

Analysis of Fractal Dimension, Lacunarity and Hence the Suitable Image Quality Metric of Noisy Digital Images

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ABSTRACT: Fractal geometry is one among the variously used important mathematical concept. But its influence on noise of digital images is a topic that has not been covered to a large extent yet. This paper shows the effect of fractal dimension and lacunarity to measure the roughness of noisy digital image when contaminated with varying intensities of Gaussian noise as additive noise, Periodic noise as multiplicative noise and Poisson noise as distributive noise. It further computes the suitable image quality metric. Fractal dimension successfully reveals the roughness of an image, while lacunarity reveals homogeneity or heterogeneity of the image. A resemblance between the two has also been derived. FSIM comes out to be the suitable IQA metric.

KEYWORDS: Fractal dimension, Lacunarity, Gaussian noise, Poisson noise, Periodic noise, IQA, FSIM, FSIMc, SSIM.

I. INTRODUCTION

Fractal dimension and lacunarity, i.e, fractal geometry has been used in many fields of image processing, like, texture analysis, roughness estimation, image segmentation, etc. Mandelbrot introduced and popularized the concepts of fractal geometry in the year 1977.[1] Fractal geometry begins where Euclidean geometry fails. Euclidean geometry handles simple structures but fails to handle complex ones. Complex structures, like, natural coastlines, landscapes, valleys, etc. Fractal geometry serves the purpose. Fractal geometry have been highly used in fields like remote image analysis, texture analysis, object detection, face recognition, human eye tracking, mobile object tracking, video processing. Fractal geometry in noisy digital images is still such a topic which is not yet properly analyzed. T. Pant et al. in his paper has discussed the effect of only one feature of fractal geometry, the fractal dimension on noisy digital images.[12] In this paper, the effect of of two fractal geometrical features, fractal dimension and lacunarity are estimated on noisy digital images injected with varying intensities of Poisson noise, Gaussian noise and Periodic noise. Further, to assess the image quality, it estimates the value of image quality assessment (IQA) metric, Feature Similarity Index (FSIM) and structural similarity (SSIM) for the digital images and compares their RMSE value.

A. FRACTAL DIMENSION

‘Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line...’ Mandelbrot coined the statement and introduced one of the vital scientific concepts of the 20th century, fractal geometry. [1,6]

Fractal is defined as an object having two properties: **self similarity** and **fractal dimension**. Fractal grammatically means irregular and fragmented.

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Self similarity refers to the fact that the object resembles itself whether scaled down or scaled up, i.e., if the object is divided into parts and again integrated, the original object is obtained. Fractal dimension is also called self similar dimension, D.

$$D = \frac{\log N_r}{\log \frac{1}{r}} \tag{1}$$

where, N_r is the number of self similar objects when the object is scaled down by ratio r . Fractals are highly important in remote image analysis since the images are rich in objects like water bodies, land covers, etc.[5] Fractals are also highly important in fields of texture analysis. They are also used in object detection, face recognition, pattern recognition, and many more owing to a very interesting property, the fractal signature. Fractal signature is an unique and important property of fractals.[16] It is useful in almost all applications. **Fractal features** include- **Fractal dimension, self similarity, scaling behavior, lacunarity, Hurst exponent**. Fractal analysis is the study of analysis of all these fractal features.[8]

B. LACUNARITY

Fractal Dimension estimates the geometric complexity of the object's shape. Different objects can have the same value for D though having different appearance. So, Mandelbrot in 1982 proposed Lacunarity to deal with the mentioned problem.[1,8] The difference in the appearance of the object is owing to the different lacunarity in the object.

Lacunarity means the gap-distribution of the object. Objects with low lacunarity means the objects are homogeneous, whereas, high lacunarity means the objects are heterogeneous. An object that is homogeneous at a small window scale can be heterogeneous at a large window scale. That means lacunarity is scale dependent.[15,13]

Lacunarity at scale r is defined as:

$$\Lambda_r(B) = \frac{E[(X_r^B)^2]}{(E[X_r^B])^2} \tag{2}$$

where $X_r^B(n)$ denotes the histogram of the image drawn from the values from the boxes and n is the number of pixels lying in the box.

C. FRACTAL DIMENSION AND LACUNARITY ESTIMATION USING DIFFERENTIAL BOX COUNTING (DBC)

Fractal Dimension can be estimated from the eqn 1. For natural objects not having any definite shape the fractal dimension is estimated analytically. There are a lot of methods present to estimate fractal dimension and lacunarity. In the paper, Differential Box Counting (DBC) technique has been used.

Any method that computes Fractal Dimension follows three major steps:[10]

Step 1: The quantity of the object is measured using various step sizes.

