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# Prediction and Enhancement of Dynamic Behavior of High Speed Spindles

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**ABSTRACT**: Stability lobe diagram (SLD) helps in avoiding chatter in high speed machining. The stability lobe diagram has been predicted for the spindle of 32kW and 25,000 rpm. The spindle was modelled with beam elements using FEA tool. The natural frequency of the system was found to be 921Hz by modal analysis. A parametric study involving bearing preload, bearing spacing, span, material of spindle, diameter of spindle was done. An experiment was conducted to verify the variation in natural frequency of the system due to the change in spacing. A substantial shift in frequency was observed by altering the spacing. The cumulative effect of all the design variables resulted in a frequency enhancement from 921Hz to 1300 Hz. The variation of spindle stiffness and natural frequency with speed was plotted for various bearing preloads and design variables. A regression analysis is done to develop an empirical relation to predict the natural frequency of the spindle. Pareto chart was plotted to find the most influencing factor among the design variables. The frequency response function of the spindle system was found by Harmonic analysis. Stability lobe diagram was predicted using MATLAB. Real time machining was done to validate the stability lobe. Parametric study of stability lobe has been done and a significant increase in permissible depth of cut has been observed from stability lobe diagram due to the increase in natural frequency.

KEYWORDS: Stability Lobe Diagram, High Speed Machining, High Speed Spindles, Dynamic behavior of spindles.

### I. INTRODUCTION

Parameters affecting the natural frequency of the spindle were found and the relation among them was found using regression analysis. The natural frequency of the spindle was determined using finite element analysis method. By choosing the optimum value of the above parameters natural frequency of the spindle is increased. Stability lobe was predicted for the actual and optimised spindle. The method is validated by a real time machining process in a vertical milling machine.

#### II. LITERATURE REVIEW

The two-dimensional stability lobe theory deals with the stability of solutions for dynamical cutting systems, which usually stands for the spindle speed and axial depth. As a function of speed and depth of cut, the border between a stable cut (i.e., chatter-free) and an unstable one (i.e., with chatter) can be visualized in a chart called stability lobe diagram (SLD). Stability studies have two important research approaches. There are two approaches in stability lobe. One involves the machine behaviour, assuming a rigid work piece. The other considers the effects of work piece alone considering the spindle system to be rigid.

The methods used to predict the stability lobes are:

- 1. Experimental method
  - 2. Semi analytical method
  - 3. Analytical method

Experimental method aims to obtain stability lobe diagram by conducting a series of experiments on work piece by machining it using a milling machine tool. In semi analytical method most of the parameters required to obtain stability lobes are calculated analytically. The modal parameters of spindle/tool holder/tool systems are obtained



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experimentally using impulse hammer test. The aim of analytical approach is to obtain the transfer functions of structure, which is required in the stability model and to eliminate the need for series of experimental impact testing at various points on the work piece. Researchers have predicted chatter during machining and established 2D stability lobe diagrams plotted between axial depths of cut and spindle speed. Stability lobe diagram is helpful in selecting the proper spindle speeds for chatter free machining. The experimental method involves lot of cost and time, whereas the analytical method eliminates need for experimentation. The prerequisite for establishing stability lobe is the frequency response function (FRF), which can be done by experimental and numerical methods.

#### III. PREDICTION AND ENHANCEMENT OF NATURAL FREQUENCY OF HIGH SPEED SPINDLE

The natural frequency of the chosen 32 kW spindle is found to be 921 Hz by FEM approach. Eight design variables were selected and their effect on spindles natural frequency was found. The vital parameters contributing to natural frequency are bearing preload, material of spindle, spacing between bearings. But certain parameters (spacing between bearings) have negative influence on the dynamic behavior of the spindle. Increasing the value of these parameters led to decrease in natural frequency of the spindle. The cumulative effect of all the design variables resulted in a frequency enhancement from 921 Hz to 1300 Hz. Thus the augmented natural frequency of the spindle permits the usage of three fluted cutter (without resonance).

