

Analysis of FM/M (a,b)/1/MWV Queueing Model

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ABSTRACT: In this paper we analyse the general bulk service queueing model to find the mean queue length (L_q), probability that the system is in vacation (P_v) and the probability that the system is in busy state (P_{busy}) are expressed in terms of crisp value for FM/M(a,b)/1 under multiple working vacations with fuzzy numbers. The bulk service rule is applied. The most general bulk service rule is introduced by Neuts[14]. The batches are served according to FCFS discipline. By considering the arrival rate, service rate for busy period, service rate for vacation period, exponential distribution of vacation parameters are as trapezoidal fuzzy numbers and the basic idea is to convert all these fuzzy numbers into crisp values by applying Robust Ranking Technique. Further Robust Ranking technique is used to find the expected mean queue length. Moreover, the analytical results are numerically illustrated for L_q , P_v and P_{busy} under crisp environment for the different values of the parameters.

KEYWORDS: Bulk Service, Robust Ranking Technique, Multiple Working Vacation, Fuzzy number and Crisp value.

I. INTRODUCTION

In queueing models, there are situations particularly in transportation system, where the service provided is such that a group of customers can be served simultaneously. Examples include shuttle bus service, freight trains, express elevators, tour guides and batch servicing in manufacturing process. The bulk service rule introduced by Neuts[14] is the most general one and this has been further examined by many authors. Neuts[15] evaluated the matrix analytic methods in queueing theory. The analytic solutions for M/M(a,b)/1 queue, $E_k/M(a,b)/1$ queue and GI/M(a,b)/1 queue with single servers and multiple vacation has obtained by Afthab Begum[1] in her Ph.D thesis. The theory of batch service queues commenced with the works of Bailey[2]. He considered a queue with poisson arrival and fixed size of service. Later many authors have examined a variety of extensions of the basic model. Borthakur et al[3] has examined the bulk service rule.

Later Servi and Finn[16] introduced a class of semi vacation policies, in which servers work at a lower rate rather than completely stopping primary service during vacation, such a vacation called working vacation(WV). The M/M/1 queueing system with working vacation has been analysed by Servi and Finn[16] and Tian et al[17]. Most of the general bulk service queueing models with servers vacation have been analysed by many authors using matrix geometric method. Tian, Li and Zhang[18] provided a survey of the results of working vacation queues. Wu and Tackagi[19] later extended Servi and Finn's M/M/1/WV to M/G/1/MWV model. Moreover, Julia Rose Mary and Afthab Begum[4] have analysed the Markovian M/M(a,b)/1 queueing model under multiple working vacation and derived the steady state probability distribution and the mean queue length for the model.

In literature we have many methods for converting fuzzy into crisp for which Robust ranking technique is the most successful and very convenient method for converting fuzzy to crisp values. Kao et al[7] developed a general approach for queueing systems in a fuzzy environment based on Zadeh's extension principle, which reduces fuzzy queue into family of crisp values. Li and Lee[8] investigated analytical results for two fuzzy queues using a general approach based on Zadeh's extension principle. The M/M/1 queueing system with working vacation has been analysed by Liu et al[9]. Analytic solution of bulk service queueing models can be found in Madhil and Choudhury[10] and Medhi[11]. Nagi and Lee[12] proposed a procedure using α -cut and two variables simulations to analyse fuzzy queues, using

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parametric programming. Moreover, A. NagoorKhani and V. AshokKumar[13] have analysed bulk arrival fuzzy queues with fuzzy outputs.

Ranking Technique has been discussed by researchers like R.R Yager and S.P.Chen[20]. Zhang and Xu[21] have analysed the M/M/1 queue with multiple working vacations and N-Policy. Recently Julia Rose Mary and Angel Jenitta[5] have studied the cost analysis for bi level threshold policy and single vacation of an unreliable server with fuzzy parameters and Julia Rose Mary and Majula Christina[6] have also investigated analysis of total average cost for $M_{(m,n)}^x/M/1/BD/MV$ with fuzzy parameters using Robust Ranking technique. With the help of these available literatures we determine M/M(a,b)/1/MWV with fuzzy parameters.

