

(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016

# Forced Vibration Responses of Functionally Graded Viscoelastic Beams under Thermal Environment<sup>\*</sup>

Şeref Doğuşcan Akbaş

Associate Professor, Dr., Department of Civil Engineering, Bursa Technical University, Bursa, Turkey

<sup>\*</sup>This paper was presented at 1<sup>st</sup> International Conference on Engineering Technology and Applied Sciences on 21–22April 2016 in Afyon, Turkey [20].

**ABSTRACT**: This paper presents the forced vibration analysis of viscoelastic simple supported beam composed of functionally graded material (FGM) under temperature environment with considering the temperature dependent physical properties. The FGM viscoelastic beam is excited by a transverse triangular force impulse modulated by a harmonic motion and temperature rising. Material properties of the beam change in the thickness direction according to power-law distributions. The damping effect is considered by using the Kelvin–Voigt viscoelastic model. The considered problem is investigated within the Euler-Bernoulli beam theory by using finite element method. The obtained system of differential equations is reduced to a linear algebraic equation system and solved in the time domain by using Newmark average acceleration method. In the study, the effects of the different material distributions, temperature rising and forced dynamic load on the dynamic response of the FGM viscoelastic beams are investigated in detail. Also, the difference between the the temperature dependent physical properties and the temperature independent physical properties investigated for dynamic responses of FGM beam.

KEYWORDS: Functionally Graded Material; Viscoelastic Beam; Forced Vibration; Thermal Effect.

#### I. INTRODUCTION

Functionally graded material (FGM) is a class of composites that has a continuous variation of material properties from one surface to another. These materials can be fabricated by varying the percentage contents of two or more materials such that the new materials have the desired property gradation in spatial directions. Typically, in an FGM, one face of a structural component is ceramic that can resist severe thermal loading and the other face is metal that has excellent structural strength. FGMs consisting of heat-resisting ceramic and fracture-resisting metal can improve the properties of thermal barrier systems because cracking and delamination, often observed in conventional layered composites, can be reduced by proper smooth transition of material properties. The technology of FGM was an original material fabrication technology proposed in Japan in 1984 by Sendai Group. Since the concept of FGM was introduced in 1980s, these new kinds of materials have been employed in many engineering applications, such as aircrafts, space vehicles, defense industries, electronics and biomedical sectors, to eliminate stress concentrations, to relax residual stresses, and to enhance bonding strengths. With the increased use of FGM, an understanding the mechanical behavior of GM structures is important to the structural designers.

In the literature, much more attention has been given to the dynamic behaviour of FGM beams [1-9]. However, the research on the dynamic behaviour of FGM beams under the thermal effect is not studied broadly; Rajesh et al. [10] studied buckling and vibration behavior of a FGM sandwich beam having constrained viscoelastic layer in thermal environment by using finite element formulation. Free vibration of statically thermal postbuckled FGM beams with surface-bonded piezoelectric layers subject to both temperature rise and voltage is studied by Li et al. [11]. Pradhan and Murmu [12] investigated thermo-mechanical vibration analysis of FGM beams and FGM sandwich beams. Thermo-mechanical buckling and nonlinear free vibration analysis of FGM beams resting on nonlinear elastic foundation are



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016

investigated by Fallah and Agdahm [13]. Esfahani et al. [14] studied vibrations of a FGM beam under in-plane thermal loading in the prebuckling and postbuckling regimes.Zahedinejad [15] investigated free vibration of FGM beams with various boundary conditions resting on a two-parameter elastic foundation in the thermal environment. Şimşek and Kocatürk [16] studied free and forced vibration of a FG beam subjected to a concentrated moving harmonic load. Şimşek [17,18] linear and nonlinear dynamic behavior of FG beams under action of a moving harmonic load.Akbaş [19,20,21] studied dynamic behavior of FG beams in thermal environment.

