrate the method of obtaining Diophantine quadruples with property D(1) and D(4) wherein the members of the quadruples are represented by derived k-Fibonacci and derived k-Lucas numbers.

Further, we derive telescoping series for derived k-Fibonacci and derived k-Lucas sequences.

II. METHOD OF ANALYSIS

Definition :1 Derived k-Fibonacci sequence

For any positive real number k, the derived k-Fibonacci sequence $\{F_{k,n}\}_{n=1}^{\infty}$ is defined as, $F_{k,0} = 0, F_{k,1} = 1$ and $F_{k,n+1} = kF_{k,n} - F_{k,n-1}$ for $n \geq 1$. Binet form for $F_{k,n}$ is

$$F_{k,n} = \frac{r_1^n - r_2^n}{r_1 - r_2} \text{ where } r_1 + r_2 = k, r_1 r_2 = 1$$

Definition :2 Derived k-Lucas sequence

For any positive real number k, the derived k-Lucas sequence $\{L_{k,n}\}_{n=1}^{\infty}$ is defined as, $L_{k,0} = 2, L_{k,1} = k$ and $L_{k,n+1} = kL_{k,n} - L_{k,n-1}$ for $n \geq 1$. Binet form for $L_{k,n}$ is

$$L_{k,n} = r_1^n + r_2^n \text{ where } r_1 + r_2 = k, r_1 r_2 = 1$$

First 15 derived k-Fibonacci numbers are given below.

$$F_{k,2} = k$$

$$F_{k,3} = k^2 - 1$$

$$F_{k,4} = k^3 - 2k$$

$$F_{k,5} = k^4 - 3k^2 + 1$$

$$F_{k,6} = k^5 - 5k^3 + 3k$$

$$F_{k,7} = k^6 - 5k^4 + 6k^2 - 1$$

$$F_{k,8} = k^7 - 6k^5 + 10k^3 - 4k$$

$$F_{k,9} = k^8 - 7k^6 + 15k^4 - 10k^2 + 1$$

$$F_{k,10} = k^9 - 8k^7 + 21k^5 - 20k^3 + 5k$$

$$F_{k,11} = k^{10} - 9k^8 + 28k^6 - 35k^4 + 15k^2 - 1$$

$$F_{k,12} = k^{11} - 10k^9 + 36k^7 - 56k^5 + 35k^3 - 6k$$

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$$F_{k,13} = k^{12} - 11k^{10} + 45k^8 - 84k^6 + 70k^4 - 21k^2 + 1$$

$$F_{k,14} = k^{13} - 12k^{11} + 55k^9 - 120k^7 + 126k^5 - 56k^3 + 7k$$

$$F_{k,15} = k^{14} - 13k^{12} + 66k^{10} - 165k^8 + 210k^6 - 126k^4 + 28k^2 - 1$$

First 15 derived k-Lucas numbers are given below.

$$L_{k,2} = k^2 - 2$$

$$L_{k,3} = k^3 - 3k$$

$$L_{k,4} = k^4 - 4k^2 + 2$$

$$L_{k,5} = k^5 - 5k^3 + 5k$$

$$L_{k,6} = k^6 - 6k^4 + 9k^2 - 2$$

$$L_{k,7} = k^7 - 7k^5 + 14k^3 - 7k$$

$$L_{k,8} = k^8 - 8k^6 + 20k^4 - 16k^2 + 2$$

$$L_{k,9} = k^9 - 9k^7 + 27k^5 - 30k^3 + 9k$$

$$L_{k,10} = k^{10} - 10k^8 + 35k^6 - 50k^4 + 25k^2 - 2$$

$$L_{k,11} = k^{11} - 11k^9 + 44k^7 - 77k^5 + 55k^3 - 11k$$

$$L_{k,12} = k^{12} - 12k^{10} + 54k^8 - 112k^6 + 105k^4 - 36k^2 + 2$$

$$L_{k,13} = k^{13} - 13k^{11} + 65k^9 - 156k^7 + 182k^5 - 91k^3 + 13k$$

$$L_{k,14} = k^{14} - 14k^{12} + 77k^{10} - 210k^8 + 294k^6 - 196k^4 + 49k^2 - 2$$

$$L_{k,15} = k^{15} - 15k^{13} + 90k^{11} - 275k^9 + 450k^7 - 378k^5 + 140k^3 - 15k$$

