

Role of One-Dimensional Ground Water Recharge Model in Agriculture

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ABSTRACT: One-dimensional flow in ground water recharge plays an important role in agriculture and drainage system. If the entire pore spaces are not completely filled with flowing liquid, due to content changes as a function of time, unsteady and unsaturated flow of water through soils takes place. The underground removal of seepage and waste water are related to these type of flows, which are governed by partial differential equation. The present paper contains the mathematical formulation of the problem and solution of governing equation is obtained by Successive over relaxation method.

KEYWORDS: One-dimensional, Ground-water, Porous Media,

I. INTRODUCTION

Hydrologist, agriculturalists, scientists and engineers are concerned with the study of one-dimensional ground-water flow. It plays an important role in water resource science, soil engineering and agricultural sciences. The mathematical model is based on the hydrological situation of one dimensional vertical ground water recharge by spreading.

Many Authors discussed this problem in different ways. Verma [1] and Mishra [2] applied similarity transformation. Verma [3] also used Laplace transformation to obtain solution of this model. He considered the average diffusivity coefficient over the whole range of moisture content as constant, and a variation of permeability with moisture content is assumed to be linear. He [4] has also given a mathematical solution of one-dimensional groundwater recharge for a very deep water table. De Wiest [5,6] represented this problem in different ways. Mehta [7] used singular perturbation technique to obtain the solution. Klute [8] used numeric technique to obtain the solution. Ferris [9] gave quantitative approach to groundwater investigation. Klute [10] and Richardson [11] contributed different aspects in this model.

II. STATEMENT OF THE PROBLEM

Consider a large basin of such geological configuration whose sides are limited by rigid boundaries and the bottom by a thick layer of water table. The groundwater recharge takes place over this basin. The flow is assumed vertically downwards through unsaturated porous media.

The diffusivity co-efficient is assumed to be a constant. Also it is considered equivalent to its average value over the whole range of moisture content. The permeability of the media is treated as continuous linear function of the moisture content. The governing equation of this problem is a partial differential equation.

III. MATHEMATICAL FORMULATION

Equation of continuity for an unsaturated medium is given by [8]

$$\frac{\partial}{\partial t}(\rho_s \theta) = -\text{div} \mathbf{V} \quad (1)$$

Where $\text{grad} \phi$ is the gradient of the moisture potential, $\mathbf{V}$ the volume flux of moisture, and k the co-efficient of aqueous conductivity.

From Darcy's law, the motion of water in a porous medium, we get,

$$\vec{V} = -k \text{grad} \phi \quad (2)$$

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From equations (1) and (2) we obtain,

$$\frac{\partial}{\partial t}(\rho_s \theta) = \text{div}(\rho k \text{grad} \varphi) \quad (3)$$

Where $M = \rho \vec{V}$, ρ is the flux density.

Only vertical direction is considered that of the flow is considered.

$$\rho_s \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(\rho k \frac{\partial \psi}{\partial z} \right) - \frac{\partial}{\partial z} (\rho k g) \quad (4)$$

Where ψ the capillary pressure potential, g is the gravitational constant and $\varphi = \psi - gz$ The positive direction of the z -axis is considered in the same as that of the gravity.

Considering θ and ψ to be connected by a single valued function, from (4),

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D \frac{\partial \theta}{\partial z} \right) - \frac{\rho g}{\rho_s} \frac{\partial k}{\partial z} \quad (5)$$

Where $D = \frac{\rho k}{\rho_s} \frac{\partial \psi}{\partial \theta}$ which is called diffusivity co-efficient.

Replacing D by average value D_a

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D_a \frac{\partial \theta}{\partial z} \right) - \frac{\rho g}{\rho_s} \frac{\partial k}{\partial z} \quad (6)$$

If Permeability k is assumed to be a constant, then (6) becomes

$$\frac{\partial \theta}{\partial t} = D_a \frac{\partial^2 \theta}{\partial z^2} \quad (7)$$

$$\text{Let } \frac{z}{L} = X, \quad \frac{t D_a}{L^2} = T$$

$$\frac{\partial \theta}{\partial T} = \frac{\partial^2 \theta}{\partial X^2}$$

Which is the dimensionless form of (7).

X = Penetration depth (dimensionless), T = Time (dimensionless)

β = Flow parameter (cm^2), ρ = Mass density of water (gm)

ρ_s = Bulk density of the medium on dry weight basis (gm cm^{-3})

D_a = Average value of the diffusivity co-efficient over the whole range of moisture content ($\text{cm}^2 \text{sec}^{-1}$)

The appropriate boundary conditions are

$$\theta(0, T) = \theta_0, \quad \theta(1, T) = 1 \quad (8)$$

$$\theta(X, 0) = \theta_1 \quad (9)$$

Initially, The moisture content throughout the region is assumed to be θ_1 , at the layer $z = 0$ it is θ_0 , and at the water table ($z = L$) it is assumed to remain 100% throughout the process of investigation. Neglecting the effect of capillary action at the stationary groundwater level as it is very small.