In this study, the effect of material-temperature dependent on the forced vibration analysis of viscoelastic simple supported FG beam under a transverse triangular force impulse modulated by a harmonic motion and temperature rising is studied. The considered problem is investigated within the Euler-Bernoulli beam theory by using energy based finite element method with the Kelvin–Voigt model. The material properties of the beam are temperature-dependent and change in the thickness direction of the beamaccording to power-law distributions. In the study, the effects of material distributions and temperature rising on the dynamic response of the FGM viscoelastic beams are investigated in detail. Also, the difference between the the temperature dependent physical properties and the temperature independent physical properties investigated for dynamic responses of FGM beam.

#### **II. THEORY AND FORMULATIONS**

Consider a simple supported FG beam of length *L*, width *b*, thickness *h* as shown in Figure 1. The beam is excited by a transverse triangular force (*P*) impulse modulated by a harmonic motion at the midpoint of the beam and under uniform temperature rising  $\Delta T$  as seen from figure 1.



Fig. 1 A simple supported FG beam subjected to a force impulse under uniform temperature rising.

In this study, the material properties are both temperature-dependent and position-dependent. The effective material properties of the functionally graded beam, *P*,i.e., Young's modulus *E*, and mass density  $\rho$  vary continuously in the height direction (*Y* axis) and a function of temperature *T* (see Touloukian [22]) as follows;

$$P(T) = P_0 (P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3)$$
(1)

where  $P_T$  and  $P_B$  are the material properties of the top and the bottom surfaces of the beam that depends on temperature (T),  $T=T_0+\Delta T$ , where  $T_0$  is installation temperature and  $\Delta T$  is the uniform temperature rise. In this study, the material properties of the FG beam vary continuously in the thickness direction (*Y* axis) according to the power law distribution as follows;

$$P(Y,T) = \left(P_T(T) - P_B(T)\right) \left(\frac{Y}{h} + \frac{1}{2}\right)^n + P_B(T)(2)$$



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016

It is clear from equation (2) that when Y=-h/2,  $P=P_B$ , and when Y=h/2,  $P=P_T$ . Where *n* is the non-negative power-law exponent which dictates the material variation profile through the height direction of the beam. In Eq. (2),  $P_{-I}$ ,  $P_0$ ,  $P_1$ ,  $P_2$  and  $P_3$  indicate the coefficients of temperature *T* and are unique to the constituent materials. In this study, the unit of the temperature is Kelvin (K), the unit of the Young's modulus *E* is Pascal (Pa) and the unit of the mass density  $\rho$  is  $kg/m^3$ .

The beams considered in numerical examples are made of Zirconia and Aluminum Oxide. The bottom surface of the functionally graded beam is Zirconia and the top surface of the functionally graded beam is Aluminum Oxide. The coefficients of temperature T for Zirconia and Aluminum Oxide are given from Reddy and Chin [23].

According to the coordinate system (X, Y, Z) shown in figure 1, based on Euler-Bernoulli beam theory, the axial and the transverse displacement field are expressed as

$$u(X, Y, t) = u_0(X, t) - Y \frac{\partial v_0(X, t)}{\partial X}$$
(3)  

$$v(X, Y, t) = v_0(X, t)$$
(4)  

$$w(X, Y, t) = 0$$
(5)

where u, v, w are x, y and z components of the displacements, respectively. Also,  $u_0$  and  $v_0$  are the axial and the transverse displacements in the mid-plane, t indicates time.