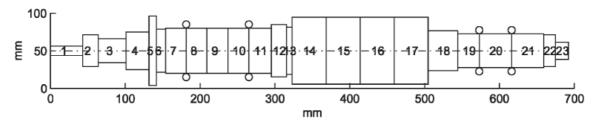


Figure 1 Layout of the spindle system

#### IV. REGRESSION ANALYSIS

The main objective of the multiple regression analysis is to develop an empirical relation to predict the natural frequency of a motorized high-speed spindle system. The model developed will also determine the dependence of natural frequency rise on rear bearing preload, bearing spacing, bearing span and material of spindle. The results of finite element analysis on natural frequency rise obtained for 32 kW and 25,000-rpm spindle presented in Chapter 3 is used to develop the regression model. The regression model attempted in this work is the multiple linear regression which is the process of developing an empirical relation for the dependent variable y, which is a linear function of two or more independent variables. For example, y might be a linear function of  $x_1$ ,  $x_2$  and  $x_3$ , as in

$$y = a_0 + a_1 \ x_1 + a_2 \ x_2 + a_3 \ x_3$$

Such an equation is useful when fitting experimental or numerical data where the variable being studied is often a function of two or more variables.

The multiple linear regression model for the present problem is expressed as

- $f = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + a_5 x_5 + a_6 x_6 + a_7 x_7 + a_8 x_8$
- f frequency (Hz)
- $x_1$  Rear bearing preload (N)
- x<sub>2</sub> Spacing between front bearings (mm)



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- $x_3-Spacing \ between \ front \ bearings \ (mm)$
- $x_4$  Span between bearings (mm)
- x<sub>5</sub> Spacing between midspan of bearings and free end of cutter (mm)
- $x_6$  Material of spindle (N/m<sup>2</sup>)
- $x_7$  Diameter of spindle (mm)
- $x_8$  Length of spindle (mm)

 $a_0, a_1...a_8$  – Regression coefficients

There are eight factors and three levels were given for each factor. Taguchi design was chosen and it gave twenty seven experiments. The natural frequency was found for the cases by FEM approach. Table 1 indicates the level taken for each factor. Table 2 shows the data set L27 Taguchi design.

Table 1 Level for factors

а	b	С	d	е	f	g	h
175	18	17.5	330	390	1.00E+11	11.1125	525
427.5	69.4	51.25	370	410	2.60E+11	27.78125	695
680	120.8	85	410	430	4.20E+11	44.45	865

- a Rear bearing preload (N)
- b Rear bearing spacing (mm)
- c Front bearing spacing (mm)
- d Span between bearings (mm)
- e Spacing between mid-span of bearings and free end of cutter (mm)
- f Material of spindle  $(N/m^2)$
- g Diameter of spindle (mm)
- h Length of spindle (mm)



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Table 2 Data Set

a	b	c	d	e	f	g	h	Frequency
175	18	17.5	330	390	1.00E+11	11.1125	525	794
175	18	17.5	330	410	2.60E+11	27.7813	695	1097
175	18	17.5	330	430	4.20E+11	44.45	865	1212
175	69.4	51.25	370	390	1.00E+11	11.1125	695	620
175	69.4	51.25	370	410	2.60E+11	27.7813	865	640
175	69.4	51.25	370	430	4.20E+11	44.45	525	723
175	120.8	85	410	390	1.00E+11	11.1125	865	400
175	120.8	85	410	410	2.60E+11	27.7813	525	623
175	120.8	85	410	430	4.20E+11	44.45	695	585
427.5	18	51.25	410	390	2.60E+11	44.45	525	813
427.5	18	51.25	410	410	4.20E+11	11.1125	695	921.5
427.5	18	51.25	410	430	1.00E+11	27.7813	865	574
427.5	69.4	85	330	390	2.60E+11	44.45	695	761
427.5	69.4	85	330	410	4.20E+11	11.1125	865	886
427.5	69.4	85	330	430	1.00E+11	27.7813	525	597
427.5	120.8	17.5	370	390	2.60E+11	44.45	865	792
427.5	120.8	17.5	370	410	4.20E+11	11.1125	525	901
427.5	120.8	17.5	370	430	1.00E+11	27.7813	695	569
680	18	85	370	390	4.20E+11	27.7813	525	918
680	18	85	370	410	1.00E+11	44.45	695	605
680	18	85	370	430	2.60E+11	11.1125	865	766
680	69.4	17.5	410	390	4.20E+11	27.7813	695	976
680	69.4	17.5	410	410	1.00E+11	44.45	865	596.8
680	69.4	17.5	410	430	2.60E+11	11.1125	525	845
680	120.8	51.25	330	390	4.20E+11	27.7813	865	810
680	120.8	51.25	330	410	1.00E+11	44.45	525	591
680	120.8	51.25	330	430	2.60E+11	11.1125	695	795



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Thus the natural frequency was found for each of the 27 case given and the maximum frequency was found to be 1212Hz.