II. MODEL DESCRIPTION

In this model it is assumed that the arrival process is Poisson with parameter λ . The server processes the customers in batches according to the GBSR. According to the rule the server starts service only when a minimum of 'a' customers are present in the system. If the server after a service completion finds 'a' or more but at most 'b' customers present in the system then he takes them all in a batch, and if he finds more than b, then he takes in the batch the first 'b' customers for service, while others wait. Each batch service contains a minimum 'a' server and a maximum 'b' servers.

Whenever the server completes a service and finds less than 'a' customer in the queue he begins a vacation which is an exponentially distributed random variable V with parameter η . After completing a vacation if the system length is still less than 'a' he takes another vacation and the vacations are continued until the server finds atleast 'a' customers in the queue. Suppose during vacation if the queue size becomes atleast 'a' the server starts his service under the GBSR with service rate μ_v which is different from the regular service rate μ . When the vacation ends he switches his service rate from μ_v to μ .

Suppose the arrival rate λ , service rate μ for busy period, service rate μ_v for vacation period, exponential distribution of vacation parameters η . Minimum number of customers in the queue is a, and a maximum number of customers in the queue is b are approximately known and can be represented as fuzzy set $\bar{\lambda}, \bar{\mu}, \bar{\mu}_v$, and $\bar{\eta}$ where,

$$\bar{\lambda} = \{t, \theta_{\bar{\lambda}}(t) / \theta_{\bar{\lambda}}(t) / t \in S(\bar{\lambda})\}, \bar{\mu} = \{x, \theta_{\bar{\mu}}(x) / x \in S(\bar{\mu})\}, \bar{\mu}_v = \{s, \theta_{\bar{\mu}_v}(s) / s \in S(\bar{\mu}_v)\} \text{ and } \bar{\eta} = \{u, \theta_{\bar{\eta}}(u) / u \in S(\bar{\eta})\}$$

Here $\theta_a(b)$ and $S(a)$ denote the membership function and support of a where $a = \bar{\lambda}, \bar{\mu}, \bar{\mu}_v, \bar{\eta}$ are fuzzy numbers and $b = t, x, s, u$ are the crisp values corresponding to arrival rate, service rate for busy period, service rate for vacation period and for vacation parameter respectively.

On the basis of the concept of α -cut we develop a mathematical programming approach for deriving the α -cuts for $\bar{\lambda}, \bar{\mu}, \bar{\mu}_v$ and $\bar{\eta}$ as crisp intervals which are given by

$$\bar{\lambda}(\alpha) = \{t \in T / \theta_{\bar{\lambda}}(t) \geq \alpha\}, \bar{\mu}(\alpha) = \{x \in X / \theta_{\bar{\mu}}(x) \geq \alpha\}, \bar{\mu}_v(\alpha) = \{s \in S / \theta_{\bar{\mu}_v}(s) \geq \alpha\}, \bar{\eta}(\alpha) = \{u \in U / \theta_{\bar{\eta}}(u) \geq \alpha\}$$

where $0 < \alpha \leq 1$. Hence a fuzzy queue can be reduced to a family of crisp queues with different α -cuts $\{\lambda(\alpha) / 0 < \alpha \leq 1\}, \{\mu(\alpha) / 0 < \alpha \leq 1\}, \{\mu_v(\alpha) / 0 < \alpha \leq 1\}$ and $\{\eta(\alpha) / 0 < \alpha \leq 1\}$.