By using equation (3) and strain- displacement relation can be obtained:

$$\varepsilon_{XX} = \frac{\partial u}{\partial X} = \frac{\partial u_0(X,t)}{\partial X} - Y \frac{\partial^2 v_0(X,t)}{\partial X^2} (6)$$

The Kelvin–Voigt viscoelastic model is used for the material. The constitutive relations for the Kelvin– Voigt viscoelastic model between the stresses and strains become

$$\sigma_{xx} = E(Y,T)(\varepsilon_{xx} + \eta_1(Y,T)\dot{\varepsilon}_{xx} - \alpha_x(Y,T)\Delta T)(7)$$

where  $\sigma_{xx}$  indicates normal stresses,  $\varepsilon_{xx}$  indicates normal strains in the X direction,  $\dot{\varepsilon}_{xx}$  is the time derivatives of the normal strains,  $\alpha_x$  is the thermal expansion coefficient,  $\eta_1$  is the damping ratio of the viscoelastic FG beam, as follows

$$\eta_1(Y,T) = \frac{c(Y,T)}{E(Y,T)} (8)$$

where c is the coefficient of damping of the beam, their dependence on Y coordinate and temperature are given by equations (1-2).

Substituting equation (6) into equation (7), the stresscan be obtained as follows

$$\sigma_{\chi\chi} = E(Y,T)\left(\left(\frac{\partial u_0(X,t)}{\partial X} - Y\frac{\partial \phi(x,t)}{\partial X}\right) + \eta_1(Y,T)\frac{\partial}{\partial t}\left(\frac{\partial u_0(X,t)}{\partial X} - Y\frac{\partial \phi(x,t)}{\partial X}\right)\right) - \alpha_{\chi}(Y,T)\Delta T \quad (9)$$

The elastic strain energy  $(U_i)$  of the beam is expressed as

$$U_i = \frac{1}{2} \int_0^L \int_A \left( \sigma_{ij} \varepsilon_{ij} \right) dA \, dX \tag{10}$$

By substituting equations (6) and (9) into equation (10), elastic strain energy  $(U_i)$  can be rewritten as follows:



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016

$$U_{i} = \frac{1}{2} \int_{0}^{L} \left[ A_{11} \left( \frac{\partial u_{0}(X,t)}{\partial X} \right)^{2} - 2B_{11} \left( \frac{\partial u_{0}}{\partial X} \right) \left( \frac{\partial^{2} v_{0}}{\partial X^{2}} \right) + D_{11} \left( \frac{\partial^{2} v_{0}(X,t)}{\partial X^{2}} \right)^{2} + A_{XT} \left( \frac{\partial u_{0}}{\partial X} \right) + 2A_{YT} \left( \frac{\partial^{2} v_{0}}{\partial X^{2}} \right) + A_{XT} \alpha(Y,t) \Delta T \right] dX(11)$$

where

 $(A_{11}, B_{11}, D_{11}) = \int_{A} E(Y, T)(1, Y, Y^{2}) dA(12a)$  $(A_{XT}, A_{YT}) = \int_{A} E(Y, T) \alpha(Y, t) \Delta T(1, Y) dA(12b)$ 

Kinetic energy (V) of the FGM beam are expressed as follows:

$$V = \frac{1}{2} \int_0^L \int_A \rho(Y, T) \left[ \left( \frac{\partial u}{\partial t} \right)^2 + \left( \frac{\partial v}{\partial t} \right)^2 \right] dA \, dX(13)$$

By substituting equations (3) and (4) into equation (13), Kinetic energy (V) can be rewritten as follows:

$$T = \frac{1}{2} \int_0^L \left[ I_1 \left( \frac{\partial u_0}{\partial t} \right)^2 - 2I_2 \left( \frac{\partial u_0}{\partial t} \right) \left( \frac{\partial^2 v_0}{\partial x \, \partial t} \right)^2 + I_1 \left( \frac{\partial v_0}{\partial t} \right)^2 + I_3 \left( \frac{\partial^2 v_0}{\partial x \, \partial t} \right)^2 \right] dX(14)$$

where

$$(I_1, I_2, I_3) = \int_A \rho(Y, T) (1, Y, Y^2) dA$$
(15)

The potential energy of the external load can be written as

$$U_e = -\int_0^L [P(x,t) v(x,t)] \, dX(16)$$

The dissipation function of the microbeam at any instant t is

$$R = \frac{1}{2} \int_{V} \left[ E(Y, T) \eta_1(Y, T) \left( \frac{\partial \varepsilon}{\partial t} \right)^2 \right] (17)$$