IV. MATHEMATICAL SOLUTION

Using Crank- Nicolson method [12], we have,

$$\theta_{m,n+1} = \theta_{m,n} + \frac{k}{2h^2} (\theta_{m+1,n} - 2\theta_{m,n} + \theta_{m-1,n} + \theta_{m+1,n+1} - 2\theta_{m,n+1} + \theta_{m-1,n+1})$$

$$\text{Let } r = \frac{k}{h^2}$$

$$(1+r)\theta_{m,n+1} = \theta_{m,n} + \frac{r}{2} (\theta_{m+1,n} - 2\theta_{m,n} + \theta_{m-1,n} + \theta_{m+1,n+1} + \theta_{m-1,n+1})$$

Using S.O.R. [15] method,

$$\theta_{m,n+1} = (1 - \omega)\theta_{m,n} + \omega \left[\frac{r}{2(1+r)} (\theta_{m+1,n} + \theta_{m-1,n+1}) + \frac{\lambda_i}{(1+r)} \right] \quad (10)$$

$$\text{Where } \lambda_i = \theta_{m,n} + \frac{r}{2} (\theta_{m+1,n} - 2\theta_{m,n} + \theta_{m-1,n})$$

Choose $k = 0.01$, $h = 0.05$, $\omega = 1.8$ in (10), and $\theta_0 = 0.02$, $\theta_1 = 0.5$ in (8) and (9)

$$\theta_{m,n+1} = -0.8\theta_{m,n} + 1.8 \left[0.4(\theta_{m+1,n} + \theta_{m-1,n+1}) + \frac{\lambda_i}{5} \right] \quad (11)$$

$$\text{Where } \lambda_i = \theta_{m,n} + 2(\theta_{m+1,n} - 2\theta_{m,n} + \theta_{m-1,n})$$

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Numerical calculations for different values of saturation at different time and different length are shown in the following table.

T→	T=0.01	T=0.02	T=0.03	T=0.04
X ↓	θ			
0	0.002	0.02	0.02	0.02
0.05	0.18144	0.122049792	0.11007492	0.095994594
0.1	0.3106368	0.2157837	0.190371835	0.177300751
0.15	0.403658496	0.297857917	0.261591609	0.249146349
0.2	0.470634117	0.367453129	0.324168841	0.312359775
0.25	0.518856564	0.425122962	0.378545333	0.367513864
0.3	0.553576726	0.472089364	0.425261083	0.4151534
0.35	0.578575243	0.50982494	0.464957608	0.455873444
0.4	0.596574175	0.539816787	0.498345686	0.490325886
0.45	0.609533406	0.563442925	0.526164459	0.51919671
0.5	0.618864052	0.581916789	0.549144517	0.543175927
0.55	0.625582118	0.596271364	0.567979721	0.562930435
0.6	0.630419125	0.607365101	0.583308477	0.579083672
0.65	0.63390177	0.61589867	0.59570334	0.592202647
0.7	0.636409274	0.622436017	0.605667167	0.60279145
0.75	0.638214677	0.627425994	0.613633968	0.611289798
0.8	0.639514568	0.6312226	0.619972873	0.618075154
0.85	0.640450489	0.634102908	0.624993917	0.623467142
0.9	0.641124352	0.636282392	0.628954701	0.62608731
0.95	0.641609533	0.637927697	0.6373979	0.6329903
1	1	1	1	1

V. CONCLUDING REMARKS

The solution is obtained by putting $\theta_0 = 0.02$, $h = 0.05$, $k = 0.01$. The sides of basin are limited by rigid boundaries and bottom by a thick layer of water table and also water flows only in positive direction is considered. Solution is obtained

by considering diffusivity and permeability constant. From the table, it is clear that $\theta = \theta_0 = 0.02$ at layer $x=0$ and moisture content is assumed to remain 100% at water table ($x=1$), throughout the process of investigation. Initially it is 0.02 and then increases to 1. Keeping T constant, for different values of X, moisture content is increasing and keeping X constant, the moisture content is decreasing.

VI. SCOPE OF THE PROBLEM

Increase of moisture content reduced the salinity of the soil. Fertility of soil increases with the increase in moisture content, which helps to grow up a qualitative crop.

Engineers in several fields have to study the mechanism of drainage which are applicable to the problems of water supply, land reclamation and stabilization of foundations and sub grade, petroleum production and agriculture.

The study of groundwater recharge plays an important role in agriculture and hydrology.

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