Let the confidence interval of the fuzzy sets $\lambda(\alpha), \mu(\alpha), \mu_v(\alpha)$ and $\eta(\alpha)$ be $[l_{\lambda}(\alpha), u_{\lambda}(\alpha)], [l_{\mu}(\alpha), u_{\mu}(\alpha)], [l_{\mu_v}(\alpha), u_{\mu_v}(\alpha)]$ and $[l_{\eta}(\alpha), u_{\eta}(\alpha)]$. According to the classical M/M(a,b)/1/MWV model,

the expected queue length (L_q) is given by, $L_q = AH(r, \mu) + BH(r_v, \mu) + H(r_v, \mu_v)$

$$\text{where } H(x, y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda(1-x)} \left[\frac{a(a-1) + ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right]$$

$$A = \frac{(1-r)}{\mu(1-r^a)} \left[\frac{\eta}{(1-r_v)} - \frac{B\mu(1-r_v^a)}{(1-r_v)} \right], \quad B = \frac{\eta r_v}{\mu r_v(1-r_v^b) + \lambda(r_v - 1)}$$

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By applying the fuzzy variable for arrival rate, service rate for busy, service rate for vacation, and vacation parameter then we get,

The expected queue length (L_q) is given as, $L_q = AH(q, x) + BH(p, x) + H(p, s)$

$$\text{where } H(x, y) = \frac{x}{(1-x)^2} + \frac{y}{t(1-x)} \left\{ \frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right\}$$

Similarly, the probability that the system is in vacation (P_v) is given by, $P_v = \frac{Q_0}{(1-r_v)}$

and the probability that the system is in busy state (P_{busy}) is given by, $P_{\text{busy}} = \left(\frac{A}{(1-r)} + \frac{B}{(1-r_v)} \right) Q_0$

where Q_0 is given by, $Q_0 = \frac{1}{F(r_v, \mu_v) + AF(r, \mu) + BF(r_v, \mu)}$, with $F(x, y) = \frac{1}{(1-x)} \left(1 + \frac{y}{\lambda} \left(a - \frac{x(1-x^a)}{1-x} \right) \right)$

$$A = \frac{(1-q)}{x(1-q^a)} \left[\frac{u}{(1-p)} - \frac{Bx(1-p^a)}{(1-p)} \right] \quad \text{and} \quad B = \frac{up}{xp(1-p^b) + t(p-1)}$$

III. ROBUST RANKING TECHNIQUE

By giving a convex fuzzy number \bar{a} , the Robust Ranking index is defined by,

$$R(\bar{a}) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U) d\alpha \quad \text{where } (a_\alpha^L + a_\alpha^U) \text{ is the } \alpha\text{-level cut of the fuzzy number } \bar{a}. \text{ In this paper we use Robust}$$

Ranking method for ranking the fuzzy numbers. The Robust Ranking index $R(\bar{a})$ gives the representative value of the fuzzy number \bar{a} . It satisfies the linearity and additive property.

IV. NUMERICAL EXAMPLE

By considering M/M(a,b)/1/MWV queueing system for the expected mean queue length (L_q), the corresponding parameters such as arrival rate, service rate for busy, service rate for vacation, vacation parameters are as fuzzy numbers, we have $\lambda = [0.2, 0.3, 0.4, 0.5]$, $\mu = [0.05, 0.15, 0.25, 0.35]$, $\mu_v = [0.002, 0.004, 0.006, 0.008]$ and $\eta = [0.01, 0.02, 0.03, 0.04]$ whose intervals of confidence are $[0.2 + \alpha \cdot 0.5 - \alpha]$, $[0.05 + \alpha \cdot 0.35 - \alpha]$, $[0.002 + \alpha \cdot 0.008 - \alpha]$ and $[0.01 + \alpha \cdot 0.04 - \alpha]$ respectively. The membership function of the Trapezoidal fuzzy number (0.2, 0.3, 0.4, 0.5) is given by

$$\theta(p) = \begin{cases} \frac{p-0.2}{0.1}, & 0.2 \leq p \leq 0.3 \\ 0.2, & 0.3 \leq p \leq 0.4 \\ \frac{0.5-p}{0.1}, & 0.4 \leq p \leq 0.5 \\ 0, & \text{otherwise} \end{cases}$$