By substituting equations (6) into equation (17) and reducing the volume integral to a one-dimensional integral, the dissipation function (R) can be rewritten as follows:

$$R = \frac{1}{2} \int_0^L \left[ C_1 \left( \frac{\partial}{\partial X} \left( \frac{\partial u_0}{\partial t} \right) \right)^2 - 2C_2 \left( \frac{\partial}{\partial X} \left( \frac{\partial u_0}{\partial t} \right) \right) \left( \frac{\partial^2}{\partial X^2} \left( \frac{\partial v_0}{\partial t} \right) \right) + C_3 \left( \frac{\partial^2}{\partial X^2} \left( \frac{\partial v_0}{\partial t} \right) \right)^2 \right] dX(18)$$

where

$$(C_1, C_2, C_3) = \int_A E(Y, T)\eta_1(Y, T)(1, Y, Y^2) dA$$
(19)  
Lagrangian functional of the problem is given as follows:

$$I = T - (U_{\rm i} + U_{\rm e})(20)$$

The problem is solved by using Lagrange's equations and time integration method of Newmark [24].

Total nodal displacements q which is written for a two-node beam element, each node has three degrees of freedom, are defined as follows:

$$\{q(t)\}_{e} = \left[u_{i}^{(e)}(t), v_{i}^{(e)}(t), \emptyset_{i}^{(e)}(t), u_{j}^{(e)}(t), v_{j}^{(e)}(t), \emptyset_{j}^{(e)}(t)\right]^{T} (21)$$

The displacement field of the finite element is expressed in terms of nodal displacements as follows



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016

$$\begin{aligned} u^{(e)}(X,t) &= \varphi_1^{(U)}(X) \, u_i(t) + \varphi_2^{(U)}(X) \, u_j(t) \, (22) \\ v^{(e)}(X,t) &= \varphi_1^{(V)}(X) \, v_i(t) + \varphi_2^{(V)}(X) \phi_i(t) + \varphi_3^{(V)}(X) \, v_j(t) + \varphi_4^{(V)}(X) \phi_j(t) (23) \end{aligned}$$

where  $u_i$ ,  $v_i$  and  $\phi_i$  are axial displacements, transverse displacements and rotations at the two end nodes of the beam element, respectively.  $\phi_i^{(U)}$  and  $\phi_i^{(V)}$  are shape functions for axial and transverse degrees of freedom, respectively, and are given from [19].

After substituting Equations (2.23) and (2.24) into Equation (2.21) and then using the Lagrange's equations gives the following equation;

 $\frac{\partial I}{\partial q_k^{(e)}} - \frac{\partial}{\partial t} \frac{\partial I}{\partial \dot{q}_k^{(e)}} + Q_{D_k} = 0, k=1,2,3,4,5,6(24)$  where

$$Q_{D_k} = -\frac{\partial \mathbf{R}}{\partial \dot{q}_k^{(e)}}(25)$$

 $Q_{D_k}$  is the generalized damping load which can be obtained from the dissipation function by differentiating *R* with respect to  $\dot{q}_k^{(e)}$ . Where  $\dot{q}_k^{(e)}$  indicates the time derivatives of nodal displacements *q*.

The Lagrange's equations yield the system of equations of motion for the finite element and by use of usual assemblage procedure the following system of equations of motion for the whole system can be obtained as follows

$$[K]{q(t)} + [D]{\dot{q}(t)} + [M]{\ddot{q}(t)} = {F(t)}(26)$$

where, [K] is the stiffness matrix, [D] is the damping matrix, [M] is mass matrix and  $\{F(t)\}$  is the load vector.

