

Finite Difference Method in Immiscible Double Phase Fluid Flow through Porous Media

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ABSTRACT: The present paper deals with the problem to study the stabilization of instabilities (fingers) arising in the immiscible double phase fluid flow through porous media. It contains a numerical solution of a governing equation of well-known fingering phenomenon arising in double phase flow through homogeneous porous media under certain initial and boundary conditions. Numerical solution of a governing partial differential equation is obtained using a finite difference method.

KEYWORDS: Double Phase, Fingering, Porous Media, FDM

I. INTRODUCTION

Fingers arise are not of same in size and shape. So it is difficult to obtain the governing equation. Therefore for the convenience to obtain a governing equation, only average cross section area occupied by the fingers is taken into account. Adrian E. Scheidegger and Edward F. Johnson [1] have discussed the statistical behaviour of instabilities in displacement processes in porous media. Verma (1964) [2] has examined the behaviour of fingering phenomena in a displacement process through heterogeneous porous media.

A fluid, contained in a porous media, is known as native fluid. Instead of regular displacement of whole front, when a native fluid is displaced by another fluid of lesser viscosity, perturbations occur which shoot through the porous medium at relatively great speed. This occurrence of instabilities is known as fingering phenomenon. Such a phenomenon takes place in oil reservoir.

According to (Scheidegger, A.E., 1960) [3], fingers may be stabilized in homogeneous media, by statistical view point. Many authors, have discussed this phenomenon in different ways.

II. STATEMENT OF THE PROBLEM

A physical region is considered as a square domain in the horizontal plane [4]. At the points of the opposite corners along one of the diagonals, low and high pressure is maintained. When water is injected at high pressure, oil is displaced towards the low-pressure production zone. The subject matter of investigation is to examine the displacement of oil by water through the porous medium in a given amount of time and the appearance of fingers [5]. Due to the viscous forces in the fluid phases and the capillary force at their interface, the resistance to water-oil movement arises. Viscosity of oil is larger than viscosity of water. Therefore the viscosity ratio of heavy oil to water is large. A mathematical model is based on the individual fluid pressures and capillary effects. The present study is applicable oil-rich reservoir. A qualitative study has been carried out to explore the effect of permeability variations on the flow patterns. The problem yields with the non-linear partial differential equation. Successive over Relaxation method is used to convert the Partial differential equation into difference equation [6].

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III. MATHEMATICAL FORMULATION

Let us consider porous medium of homogeneous physical characteristic. Also let us consider water injected with constant velocity into a dipping oil saturated. Therefore oil will be displaced by water. Due to the displacement a well-developed fingers will be arise.

According to Darcy's law, the leakage velocities of injected fluid (water) V_w and native fluid (oil) V_o in the inclined medium is

$$V_w = -\frac{k_w}{\mu_w} k \left[\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta \right] \quad (1)$$

$$V_o = -\frac{k_o}{\mu_o} k \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta \right] \quad (2)$$

Where ρ_w and ρ_o are densities of water and oil respectively which are considered as constant, θ is the inclination of the bed, g is acceleration due to gravity, k is the permeability of the medium, k_w and k_o are the permeability of water and oil respectively and P_w, P_o , are pressure of water and oil respectively.

The equation of continuity of the phases is given by

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad (3)$$

$$\phi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \quad (4)$$

From the definition of phase saturation we have

$$S_w + S_o = 1 \quad (5)$$

Due to the pressure discontinuity of the two phases across the common interface, the capillary pressure, is defined as

$$P_c = P_o - P_w \quad (6)$$

$$\text{or } \frac{\partial P_w}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} \quad (7)$$

Using equations (1) and (2) in equation (3) and (4) respectively. Thus we have

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left\{ \frac{k_w}{\mu_w} k \left[\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta \right] \right\} \quad (8)$$

$$\phi \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x} \left\{ \frac{k_o}{\mu_o} k \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta \right] \right\} \quad (9)$$

Substituting the value of $\frac{\partial P_w}{\partial x}$ from Eq. (7), Eq. (8) reduces to

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left\{ \frac{k_w}{\mu_w} k \left[\frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} + \rho_w g \sin\theta \right] \right\} \quad (10)$$