Table 3 Model	summary
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Regression analysis				
R square	0.8859			
R square adjusted	0.8352			
Standard error	2.03			
Observations	27			

Since the R square value is close to 0.88 it indicates that the result is almost accurate. A high R square value 0.89 indicates the closeness of the data

Factors	Coefficients	Standard error	t-statistic	p-value
а	0.0459	0.0688	0.67	0.513
b	-1.767	0.0338	-5.22	0
С	-2.703	0.515	-5.25	0
d	-1.679	0.435	-3.86	0.001
е	-0.606	0.869	-0.70	0.495
f	1.53E-08	1.53E-08	8.26	0
g	-0.83	1.05	-0.80	0.435
h	-0.042	0.102	-0.41	0.687

#### Table 4 Coefficients of the model

The empirical relation for frequency (f) is given by

 $f = 1686 + 0.0459a - 1.767b - 2.703c - 1.679d - 0.606e + 1.53x10^{-8}f - 0.83g - 0.42h$ 

The measure of the regression model's ability to predict the dependent variable is called the coefficient of multiple determination ( $R^2$ ). The  $R^2$  value is the ratio of the variance of the empirically estimated frequency rise values to the variance of all the numerical data points. The  $R^2$  value is estimated to be 0.8859. Thus 88.5% of the total variation of frequency rise is explained or accounted for by the independent variables considered.

The standard error of the estimate is interpreted to mean that, for any given value of the independent variable  $X_i$ , the dependent variable tends to be distributed about the predicted value  $Y_i$ , with a standard deviation equal to the standard error of the estimate. The standard error is estimated to be 2.03 Hz as seen in Table 4.

To decide which variables in the model are good explanatory variables of the dependent, the individual t-test (Table 4) for each variable needs to be looked at. If p-value is significant (less than 0.05), it indicates that the variable concerned is significant in the model. For instance, if the preload in front bearing is increased by 100 N, the natural frequency increases by 4Hz, if all the other variables are unchanged. Similarly for a unit decrease in span between the bearings the



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natural frequency gets improved by 1.6 Hz. The variable that contributes the most to frequency enhancement is found using Pareto chart. The purpose of the Pareto chart is to highlight the most important among a (typically large) set of factors. Figure 2 displays the pareto chart for the chosen model.

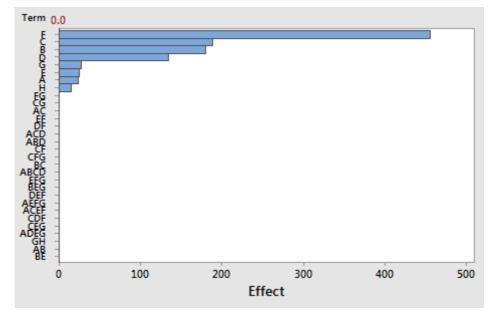


Figure 2 Pareto chart

The following is the order in which the factors influence the natural frequency

- Material of spindle
- Rear bearing spacing
- Front bearing spacing
- Span between bearings
- Diameter of spindle
- Spacing between midspan of bearings and free end of cutter
- Rear bearing preload
- Length of spindle

#### V. STABILITY LOBE DIAGRAM

A stability lobe diagram is mainly based on regenerative chatter theory is an effective tool to predict chatter. Stability diagrams can be applied in machining process to optimize the maximum depth of cut at the highest available spindle speed thus improving the material removal rate and increasing productivity. Stability is formed by series of intersected scallops-shaped borderlines of stability. The intersection of lobe denotes deepest stable cuts at various ranges of spindle speed. These depths of cuts have traditionally been found by graphical solution from the stability lobe diagram.