The α -cut of the fuzzy numbers (0.2, 0.3, 0.4, 0.5) is $(0.2 + 0.1\alpha, 0.5 - 0.1\alpha)$ for which

$$R(\bar{\lambda}) = R(0.2, 0.3, 0.4, 0.5) = \int_0^1 0.5(0.2 + 0.5) d\alpha = \int_0^1 0.5(0.7) d\alpha = 0.4. \text{ By proceeding similarly the Robust Ranking indices}$$

for the fuzzy numbers $\bar{\mu}$, $\bar{\mu}_v$ and $\bar{\eta}$ which are evaluated as $R(\bar{\mu}) = 0.2$, $R(\bar{\mu}_v) = 0.005$ and

$R(\bar{\eta})$

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(A High Impact Factor, Monthly Peer Reviewed Journal)

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)=0.025. Thus for various values of 'a' ranging from 6 to 12, the mean queue length L_q are tabulated and the values are shown in the figure.

Mean queue length for FM/M(a,b)/1/MWV model
 $\lambda=0.4, \mu=0.2, \mu_v=0.005, b=15$

η a	0.025	0.05	0.075	0.1	0.125
6	21.44	33.15	44.87	56.58	65.35
7	23.65	37.19	50.73	64.27	77.80
8	26.73	42.85	58.97	75.09	91.21
9	30.59	47.98	69.22	88.74	108.12
10	35.28	58.50	81.78	105.05	128.32
11	40.58	68.36	96.14	123.92	151.69
12	46.67	79.54	112.41	145.28	178.15

Table.1

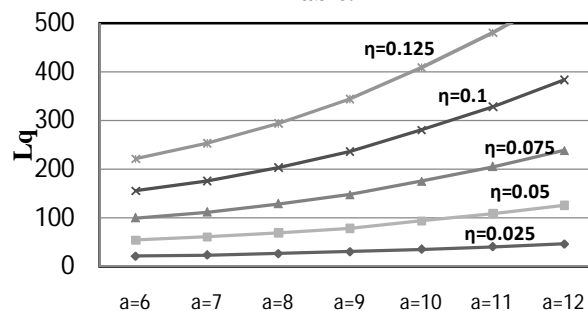


Fig.1

From the table 1 and figure.1, we conclude that, the mean queue length L_q increases when the vacation parameter η increases. Also, we find that the mean queue length (L_q) increases when the batch size increases.

Further Numerical examples are also presented to study the effect of parameters on the system characteristics. We illustrate the numerical examples for the probability that the system is in vacation (P_v) and the probability that the system is in busy state (P_{busy}). For computation we choose arbitrarily the parameters $\lambda=0.4, \mu=0.2, a=6, b=15$. Then, the probability that the system is in vacation (P_v) are evaluated for different values of μ_v and η . They are tabulated and shown in the figure, by using the formulae for P_v as

$$P_v = \frac{Q_0}{(1-r_v)}, \text{ where } Q_0 = \frac{1}{F(r_v, \mu_v) + AF(r, \mu) + BF(r_v, \mu)},$$

$$\text{with } F(x, y) = \frac{1}{(1-x)} \left(1 + \frac{y}{\lambda} \left(a - \frac{x(1-x^a)}{1-x} \right) \right), \quad A = \frac{(1-q)}{x(1-q^a)} \left[\frac{u}{(1-p)} - \frac{Bx(1-p^a)}{(1-p)} \right], \quad \text{and} \quad B = \frac{up}{xp(1-p^b) + t(p-1)}$$

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Vol. 5, Issue 2, February 2016

P_v with respect to μ_v and η for FM/M(a,b)/1/MWV model

$\mu_v \backslash \eta$	0.005	0.055	0.105	0.155	0.205	0.255
0.025	0.6816	0.5917	0.5228	0.4682	0.4240	0.3874
0.5	0.5230	0.4684	0.4241	0.3875	0.3567	0.3304
0.75	0.4243	0.3876	0.3568	0.3305	0.3078	0.2881
0.1	0.3569	0.3306	0.3079	0.2882	0.2708	0.2554
0.125	0.3080	0.2882	0.2708	0.2554	0.2417	0.2293