Step 2: A regression line is drawn in accordance to quantity vs. step size. The unit of the axes are in logarithmic scale. This log-log plot is also known as Richardson Plot.

Step 3: The slope of the line is the fractal dimension of the object under consideration.

DBC proposed by Sarkar and Chaudhuri in 1992 estimates D quite efficiently. DBC follows

$$N_r \propto \left(\frac{1}{r}\right)^{-D} \tag{3}$$

where N_r is the count of the number of boxes, r is the ratio of the size length of boxes to the side length of the image.

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An image of size $M * M$ is scaled down to a size $s * s$ where

$$\frac{M}{2} \geq s \geq 1$$

s is an integer.

Therefore,

$$r = \frac{s}{M}$$

For different values of steps, N_r is counted. In the paper, boxes of size $2 * 2$, $3 * 3$, $4 * 4$, $5 * 5$, $6 * 6$ and $7 * 7$ have been used. Then the regression line is plotted and hence the average D is estimated. Lacunarity is computed as the ratio of standard deviation of the histogram values of the image to the square of the expectation of the histogram values of the image.[7,9]

II. NOISE AND ITS TYPES

Noise is a type of disturbance which results in change in the pixel values of an image. Noise can be categorized on basis of distribution, on the nature of correlation, on the nature of noise, and on the basis of source.

Noise can be additive, multiplicative or impulse in nature. In the experiment, Poisson noise as a category of distributive noise, varying intensities of Gaussian noise as additive noise and varying intensities of Periodic noise multiplicative noise have been added. Variances used in case of Gaussian noise are 0.01, 0.001 and 0.005 and variances used in case of Periodic noise are 0.13, 0.22 and 0.78. These values are not randomly considered, several instances are tested and the most suitable ones are used.

A. POISSON NOISE

Poisson noise is generated from the image data itself instead of adding it artificially. The probability distribution function is given as

$$p(z) = \frac{(np)^z}{z!} * e^{-np}$$

n is the number of pixels and p is the ratio of noisy pixels to the total number of pixels.

B. GAUSSIAN NOISE

It is a frequently occurring additive noise in digital images. The probability distribution function is

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{z - \mu}{2\sigma^2}\right)$$

where μ is the mean and σ^2 is the standard deviation.

C. PERIODIC NOISE

It is a multiplicative noise occurring at multiples of a specific frequency. The occurrence of bars over the image indicates the presence of periodic noise.

Three images are taken and have been injected with varying intensities of Gaussian, Periodic and Poisson noise. Some of them are shown below.

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Fig. 1 Image 1 with no noise.

img1gau1



Fig. 2.. Image 1 with Gaussian noise of variance 0.001.



Fig. 3.. Image 2 with no noise.

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Fig. 4.. Image 2 with Periodic noise of variance 0.22.

III. IMAGE QUALITY ASSESSMENT PARAMETERS

A. STRUCTURAL SIMILARITY INDEX (SSIM)

SSIM is a human visual system (HVS) feature based metric proposed by Wang et. al in 2002. The HVS performs many image processing tasks which are superior than other models[22]. SSIM measures the similarity between two images. It is an improvement over methods like Mean Square Error (MSE) and Peak signal to Noise Ratio (PSNR)[18]. It is calculated over several windows of an image as,

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \quad (4)$$

where, μ_x and μ_y are the average of x and y respectively. σ_x^2 and σ_y^2 are the variance of x and y respectively. σ_{xy} is the covariance of x and y.

Constraints $c_1 = (k_1L)^2$ and $c_2 = (k_2L)^2$. L = dynamic range of pixel values = 255 (default). $k_1 = 0.01$ and $k_2 = 0.03$ (default)[18].

B. FEATURE SIMILARITY INDEX (FSIM AND FSIMC)

FSIM index understands an image based on its low level features like, the phase congruency (PC) and gradient magnitude (GM). PC contains high informations and thus is a primary feature in FSIM. GM is used to encode contrast information. Sobel or Prewitt or Scharr operator is used to compute gradient magnitude. FSIM is for grayscale images or the lumic components of the coloured images. FSIMc is for colored images.[19,21,22]

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IV. CALCULATIONS

Table 1 shows the fractal dimension and lacunarity value of the original non-noisy images.

TABLE I
FRACTAL DIMENSION AND LACUNARITY VALUE OF NON-NOISY IMAGES

	Fractal Dimension	Lacunarity
Img1	1.7980	0.0529
Img2	1.8607	0.0264
Img3	1.8263	0.0351

Table 2 shows the fractal dimension value of the noisy images.