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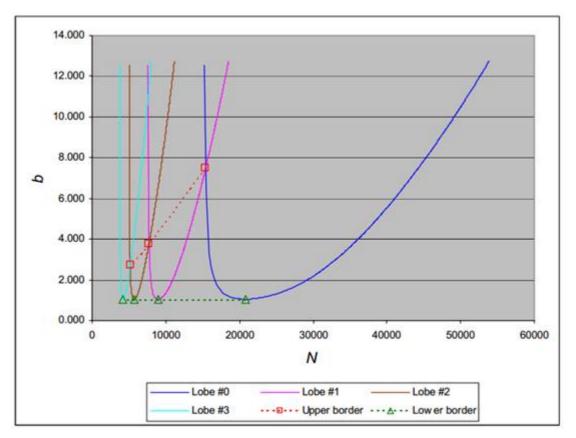


Figure 3 Stability lobe diagram for prediction of maximum chip width

Since the series of relationship curves in Figure 3 is shaped like lobes, the graph is usually called a stability lobe diagram. A stability lobe diagram shows the relationship between chip width (or depth of cut) and spindle speed, with the lobe number as a parameter. Usually, the variable on its x-axis is represented as spindle speed N, tooth frequency  $f_t$  ( $f_t = n_t N / 60$ ), or the tooth frequency ratio  $r_t$  ( $r_t = f_t / f_n$ ); the variable on its y-axis is represented as chip width b or the chip width ratio  $r_b$  ( $r_b = b/b_{min}$ ). The dimensionless relationship of  $r_b \sim r_t$  will be discussed. The lobes and their characteristics are similar for the other variables. This procedure of creating stability lobe diagram provides an easy way for machine operators to choose the optimum chip widths and high spindle speeds at the sweet spots on the diagram to obtain a high material removal rate (MRR). A stability lobe diagram is composed of a series of scallop-shaped chatter lines. It may be represented either as the relationship between the chip width to the minimum chip width and the tooth-stroke frequency to the natural frequency as  $r_b$  and  $r_t$ . The dimensionless relationship is preferable because it makes it easy to compare machining processes under different cutting parameters and conditions. The intersections or sweet spots of the consecutive series of lobes provide the optimum chip widths.

#### VI. PREDICTION OF STABILITY LOBE FOR SPINDLE SYSTEM

The following methodology has been used to compare and validate the stability lobe structure

- 1. Determination of Frequency Response Function by impact hammer test.
- 2. Prediction of FRF.
- 3. Comparison of FRF by experiment and numerical method



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- 4. Prediction and validation of stability lobe.
- 5. Experimental verification of predicted stability lobe.
- The extraction of FRF by experiment requires the following equipment
- i. LabVIEW software
- ii. Accelerometer
- iii. Impact hammer

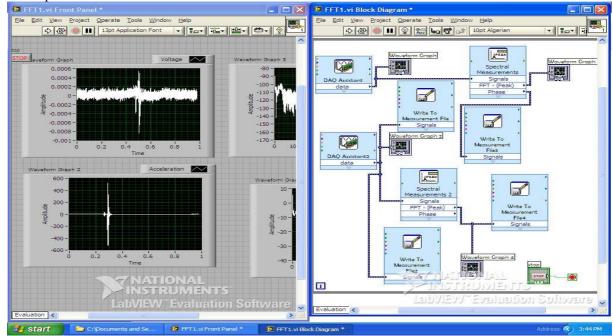


Figure 4 LabVIEW programming structure

The program is coded in Labview to measure the frequency. An accelerometer is a device that measures proper acceleration. The proper acceleration measured by an accelerometer is not necessarily the coordinate acceleration. Instead, the accelerometer observes the acceleration associated with the phenomenon of weight experienced by any test mass at rest in the frame of reference of the accelerometer device. The accelerometer is shown in Figure 5.

#### **Specifications of Accelerometer:**

**Range:** 500 m/s<sup>2</sup> **Sensitivity:** 96.72 mVolts/g

Frequency: 0.3 – 5000 Hz Weight: 4.31 g



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Figure 5 Accelerometer

This is placed on to the component which is subjected to vibration. The vibration pattern is then observed in terms of frequency domain in the LabVIEW software.

The impact hammer used for determination of FRF is shown in Figure 6.



Figure 6 Impact hammer

### Specifications:

General-Purpose Modal Analysis Impact Hammer

Measurement range: 4800 N

Hammer mass: 0.34 lb (0.16 kg)

Sensitivity (±15%): 22.7 mV/N



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The impact hammer input is read in the form of voltage, which is then converted into force by the scale factor. Dropping a coin of known mass on the hammer validates this input. Since the force of impact is theoretically obtained using the relations, it is used for validating the input of hammer by verifying both to be same.

The front bearing (FB) preload is increased for 32 kW spindle system. The following are the different stability lobes: The stability lobe of the actual spindle system is shown in Figure 7.