Now adding Eq. (9) and (10)

$$\phi \frac{\partial (S_w + S_o)}{\partial t} = \frac{\partial}{\partial x} \left\{ k \left[\frac{\partial P_o}{\partial x} \left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o} \right) - \frac{k_w}{\mu_w} \frac{\partial P_c}{\partial x} + g \sin\theta \left(\rho_w \frac{k_w}{\mu_w} + \rho_o \frac{k_o}{\mu_o} \right) \right] \right\} \quad (11)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 8, August 2016

From (5), we have $\frac{\partial(S_w+S_o)}{\partial t} = 0$

Using this in equation (11), we have

$$\frac{\partial}{\partial x} \left\{ k \left[\frac{\partial P_o}{\partial x} \left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o} \right) - \frac{k_w}{\mu_w} \frac{\partial P_c}{\partial x} + g \sin\theta \left(\rho_w \frac{k_w}{\mu_w} + \rho_o \frac{k_o}{\mu_o} \right) \right] \right\} = 0$$

Integrating this equation with respect to 'x', we get

$$k \left[\frac{\partial P_o}{\partial x} \left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o} \right) - \frac{k_w}{\mu_w} \frac{\partial P_c}{\partial x} + g \sin\theta \left(\rho_w \frac{k_w}{\mu_w} + \rho_o \frac{k_o}{\mu_o} \right) \right] = \text{constant}$$

$$k \left[\frac{\partial P_o}{\partial x} \left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o} \right) - \frac{k_w}{\mu_w} \frac{\partial P_c}{\partial x} + g \sin\theta \left(\rho_w \frac{k_w}{\mu_w} + \rho_o \frac{k_o}{\mu_o} \right) \right] = -a \tag{12}$$

Where a is the constant of integration.

From equation (12), obtaining value of $\frac{\partial P_o}{\partial x}$

$$\frac{\partial P_o}{\partial x} = \frac{-a + k \frac{k_w}{\mu_w} \frac{\partial P_c}{\partial x} - k g \sin\theta \left(\rho_w \frac{k_w}{\mu_w} + \rho_o \frac{k_o}{\mu_o} \right)}{k \left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o} \right)} \tag{13}$$

Substituting the value of $\frac{\partial P_o}{\partial x}$ in equation (10) and simplifying, we have

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left\{ \left[\frac{a}{\left(1 + \frac{k_o \mu_w}{\mu_o k_w} \right)} + \frac{k \frac{k_o}{\mu_o} \left[\frac{\partial P_c}{\partial x} + g \sin\theta (\rho_o - \rho_w) \right]}{\left(1 + \frac{k_o \mu_w}{\mu_o k_w} \right)} \right] \right\} = 0 \tag{14}$$

The value of the pressure of oil (P_o) is given by [7]

$$P_o = \bar{P} + \frac{1}{2} P_c \tag{15}$$

Where \bar{P} is the mean pressure which is constant. Therefore, we have

$$\frac{\partial P_o}{\partial x} = \frac{1}{2} \frac{\partial P_c}{\partial x} \tag{16}$$

Substituting the value of $\frac{\partial P_o}{\partial x}$ in equation (12) we get

$$k \left[\frac{1}{2} \frac{\partial P_c}{\partial x} \left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o} \right) - \frac{k_w}{\mu_w} \frac{\partial P_c}{\partial x} + g \sin\theta \left(\rho_w \frac{k_w}{\mu_w} + \rho_o \frac{k_o}{\mu_o} \right) \right] + a = 0$$

$$a = \left[\frac{1}{2} \frac{\partial P_c}{\partial x} k \left(\frac{k_w}{\mu_w} - \frac{k_o}{\mu_o} \right) - g k \sin\theta \left(\rho_w \frac{k_w}{\mu_w} + \rho_o \frac{k_o}{\mu_o} \right) \right] \tag{17}$$