The components of the stiffness matrix [K] :

$$[K] = \begin{bmatrix} [K^{A}] & [K^{B}] \\ [K^{B}]^{T} & [K^{D}] \end{bmatrix}, \qquad (27)$$
  
where  
$$[K^{A}] = \int_{0}^{L_{e}} A_{11} \left[ \frac{\partial \varphi^{(U)}}{\partial X} \right]^{T} \left[ \frac{\partial \varphi^{(U)}}{\partial X} \right] dX, \qquad (28a)$$
  
$$[K^{B}] = -\int_{0}^{L_{e}} B_{11} \left[ \frac{\partial^{2} \varphi^{(V)}}{\partial X^{2}} \right]^{T} \left[ \frac{\partial \varphi^{(U)}}{\partial X} \right] dX, \qquad (28b)$$
  
$$[K^{D}] = \int_{0}^{L_{e}} (D_{11}) \left[ \frac{\partial^{2} \varphi^{(V)}}{\partial X^{2}} \right]^{T} \left[ \frac{\partial^{2} \varphi^{(V)}}{\partial X^{2}} \right] dX, \qquad (28c)$$

where  $L_e$  indicates the length of the finite beam element. The mass matrix [M] can be expressed as a sum of four submatrices as shown below:

$$[M] = [M_U] + [M_V] + [M_\theta] + [M_{U\theta}] \quad (29)$$

where  $[M_U]$ ,  $[M_V]$  and  $[M_{\theta}]$  are the contribution of u, v and  $\theta$  degree of freedom to the mass matrix,  $[M_{U\theta}]$  is coupling mass matrix due to coupling between u and  $\theta$ . Explicit forms of [M] are given as follows



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016

$$\begin{bmatrix} M_U \end{bmatrix} = \int_0^{L_e} I_1 \left[ \varphi^{(U)} \right]^T \left[ \varphi^{(U)} \right] dX \quad (30a)$$
  
$$\begin{bmatrix} M_V \end{bmatrix} = \int_0^{L_e} I_1 \left[ \varphi^{(V)} \right]^T \left[ \varphi^{(V)} \right] dX \quad (30b)$$
  
$$\begin{bmatrix} M_\theta \end{bmatrix} = \int_0^{L_e} I_1 \left[ \frac{\partial \varphi^{(V)}}{\partial X} \right]^T \left[ \frac{\partial \varphi^{(V)}}{\partial X} \right] dX \quad (30c)$$
  
$$\begin{bmatrix} M_{U\theta} \end{bmatrix} = -\int_0^{L_e} I_2 \left[ \left[ \varphi^{(U)} \right]^T \left[ \frac{\partial \varphi^{(V)}}{\partial X} \right] + \left[ \frac{\partial \varphi^{(V)}}{\partial X} \right]^T \left[ \varphi^{(U)} \right] \right] dX \quad (30d)$$

The components of the damping matrix [D] :

 $\begin{bmatrix} D \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} D^A \end{bmatrix} & \begin{bmatrix} D^B \end{bmatrix} \\ \begin{bmatrix} D^B \end{bmatrix}^{\mathrm{T}} & \begin{bmatrix} D^D \end{bmatrix} \end{bmatrix},$  (31) where

$$\begin{bmatrix} D^{A} \end{bmatrix} = \int_{0}^{L_{e}} C_{1} \left[ \frac{\partial \varphi^{(U)}}{\partial X} \right]^{T} \left[ \frac{\partial \varphi^{(U)}}{\partial X} \right] dX, \qquad (32a)$$
$$\begin{bmatrix} D^{B} \end{bmatrix} = -\int_{0}^{L_{e}} C_{2} \left[ \frac{\partial^{2} \varphi^{(V)}}{\partial X^{2}} \right]^{T} \left[ \frac{\partial \varphi^{(U)}}{\partial X} \right] dX, \qquad (32b)$$
$$\begin{bmatrix} D^{D} \end{bmatrix} = \int_{0}^{L_{e}} \left( C_{3} + \frac{1}{4} l^{2} C_{4} \right) \left[ \frac{\partial^{2} \varphi^{(V)}}{\partial X^{2}} \right]^{T} \left[ \frac{\partial^{2} \varphi^{(V)}}{\partial X^{2}} \right] dX, \qquad (32c)$$

The load vector  $\{F(t)\}$  is expressed as

 $\{F(t)\} = \int_{x=0}^{L_{\rm e}} \{\varphi(X)\}^{\rm T} F(X, t) \, dX(33)$ 

The motion equations which are given by equation (26) are solved in the time domain by using Newmark average acceleration method (Newmark [24]).