Table. 2

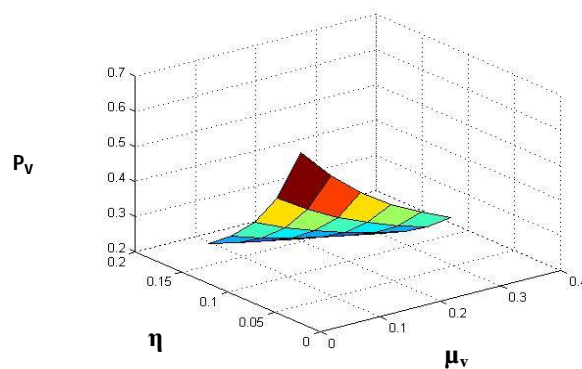


Fig.2

From the table 2 & Fig 2. We observe that P_v decreases when μ_v and η increases.

Moreover, Probability that the server is in busy state (P_{busy}) is evaluated by using the formulae

$$P_{\text{busy}} = \left(\frac{A}{(1-r)} + \frac{B}{(1-r_v)} \right) Q_0 \quad \text{with} \quad Q_0 = \frac{1}{F(r_v, \mu_v) + AF(r, \mu) + BF(r_v, \mu)},$$

$$\text{where } F(x, y) = \frac{1}{(1-x)} \left(1 + \frac{y}{\lambda} \left(a - \frac{x(1-x^a)}{1-x} \right) \right), \quad A = \frac{(1-q)}{x(1-q^a)} \left[\frac{u}{(1-p)} - \frac{Bx(1-p^a)}{(1-p)} \right] \quad \text{and} \quad B = \frac{up}{xp(1-p^b) + t(p-1)}$$

By using the above formula for various values for μ_v and η of FM/M(a,b)/1/MWV model, P_{busy} is calculated and the values are tabulated in table 3. The tabulated values are shown in figure 3.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly Peer Reviewed Journal)

Vol. 5, Issue 2, February 2016

P_{busy} with respect to μ_v and η for FM/M(a,b)/1/MWV model

$\mu_v \backslash \eta$	0.005	0.055	0.105	0.155	0.205	0.255
0.025	0.1586	0.1377	0.1216	0.1089	0.0986	0.0901
0.5	0.2433	0.2179	0.1973	0.1803	0.1660	0.1537
0.75	0.2961	0.2705	0.2490	0.2307	0.2149	0.2011
0.1	0.3321	0.3077	0.2865	0.2681	0.2520	0.2376
0.125	0.3583	0.3353	0.3150	0.2971	0.2811	0.2667

Table. 3

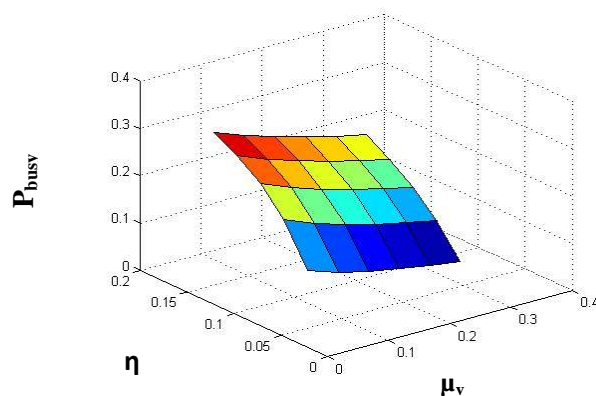


Fig. 3

The combined effect of μ_v and η are presented in the table 3 & Fig 3. We note that P_{busy} decreases and increases with respect to increase in η and μ_v .

V. CONCLUSION

In this paper we dealt with a bulk service queueing model with poisson arrival. The arrival rate, service rate for busy period, service rate for vacation period and vacation parameter are fuzzy number which are more realistic and general in nature. Moreover the fuzzy problem has been converted into crisp problem by using Robust Ranking Technique. By increasing the value of vacation parameter and batch size we find that the mean queue length is increasing likewise for P_v and P_{busy} . Moreover, we find that by applying Robust Ranking Method the solution of fuzzy problem is more efficient.

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