Properties of derived k- Fibonacci and derived k- Lucas sequences:

1. $F_{k,n+1} - F_{k,n-1} = L_{k,n}$
2. $L_{k,n+1} - L_{k,n-1} = (k^2 - 2)F_{k,n}$
3. $F_{k,2n} = F_{k,n}L_{k,n}$
4. $L_{k,2n} + 2 = L_{k,n}^2$
5. $L_{k,3n} + 3L_{k,n} = L_{k,n}^3$
6. $F_{k,n+1}L_{k,n-1} - F_{k,2n} = k$
7. $L_{k,2n} = (k^2 - 4)F_{k,n}^2 + 2$
8. $L_{k,2n+1} = (k^2 - 4)F_{k,n}F_{k,n+1} + k$

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9. $L_{k,2n} = (k^2 - 4)F_{k,n+1}F_{k,n-1} + k^2 - 2$
10. $F_{k,n}L_{k,m} + F_{k,m}L_{k,n} = 2F_{k,m+n}$
11. $F_{k,m}L_{k,n} - F_{k,n}L_{k,m} = 2F_{k,m-n}$
12. $L_{k,m}L_{k,n} = L_{k,m+n} + L_{k,m-n}$
13. $L_{k,m+n}L_{k,m-n} = L_{k,2m} + L_{k,2n}$
14. $F_{k,m+n} - F_{k,m-n} = L_{k,m}F_{k,n}$
15. $(k^2 - 4)F_{k,n}^2 L_{k,2n} + 1 = [L_{k,2n} - 1]^2$
16. $L_{k,n}^2 - (k^2 - 4)F_{k,n}^2 = 4$
17. $L_{k,n}^4 - (k^2 - 4)^2 F_{k,n}^4 = 8L_{k,2n}$
18. $(k^2 - 4)F_{k,n}^2 L_{k,n}^2 + 4 = L_{k,2n}^2$
19. $L_{k,4n} + L_{k,4n+4} = (k^2 - 4)[F_{k,2n}^2 + F_{k,2n+2}^2] + 4$
20. $L_{k,4n} = [L_{k,n}^2 - 2]^2 - 2$

Construction of Diophantine quadruples with property D (1)

Let $a = (k + 2)F_{k,n}L_{k,2n}$, $b = (k - 2)F_{k,n}$ be two numbers such that $ab + 1 = [L_{k,2n} - 1]^2 = p^2$ (say).
Therefore, the pair (a, b) represents Diophantine 2-tuple with property D(1).

Let $c = a + b + 2L_{k,2n} - 2$

It is noted that $ac + 1 = [(k + 2)F_{k,n}L_{k,2n} + L_{k,2n} - 1]^2 = q^2$ (say)

$$bc + 1 = [(k - 2)F_{k,n} + L_{k,2n} - 1]^2 = r^2$$
 (say)

Thus, the triple (a, b, c) is Diophantine 3-tuple with property D(1).

If d is the fourth tuple then, by Euler's formula

$$d = 4(L_{k,2n} - 1)^3 + 4(L_{k,2n} - 1)^2[(k + 2)F_{k,n}L_{k,2n} + (k - 2)F_{k,n}] + 4(L_{k,2n} - 1)(k^2 - 4)F_{k,n}^2 L_{k,2n}$$

Thus, the quadruple (a, b, c, d) represents Diophantine quadruple with property D(1) as the product of any two members of the above set added with unity is a perfect square.

In a similar manner, one can observe that the quadruple

$$\{(k^2 - 4)F_{k,n}, F_{k,n}L_{k,2n}, (k^2 - 4)F_{k,n} + F_{k,n} L_{k,2n} + 2L_{k,2n} - 2,$$

$4(L_{k,2n} - 1)^3 + 4(L_{k,2n} - 1)^2[F_{k,n}L_{k,2n} + (k^2 - 4)F_{k,n}] + 4(L_{k,2n} - 1)(k^2 - 4)F_{k,n}^2 L_{k,2n}\}$ is Diophantine quadruple with property D (1).