#### III. NUMERICAL RESULTS

In this section, various numerical examples are presented and discussed to investigate the effects of the different material distributions and temperature rising on the forced vibration responses of the FG beam in figures. The beams considered in numerical examples are made of Zirconia and Aluminum Oxide. The bottom surface of the FG beam is Zirconia and the top surface of the FG beam is Aluminum Oxide. When the power index n=0, the beam material is reduced to full Aluminum Oxide (homogeneous Aluminum Oxide). The coefficients of temperature T for Zirconia and Aluminum Oxide are listed in Reddy and Chin [23]. In this study, the unit of the temperature is Kelvin (K). In numerical examples, the initial temperature (installation temperature) of the beam is assumed to be  $T_0= 300 K$ . In numerical calculations, the number of finite elements is taken as 120 elements. Unless otherwise stated, it is assumed that the width of the beam is b=0.1m, height of the beam is h=0.1m and length of the beam is L=3m in the numerical results. In the numerical integrations, five-point Gauss integration rule is used.

Numerical calculations in the time domain are made by using Newmark average acceleration method. The system of linear differential equations which are given by equation (26), is reduced to a linear algebraic system of equations by



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016

using average acceleration method. The beam is excited by a transverse triangular force impulse (with a peak value 1 N) modulated by a harmonic function (Figure 2).



Fig. 2 The shape of the excitation impulse in the a) time domain and b) frequency domain.

Young's Modulus versus temperature rising is illustrated in figure3 by using equation (1) for Zirconia and Aluminum Oxide.



Fig. 3 Young's Modulus versus temperature rising for Zirconia and Aluminum Oxide.

As seen from figure 3, with increase in temperature, Young's modulus decreases. With the temperature increase, the intermolecular distances of the material increase and intermolecular forces decrease. As a result, the strength of the material decreases.

In figure4, the variations of material properties (Young's Modulus) distributions along the height of the beam are presented for some given values of the power-law exponent *n* for the temperature environment T = 400 K.



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016



Fig. 4 Young's Modulus distributions along the height of the beam for some given values of the power-law exponent n for the temperature environment T= 400 K.

As seen from figure4 that, when the material power law index n increases, Young's modulus decreases. Because when the material power law index n increase, the material of the beam get close to the bottom material (pure Zirconia) and it is known from the physical properties of the pure Zirconia and the top material (Aluminum Oxide) that the Young modulus of pure Aluminum Oxide is approximately greater than that of Zirconia.

In figures 5, 6, 7 and 8, the effect of the different material distribution on the forced vibration responses of the FG beam is presented for temperature T=300 K for various values of the power law index n=0, 0.5, 1 and 5, respectively. For these figures, the vertical displacements of the midpint of the FG beam are presented for the investigation of the forced vibration.



Fig. 5 Vertical displacements at the midpoint of the FG beam for values of n = 0.



Fig. 6 Vertical displacements at the midpoint of the FG beam for values of n = 0.5.



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016



Fig. 7 Vertical displacements at the midpoint of the FG beam for values of n = 1.



Fig. 8 Vertical displacements at the midpoint of the FG beam for values of n = 5.

It is seen from figures 5-8 that with increase the power law index n, amplitude of waves increases seriously. This is because by increasing in n, the elasticity modulus and mass density of the beam decrease according to equation (2). As result, the strength of the material decreases. Hence, the beam becomes more flexible. Also, it is seen figures 5-8 that the waves gradually disappear with increase in time because of the damping effect. It shows that the material distributions play important role in the forced vibration response of the FG beam.