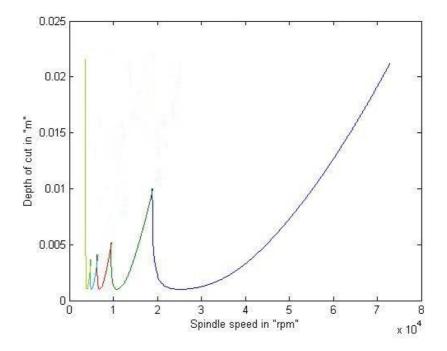


Figure 7 Stability lobe diagram of actual spindle system

Thus the actual spindle is limited to a depth of cut of 1mm when operated at a speed of 25000 rpm (as the natural frequency is 921 Hz).



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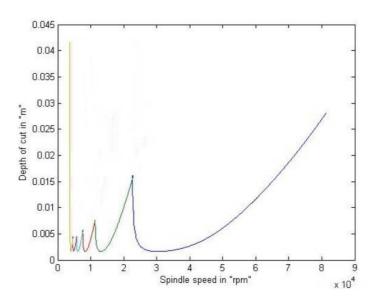


Figure 8 Stability lobe diagram of spindle system with front bearing preload of 3290 N

The front bearings are preloaded to the maximum permissible extent (3290 N). This resulted in an increase in natural frequency from 920 Hz to 970 Hz. There is a significant increase in the minimum permissible depth of cut.

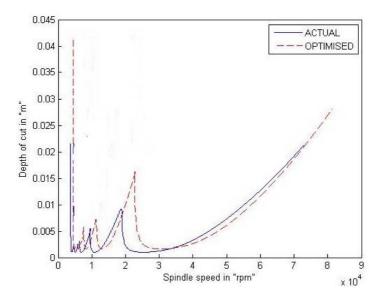


Figure 9 Comparison of stability lobe of actual and optimised spindle system

Therefore optimising the spindle system results in a increase in permissible depth of cut.



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Table 5 Comparison of dynamic properties

Sl No.	Specification	Natural frequency (Hz)	Modal stiffness (kN/m)	Permissible b <sub>min</sub> (mm)
1	Actual spindle system	1230	27,120	1
2	Optimised spindle system	1270	40,500	1.3
3	Optimised spindle system with FB preload 1630N	1375	41,700	1.6
4	Optimised spindle system with FB preload 2490N	1440	42,500	1.8
5	Optimised spindle system with FB preload 3290N	1480	43,400	2

Table 5 indicates the natural frequency, dynamic stiffness and permissible depth of cut for the various cases. There is an increase in natural frequency of the spindle from 1230 Hz to 1480Hz which in turn results in an increase in dynamic stiffness from 27,120kN/m to 43,400kN/m. Therefore the permissible depth of cut (which depends on dynamic stiffness) improves from 1mm to 2 mm. The stability lobe has been generated using numerical method technique and validated through experiment. The shift in lobe due to the change in front bearing preload has also been predicted. The chatter has been found experimentally when depth of cut changes from 1mm to 5 mm (the spindle speed being 10,000 rpm) and the corresponding roughness values are 0.533 microns and 2.594 microns respectively.

### EXPERIMENTAL VALIDATION

The experimental setup is shown in Figure 10 where the accelerometer is attached to the spindle in Makino S33 machine. The spindle is excited using an impact hammer on the side opposite to where the accelerometer is pasted.

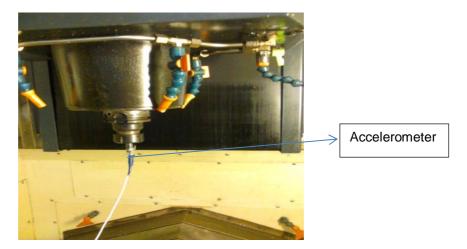


Figure 10 Accelerometer setup



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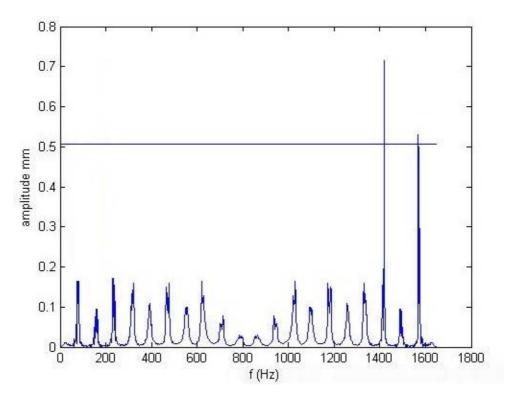
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The excitation signal from accelerometer is captured through data acquisition card and then analysed by using LabVIEW software. The impact hammer incorporates a force transducer in aluminium tip to measure force input during hammer strike. Because the setup and measurement time is short, it is referred to as impact testing. The force and acceleration signals are extracted to predict the system response using MATLAB software.