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Construction of Diophantine quadruples with property D (4)

Let $a = (k + 2)F_{k,n}^2$, $b = (k - 2)L_{k,n}^2$ be two numbers such that $ab + 4 = L_{k,2n}^2 = p^2$ (say). Therefore, the pair (a, b) represents Diophantine 2-tuple with property D(4).

$$\text{Let } c = a + b + 2L_{k,2n}$$

It is noted that $ac + 4 = [(k + 2)F_{k,n}^2 + L_{k,2n}]^2 = q^2$ (say)

$$bc + 4 = [(k - 2)L_{k,n}^2 + L_{k,2n}]^2 = r^2$$
 (say)

Thus, the triple (a, b, c) is Diophantine 3-tuple with property D(4).

If d is the fourth tuple then, by Euler's formula

$$d = (k + 2)F_{k,n}^2 L_{k,2n}^2 + (k - 2)L_{k,n}^2 L_{k,2n}^2 + 2L_{k,2n}^3 - 4L_{k,2n}$$

Thus, the quadruple (a, b, c, d) represents Diophantine quadruple with property D(4) as the product of any two members of the above set added with 4 is a perfect square.

In a similar manner, one can observe that the quadruple

$\{(k^2 - 4)F_{k,n}^2, L_{k,2n}^2, 4L_{k,2n}, 3L_{k,2n}^3 + (k^2 - 4)F_{k,n}L_{k,2n} * (F_{k,n} + L_{k,2n} + 1)\}$ is Diophantine quadruple with property D(4).

Note:

Also, one can observe that the quadruple $\{(k + 2)F_{k,2n}, (k - 2)F_{k,2n}, 2kF_{k,2n} + 2L_{k,2n},$

$2L_{k,2n}^3 + 2kF_{k,2n}L_{k,2n}^2 - 4L_{k,2n}\}$ is Diophantine quadruple with property D(4).

III. TELESCOPING SERIES FOR DERIVED K- FIBONACCI AND DERIVED K- LUCAS SEQUENCES:

In this section, we present theorems exhibiting telescoping series for derived k- Fibonacci and derived k- Lucas sequences.

Theorem: 1

If $m, n \geq 0$, then

$$\sum_{i=1}^n \frac{1}{F_{k,im} F_{k,(i+1)n}} = \frac{1}{2F_{k,m}} \left[\frac{L_{k,m}}{F_{k,m}} - \frac{L_{k,m(n+1)}}{F_{k,m(n+1)}} \right]$$

Proof:

Replacing n by nm and m by (n+1)m in property (11), we have

$$2F_{k,m} = F_{k,(n+1)m} L_{k,nm} - L_{k,(n+1)m} F_{k,nm} \tag{1}$$

which we obtain

$$\frac{F_{k,m}}{F_{k,nm} F_{k,(n+1)m}} = \frac{1}{2} \left[\frac{L_{k,nm}}{F_{k,nm}} - \frac{L_{k,m(n+1)}}{F_{k,m(n+1)}} \right] \tag{2}$$

Replacing n by i in (2) and taking the summation from i=1 to n, we have

$$\begin{aligned} \sum_{i=1}^n \frac{1}{F_{k,im} F_{k,(i+1)n}} &= \frac{1}{2F_{k,m}} \sum_{i=1}^n \left[\frac{L_{k,im}}{F_{k,im}} - \frac{L_{k,m(i+1)}}{F_{k,m(i+1)}} \right] \\ &= \frac{1}{2F_{k,m}} \left[\frac{L_{k,m}}{F_{k,m}} - \frac{L_{k,m(n+1)}}{F_{k,m(n+1)}} \right] \quad \text{(on expanding)} \end{aligned}$$

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Theorem: 2

If $m, n \geq 0$, then

$$\sum_{i=0}^n \frac{1}{L_{k,im} L_{k,(i+1)m}} = \frac{F_{k,m(n+1)}}{2F_{k,m} L_{k,(n+1)m}}$$

Proof:

From (1), we have

$$\frac{F_{k,m}}{L_{k,nm} L_{k,m(n+1)}} = \frac{1}{2} \left[\frac{F_{k,m(n+1)}}{L_{k,m(n+1)}} - \frac{F_{k,nm}}{L_{k,nm}} \right] \tag{3}$$