In order to see the effects of temperature and the difference between of temperature dependent and independent physical properties of the FG beam, the vertical displacements of the midpoint of the FG beam are illustrated for different temperature value in figures 9, 10, 11 and 12.



Fig. 9 Vertical displacements at the midpoint of the FG beam for the temperature T=300 K.



(An ISO 3297: 2007 Certified Organization) Vol. 5, Special Issue 12, May 2016



Fig. 10 Vertical displacements at the midpoint of the FG beam for the temperature T=500 K.



Time (s)  $\mathbf{x} \cdot \mathbf{10}^{-4}$ Fig. 12 Vertical displacements at the midpoint of the FG beam for the temperature T=1000 K.

As seen from figures 9-12 that, with increase temperature, the amplitude of the additional waves increases. This is because: with increase in the temperature, the strength of the material decreases. Also, it is seen from figures 9-12 that with increase temperature, with increase in temperature, the difference between the displacements for the temperature dependent and independent physical properties increases. the transversal displacements for the temperature-dependent physical properties are greater than those for the temperature-independent physical properties. This situation may be explained as follows: In the temperature-dependent physical properties, with the temperature increase, the intermolecular distances of the material increase and intermolecular forces decrease. As a result, the strength of the material decreases. Hence, the beam becomes more flexible in the case of the temperature-dependent physical properties. Another result of the figures 9-12 that with the increase the time, he difference between the displacements for the temperature dependent and independent physical properties gradually interefere with each other.

#### IV. CONCLUSIONS

In this study, the effect of the temperature on the forced vibration responses of a functionally graded simple supported beam is studied considering the temperature dependent material properties. The effects of temperature and different material distributions on the forced vibration responses of the FGM beams are investigated in detail. It is



(An ISO 3297: 2007 Certified Organization)

#### Vol. 5, Special Issue 12, May 2016

observed from the investigations that the material distributions and temperature rising play important role in the dynamics responses of the FG beams. Also, it is observed from the investigations that there are significant differences of the analysis results for the temperature dependent and independent physical properties in the dynamic responses of the FG beam. Hence, the temperature-dependent physical properties must be taken into account for obtaining more realistic results. Otherwise an important error is inevitable.

#### ACKNOWLEDGMENTS

This work has been supported by Research Fund of the Bursa Technical University. Project Number: 2016-01-002.