TABLE II
FRACTAL DIMENSION VALUE OF NOISY IMAGES

Fractal Dimension	Gaussian	noise		Periodic	noise		Poisson
variances	0.001	0.005	0.01	0.13	0.22	0.78	
Img1	1.95	2.042	2.004	1.7515	1.8150	2.1322	1.9475
Img2	1.9443	1.975	1.9343	1.8373	1.8369	2.0265	1.9621
Img3	1.9208	2.0012	1.9596	1.6914	1.7714	2.0963	1.9895

Table 3 shows the lacunarity value of the noisy images.

TABLE III
LACUNARITY VALUE OF NOISY IMAGES

Lacunarity	Gaussian	noise		Periodic	noise		Poisson
variances	0.001	0.005	0.01	0.13	0.22	0.78	
Img1	0.0300	0.014	0.014	0.0377	0.0313	0.166	0.0336
Img2	0.0186	0.115	0.0083	0.0259	0.024	0.0163	0.0183
Img3	0.0399	0.0238	0.0187	0.0374	0.04	0.0318	0.034

Table 4 shows the FSIM, FSIMc, and SSIM of all the noisy images.

TABLE IV
FSIM, FSIMc, SSIM VALUE OF NOISY IMAGES

IMAGES	FSIM	FSIMc	SSIM
IMG1GAU1	0.5745	0.5453	0.2829
IMG1GAU2	0.5831	0.5536	0.2479
IMG1GAU3	0.5844	0.5550	0.2247
IMG1PER1	0.5682	0.5392	0.2956
IMG1PER2	0.5902	0.5606	0.2225
IMG1PER3	0.5892	0.5593	0.2467
IMG1POISSON	0.5769	0.5477	0.2641
IMG2GAU1	0.5771	0.5545	0.2139
IMG2GAU2	0.5795	0.5567	0.1984
IMG2GAU3	0.5778	0.5544	0.1887
IMG2PER1	0.5755	0.5532	0.2229
IMG2PER2	0.5793	0.5563	0.1914

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IMG2PER3	0.5850	0.5620	0.2024
IMG2POISSON	0.5792	0.5565	0.2085
IMG3GAU1	0.6191	0.6002	0.6078
IMG3GAU2	0.5816	0.5643	0.5251
IMG3GAU3	0.5586	0.5421	0.4691
IMG3PER1	0.6332	0.6136	0.6282
IMG3PER2	0.5587	0.5422	0.4659
IMG3PER3	0.5852	0.5677	0.5180
IMG3POISSON	0.6020	0.5838	0.5719

Table 5 shows the Root Mean Square Error (RMSE) value for FSIM, FSIMc, and SSIM of the noisy images.

TABLE V
RMSE VALUE OF NOISY IMAGES

IQA	RMSE
FSIM	3.6312e-012
FSIMc	4.8994e-012
SSIM	5.6028e-012

V. CONCLUSION

Table 1 shows the fractal dimension (FD) value of the three non-noisy images. It can be seen that $FD(I1) < FD(I3) < FD(I2)$. As fractal dimension is a measure of roughness of an image, so it can be concluded that image 1 is less rough than the other two images and Image2 is the roughest image.

It also shows the lacunarity (LAC) values. Higher the lacunarity value, more the image is heterogeneous. While, the less the value, the image is homogeneous. $LAC(I1) > LAC(I3) > LAC(I2)$. Image1 is more heterogeneous than all the other images and Image2 is the most homogeneous image than the other images.

Table 2 shows the fractal dimension value of the noisy images. It can be seen that with the increase in the content of noise the fractal dimension also increases. It means, the image with lesser noise content is less rough than the image with higher noise content. The fact can be clearly visualized from table 2 for both Gaussian noise and Periodic noise. For image 1 the less rough version of it is the one that is corrupted with periodic noise of variance 0.13. For image 2 the less rough version of it is the one that is corrupted with periodic noise of variance 0.22. For image 3 the less rough version of it is the one that is corrupted with periodic noise of variance 0.13.

Table 3 shows the lacunarity value of the noisy images. More the lacunarity, more is heterogeneity of the image. Image 1 with periodic noise of variance 0.78 is the most heterogeneous image than all other corrupted forms of image 1. Image 2 with gaussian noise of variance 0.005 is the most heterogeneous image than all other corrupted forms of image 2. Image 3 with gaussian noise with variance 0.001 is the most heterogeneous image than all other corrupted forms of image 3.

Table 4 shows the FSIM, FSIMc and SSIM values of all the noisy images.

Table 5 concludes the suitable IQA metric according to Root Mean Square Error (RMSE). RMSE is defined as the root mean square difference of value obtained and the provided subjective value. Low RMSE value means a good IQA metric. Less the value of RMSE, the more good is the metric. So, the most suitable metric for this particular is FSIM.

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