Using value of "a" in equation (14),

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 8, August 2016

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left\{ \left[\frac{\left[\frac{1}{2} \frac{\partial P_c}{\partial x} k \left(\frac{k_w}{\mu_w} - \frac{k_o}{\mu_o} \right) - gk \sin\theta \left(\rho_w \frac{k_w}{\mu_w} + \rho_o \frac{k_o}{\mu_o} \right) \right]}{\left(1 + \frac{k_o \mu_w}{\mu_o k_w} \right)} + \frac{k \frac{k_o}{\mu_o} \left[\frac{\partial P_c}{\partial x} + g \sin\theta (\rho_o - \rho_w) \right]}{\left(1 + \frac{k_o \mu_w}{\mu_o k_w} \right)} \right] \right\} = 0$$

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left(\frac{1}{2} \frac{\partial P_c}{\partial x} k \frac{k_w}{\mu_w} - gk \frac{k_w}{\mu_w} \sin\theta \rho_w \right) = 0 \quad (18)$$

For mathematical analysis, assuming a the condition for capillary pressure depending upon phase saturation as [7]

$$P_c = -\beta S_w, \text{ Where } \beta \text{ is constant} \quad (19)$$

Using (19) in equation (18),it becomes

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left\{ -\frac{\beta}{2} \frac{\partial S_w}{\partial x} \frac{k k_w}{\mu_w} \right\} = 0 \quad (20)$$

The capillary pressure P_c is very small (as zero or b as zero), assuming to be zero, eq.(20) becomes

$$\varphi \frac{\partial S_w}{\partial t} - \frac{\beta k k_w}{2 \mu_w} \frac{\partial^2 S_w}{\partial x^2} = 0 \quad (21)$$

Equation (21) is the equation of motion for saturation, with the boundary condition.

$$S_w(0, t) = s_1, S_w(L, t) = s_2, S_w(x, 0) = 0, 0 \leq x \leq L \quad (22)$$

Which is a governing equation for fingering phenomena.

$$\text{Let } \xi = \frac{x}{L}, \quad T = \frac{\beta k k_w}{2 \mu_w} t$$

$$\frac{\partial S_w}{\partial T} - \frac{\partial^2 S_w}{\partial \xi^2} = 0 \quad (23)$$

With $S_w(0, T) = s_1, S_w(1, T) = s_2, S_w(\xi, 0) = 0, 0 \leq \xi \leq 1$

IV. MATHEMATICAL SOLUTION

Using a well-known finite difference method, Successive over relaxation method to convert partial differential equation into difference equation.

First Using Crank-Nicolson method [8,9]

$$S_{w_{i,j+1}} = S_{w_{i,j}} + \frac{k}{2h^2} \left(S_{w_{i+1,j}} - 2S_{w_{i,j}} + S_{w_{i-1,j}} + S_{w_{i+1,j+1}} - 2S_{w_{i,j+1}} + S_{w_{i-1,j+1}} \right) \quad (24)$$

$$\text{Let } r = \frac{k}{h^2}$$

$$(1+r)S_{w_{i,j+1}} = S_{w_{i,j}} + \frac{r}{2} \left(S_{w_{i+1,j}} - 2S_{w_{i,j}} + S_{w_{i-1,j}} + S_{w_{i+1,j+1}} + S_{w_{i-1,j+1}} \right)$$

$$\text{Let } \lambda_i = S_{w_{i,j}} + \frac{r}{2} \left(S_{w_{i+1,j}} - 2S_{w_{i,j}} + S_{w_{i-1,j}} \right)$$

$$S_{w_{i,j+1}} = (1 - \omega)S_{w_{i,j}} + \omega \left[\frac{r}{2(1+r)} \left(S_{w_{i+1,j}} + S_{w_{i-1,j+1}} \right) + \frac{\lambda_i}{(1+r)} \right] \quad (25)$$

Equation (25) is obtained by S.O.R. method [10].

Choose $k = 0.01, h = 0.1, \omega = 1.87, s_1 = 1, s_2 = 0$

$$S_{w_{i,j+1}} = -0.87 + 1.87 \left[0.25 \left(S_{w_{i+1,j}} + S_{w_{i-1,j+1}} \right) + \frac{\lambda_i}{2} \right] \quad (26)$$

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 5, Issue 8, August 2016

Numerical calculations for different values of saturation at different time and different length are shown in the following table.