Replacing n by i in (3) and taking the summation from i=0 to n, we have

$$\begin{aligned} \sum_{i=0}^n \frac{1}{L_{k,im} L_{k,(i+1)m}} &= \frac{1}{2F_{k,m}} \sum_{i=0}^n \left[\frac{F_{k,m(i+1)}}{L_{k,m(i+1)}} - \frac{F_{k,im}}{L_{k,im}} \right] \\ &= \frac{1}{2F_{k,m}} \left[\frac{F_{k,m(n+1)}}{L_{k,m(n+1)}} - \frac{F_{k,0}}{L_{k,0}} \right] \text{ (on expanding)} \\ &= \frac{F_{k,m(n+1)}}{2F_{k,m} L_{k,(n+1)m}} \quad (\because F_{k,0} = 0) \end{aligned}$$

Theorem: 3

If $m, n \geq 0$, then

$$\sum_{i=1}^n \frac{F_{k,(2i+1)m}}{F_{k,(i+1)m}^2 F_{k,im}^2} = \frac{F_{k,nm} F_{k,(n+2)m}}{F_{k,m}^3 F_{k,(n+1)m}^2}$$

Proof:

Replacing n by nm and m by (n+1)m in property (10), we have

$$2F_{k,(2n+1)m} = F_{k,(n+1)m} L_{k,nm} + F_{k,nm} L_{k,(n+1)m} \tag{4}$$

from which we get

$$\frac{F_{k,(2n+1)m}}{F_{k,nm} F_{k,(n+1)m}} = \frac{1}{2} \left[\frac{L_{k,nm}}{F_{k,nm}} - \frac{L_{k,m(n+1)}}{F_{k,m(n+1)}} \right] \tag{5}$$

Multiplying (2) and (5), we have

$$\frac{F_{k,m} F_{k,(2n+1)m}}{F_{k,nm}^2 F_{k,(n+1)m}^2} = \frac{1}{4} \left[\frac{L_{k,nm}^2}{F_{k,nm}^2} - \frac{L_{k,m(n+1)}^2}{F_{k,m(n+1)}^2} \right]$$

Replacing n by i in the above equation and taking the summation from i=1 to n, we have

$$\begin{aligned} \sum_{i=1}^n \frac{F_{k,(2i+1)m}}{F_{k,(i+1)m}^2 F_{k,im}^2} &= \frac{1}{4F_{k,m}} \sum_{i=1}^n \left[\frac{L_{k,im}^2}{F_{k,im}^2} - \frac{L_{k,m(i+1)}^2}{F_{k,m(i+1)}^2} \right] \\ &= \frac{1}{4F_{k,m}} \left[\frac{L_{k,m}^2}{F_{k,m}^2} - \frac{L_{k,m(n+1)}^2}{F_{k,m(n+1)}^2} \right] \text{ (on expanding)} \end{aligned}$$

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$$= \frac{1}{F_{k,m}^3 F_{k,(n+1)m}^2} \left[\left(F_{k,(n+1)m} L_{k,m} + L_{k,(n+1)m} F_{k,m} \right) * \left(F_{k,(n+1)m} L_{k,m} - L_{k,(n+1)m} F_{k,m} \right) \right] \quad (6)$$

Now, replacing m by (n+1)m, n by m in property (10) and (11), we have

$$2F_{k,(n+2)m} = F_{k,(n+1)m} L_{k,m} + L_{k,(n+1)m} F_{k,m} \quad (7) \quad \text{and}$$

$$2F_{k,nm} = F_{k,(n+1)m} L_{k,m} - L_{k,(n+1)m} F_{k,m} \quad (8)$$

Using (7) and (8) in (6), we have

$$\begin{aligned} \sum_{i=1}^n \frac{F_{k,(2i+1)m}}{F_{k,(i+1)m}^2 F_{k,im}^2} &= \frac{1}{4F_{k,m}^3 F_{k,(n+1)m}^2} \left[2F_{k,(n+2)m} \quad 2F_{k,nm} \right] \\ &= \frac{F_{k,nm} F_{k,(n+2)m}}{F_{k,m}^3 F_{k,(n+1)m}^2} \end{aligned}$$

Theorem: 4

If $m, n \geq 0$, then

$$\sum_{i=0}^n \frac{F_{k,(2i+1)m}}{L_{k,(i+1)m}^2 L_{k,im}^2} = \frac{F_{k,(n+1)m}^2}{4F_{k,m} L_{k,(n+1)m}^2}$$