#### REFERENCES

- A. Chakraborty and S. Gopalakrishnan, "A spectrally formulated finite element forwave propagation analysis in functionally graded beams", Int J Solids Struct, vol. 40, no.10, pp. 2421–48, 2003.
- [2] M. Aydogduand V. Taskin, "Freevibrationanalysis of functionallygradedbeamswithsimplysupportededges", Mater Des, vol. 28,no. 5, pp. 1651-6, 2007.
- J.Yangand Y. Chen, "Freevibrationandbucklinganalyses of functionallygradedbeamswithedgecracks" Composite Structures, vol. 83, no.1, pp. 48–60, 2008.
- [4] H.J. Xiang and J. Yang, "Free and forced vibration of a laminated FGM Timoshenkobeam of variable thickness under heat conduction", Compos Part B: Eng, vol. 39,no.2, pp. 292–303, 2008.
- [5] S.C. Pradhanand T. Murmu, "Thermo-mechanicalvibration of FGM sandwichbeamundervariableelasticfoundationsusingdifferentialquadraturemethod.", J Sound Vib, vol. 321, no.1–2, pp. 342–62, 2009.
- [6] M. Şimşek, T. Kocatürk and S.D. Akbaş, "Dynamic behavior of an axially functionally graded beam under action of a moving harmonic load", Composite Structures, vol. 94,no.8,pp. 2358-2364, 2012.
- [7] S. Li, J. Hu, C. Zhai and L. Xie, "A unified method for modeling of axially and/or transversally functionally graded beams with variable crosssection profile", Mechanics based design of structures and machines, vol. 41, no.2, pp. 168-188, 2013.
- [8] A. S. Kanani, H. Niknam, A. R. Ohadi and M.M. Aghdam, "Effect of nonlinear elastic foundation on large amplitude free and forced vibration of functionally graded beam.", Composite Structures, vol. 115, pp. 60-68, 2014.
- [9] Ş.D. Akbaş, "Wave propagation in edge cracked functionally graded beams under impact force", Journal of Vibration and Control, Doi: 10.1177/1077546314547531, 2015.
- [10] R.K. hangale, and N. Ganesan, "Thermoelastic buckling and vibration behavior of a functionally graded sandwich beam with constrained viscoelastic core", Journal of Sound and Vibration, vol. 295, no.1, pp.294-316, 2006.
- [11] S. R. Li, H. D.Su, and C. J. Cheng, "Free vibration of functionally graded material beams with surface-bonded piezoelectric layers in thermal environment", Applied mathematics and mechanics, vol. 30, no.8, pp. 969-982, 2009.
- [12] S. C. Pradhan and T. Murmu, "Thermo-mechanical vibration of FGM sandwich beam under variable elastic foundations using differential quadrature method", Journal of Sound and Vibration, vol. 321, no.1, pp. 342-362, 2009.
- [13] A. Fallah and M.M. Aghdam, "Thermo-mechanical buckling and nonlinear free vibration analysis of functionally graded beams on nonlinear elastic foundation", Composites Part B: Engineering, vol.43, no.3, pp.1523-1530, 2012.
- [14] S. E. Esfahani, Y. Kiani, M. Komijani and M.R. Eslami, "Vibration of a Temperature-Dependent Thermally Pre/Postbuckled FGM Beam Over a Nonlinear Hardening Elastic Foundation", Journal of Applied Mechanics, vol.81, no.1, 011004, 2004.
- [15] P. Zahedinejad, "Free Vibration Analysis of Functionally Graded Beams Resting on Elastic Foundation in Thermal Environment", International Journal of Structural Stability and Dynamics, 1550029, 2015.
- [16] M. ŞimşekandT. Kocatürk, "Free and forced vibration of a functionally graded beam subjected to a concentrated moving harmonic load", Composite Structures, vol. 90, no.4, pp.465–73, 2009.
- [17] M. Şimşek, "Vibration analysis of a functionally graded beam under a moving mass by using different beam theories", Composite Structures, vol.92, no.4, pp.904–17, 2010.
- [18] M. Şimşek, "Non-linear vibration analysis of a functionally graded Timoshenko beam under action of a moving harmonic load", Composite Structures, vol.92, pp.2532–46, 2010.
- [19] S.D. Akbaş, "Wave propagation of a functionally graded beam in thermal environments", Steel and Composite Structures, vol.19, no.6, pp. 1421-1447, 2015.
- [20] "Forced Vibration Responses of Functionally Graded Viscoelastic Beams Under Thermal Environment",1<sup>st</sup> International Conference on Engineering Technology and Applied Sciences, Afyon, Turkey, 2016.
- [21] Ş.D. Akbaş, "Free vibration of axially functionally graded beams in thermal environment", International Journal of Engineering and Applied Sciences, vol.6, no.3, pp.37-51, 2014.
- [22] Y.S. Touloukian, "Thermophysical Properties of High Temperature Solid Materials", Macmillan, New York, 1967.
- [23] J..N. Reddy and C.D. Chin, "Thermoelastical analysis of functionally graded cylinders and plates", Journal of Thermal Stresses, vol.21,no.6, pp.593-626, 1998.
- [24] N.M. Newmark, "A method of computation for structural dynamics", ASCE Engineering Mechanics Division, vol.85, pp.67-94, 1959.