T→	T=0.01	T=0.02	T=0.003	T=0.04	T=0.05
$\xi \downarrow$	Sw				
0	1	1	1	1	1
0.1	0.4675	0.735903438	0.7360523	0.801986619	0.773296104
0.2	0.2220625	0.471966086	0.542622588	0.573288196	0.626171065
0.3	0.105479688	0.282329569	0.381472463	0.414601208	0.470335923
0.4	0.050102852	0.161725742	0.255194701	0.288140907	0.342855893
0.5	0.023798854	0.089938846	0.163597156	0.191407971	0.243503149
0.6	0.011304456	0.048952533	0.101274153	0.122154571	0.168265683
0.7	0.005369617	0.026212454	0.060930537	0.075391595	0.112874579
0.8	0.002550568	0.013856916	0.035809088	0.045028835	0.072788597
0.9	0.00121152	0.007249886	0.020392636	0.025145386	0.043204674
1	0.000575472	0.003501243	0.009979079	0.010391833	0.023113629

V. GRAPHICAL REPRESENTATION

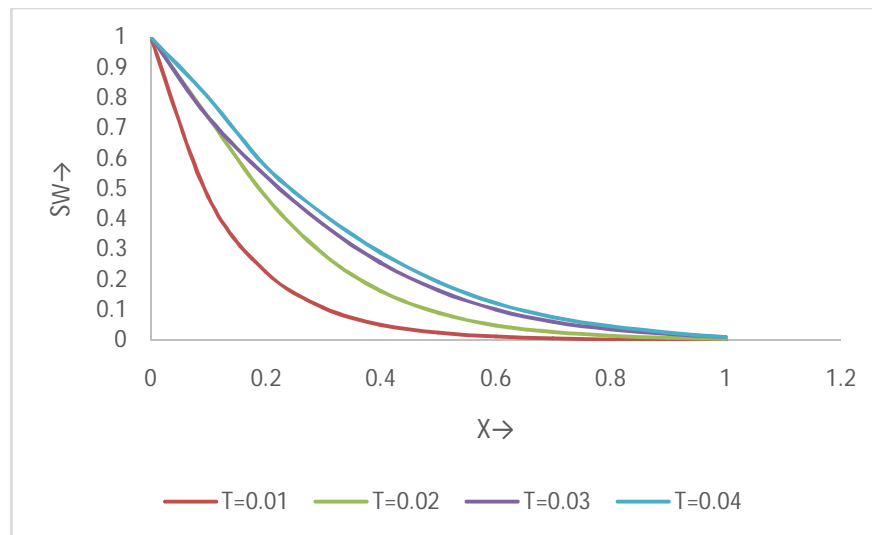


FIGURE-1

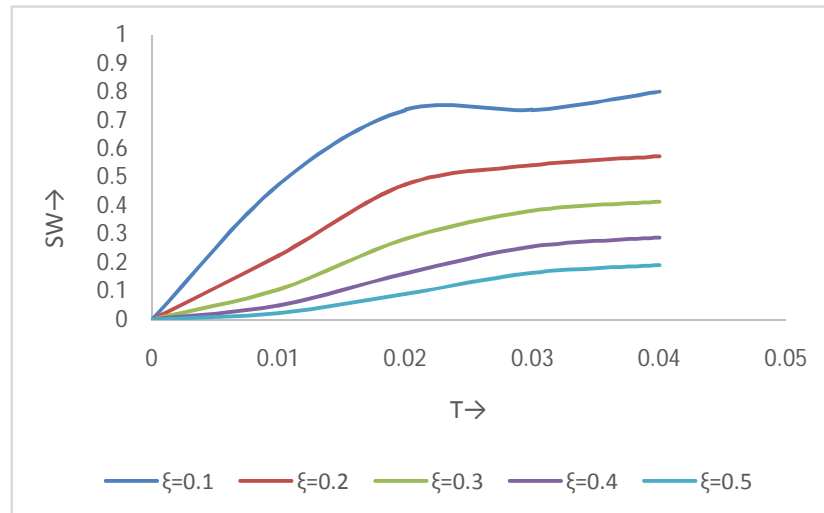


FIGURE-2

VI. CONCLUDING REMARKS

From figure-1 and figure-2 it is very clear that as time increasing, the saturation increasing and as length is increasing saturation is decreasing. The solution obtained is suitable with the physical solution of the problem.

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