Proof:

From (4), we have

$$\frac{F_{k,(2n+1)m}}{L_{k,nm} L_{k,(n+1)m}} = \frac{1}{2} \left[\frac{F_{k,m(n+1)}}{L_{k,m(n+1)}} + \frac{F_{k,nm}}{L_{k,nm}} \right] \quad (9)$$

Multiplying (3) and (9), we have

$$\frac{F_{k,m} F_{k,(2n+1)m}}{L_{k,nm}^2 L_{k,(n+1)m}^2} = \frac{1}{4} \left[\frac{F_{k,m(n+1)}^2}{L_{k,m(n+1)}^2} - \frac{F_{k,nm}^2}{L_{k,nm}^2} \right] \quad (10)$$

Replacing n by i in the above equation and taking the summation from i=0 to n, we have

$$\begin{aligned} \sum_{i=0}^n \frac{F_{k,(2i+1)m}}{L_{k,(i+1)m}^2 L_{k,im}^2} &= \frac{1}{4F_{k,m}} \sum_{i=0}^n \left[\frac{F_{k,m(i+1)}^2}{L_{k,m(i+1)}^2} - \frac{F_{k,im}^2}{L_{k,im}^2} \right] \\ &= \frac{F_{k,(n+1)m}^2}{4F_{k,m} L_{k,(n+1)m}^2} \quad (\text{on expanding}) \end{aligned}$$

Theorem: 5

If $m, n \geq 0$, then

$$\sum_{i=0}^n \frac{F_{k,(2i+1)m} [F_{k,(2i+1)m}^2 + F_{k,m}^2]}{L_{k,im}^4 L_{k,(i+1)m}^4} = \frac{F_{k,(n+1)m}^4}{8L_{k,(n+1)m}^4}$$

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Proof:

Replace n by i in (9) and (3)

$$\begin{aligned} \text{Now, } (9)^2 + (3)^2 &\Rightarrow \frac{F_{k,(2i+1)m}^2 + F_{k,m}^2}{L_{k,im}^2 L_{k,(i+1)m}^2} = \frac{1}{4} \left[\left(\frac{F_{k,m(i+1)}}{L_{k,m(i+1)}} + \frac{F_{k,im}}{L_{k,im}} \right)^2 + \left(\frac{F_{k,m(i+1)}}{L_{k,m(i+1)}} - \frac{F_{k,im}}{L_{k,im}} \right)^2 \right] \\ &= \frac{1}{2} \left[\frac{F_{k,m(i+1)}^2}{L_{k,m(i+1)}^2} + \frac{F_{k,im}^2}{L_{k,im}^2} \right] \end{aligned} \tag{11}$$

Replace n by i in (10) and multiplying the resulting equation by (11), we have

$$\frac{F_{k,(2i+1)m} [F_{k,(2i+1)m}^2 + F_{k,m}^2]}{L_{k,im}^4 L_{k,(i+1)m}^4} = \frac{1}{8} \left[\frac{F_{k,m(i+1)}^4}{L_{k,m(i+1)}^4} - \frac{F_{k,im}^4}{L_{k,im}^4} \right]$$

In the above equation taking the summation i=0 to n, we have

$$\begin{aligned} \sum_{i=0}^n \frac{F_{k,(2i+1)m} [F_{k,(2i+1)m}^2 + F_{k,m}^2]}{L_{k,im}^4 L_{k,(i+1)m}^4} &= \frac{1}{8} \sum_{i=0}^n \left[\frac{F_{k,m(i+1)}^4}{L_{k,m(i+1)}^4} - \frac{F_{k,im}^4}{L_{k,im}^4} \right] \\ &= \frac{F_{k,(n+1)m}^4}{8L_{k,(n+1)m}^4} \text{ (on expanding)} \end{aligned}$$

IV. CONCLUSION

In this paper, we have stated and derived many properties of derived k- Fibonacci and derived k- Lucas sequences through Binet’s formulae. Also, we have constructed Diophantine quadruples with property D(1) and D(4) where the numbers of the quadruples are derived k- Fibonacci and derived k- Lucas numbers. Finally, we present some telescoping series involving derived k- Fibonacci and derived k- Lucas numbers.

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