#### PREDICTION OF FRF BY FINITE ELEMENT ANALYSIS

The spindle is modeled as beam element which is defined by twenty three elements. The material properties for steel are as follows: Poisson's ratio = 0.3, Density = 7800 kg/m<sup>3</sup>. Young's modulus = 210 GPa and damping constant = 0.03. The acceleration response in time domain is converted into real and imaginary values of FRF using MATLAB software. The above response of spindle structure states the system dynamic characteristics for the applied excitation. From the obtained frequency signal it is verified that the natural frequencies for modal analysis as well as harmonic analysis the results are same with small percentage of deviation.

#### COMPARISON OF FRF BY EXPERIMENT AND FEM



#### Figure 11 Experimental FRF of spindle system

Figure above shows the FRF predicted by experiment. It is then compared with the FRF obtained from FEA and there is a slight difference between the predicted and experimental results. This is caused due to the finite element model being too stiff.



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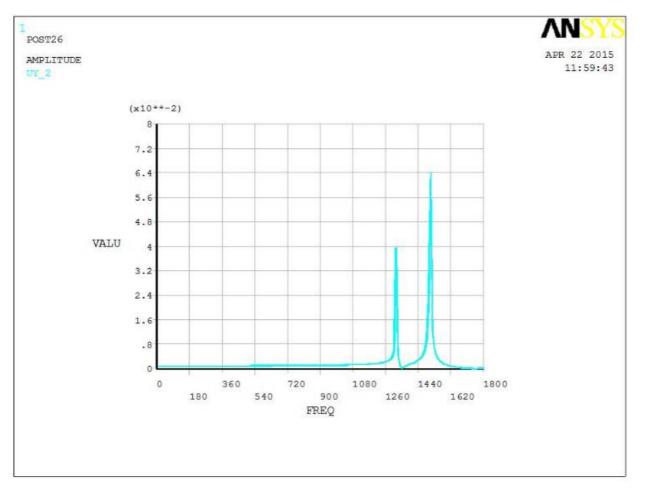


Figure 12 FRF obtained from numerical model

The natural frequency of the spindle is found to be 1420 Hz (predicted by experiment – as shown in Figure 11). The numerical approach (as shown in Figure 12) resulted in a natural frequency of 1480 Hz. There is a 4% deviation in the natural frequency of the spindle obtained by experiment and FEM. The above results will be used to develop the stability lobe for the spindle structure.



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### PREDICTION OF STABILITY LOBE

The stability lobe diagram of the spindle in Makino S33 machine is shown in Figure 13.

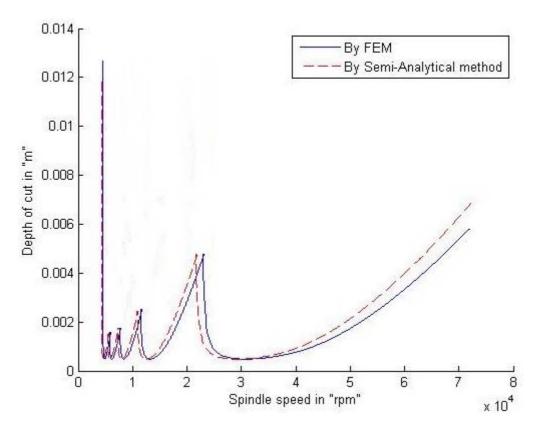


Figure 13 Comparison of stability lobe by FEM and semi analytical method

The mathematical expressions mentioned earlier are used to develop a code in software to establish the stability lobe diagram. A comparison of the stability lobes presented in Figure 13 shows a good agreement between the results of finite element analysis and experimental data. The slight discrepancy (4%) in the predicted natural frequency can be seen in the slight shift in the spindle speed (9%) calculated in the stability lobe.

#### VERIFICATION OF STABILITY LOBE

To verify the proposed stability lobe, a real machining process as shown Figure 14 is performed to check the quality of machined work piece by using Makino S33 Vertical Machining Centre (VMC) whose speed is 20,000 rpm. Solid carbide end mill of 8mm diameter and 4 teeth cutter is used.



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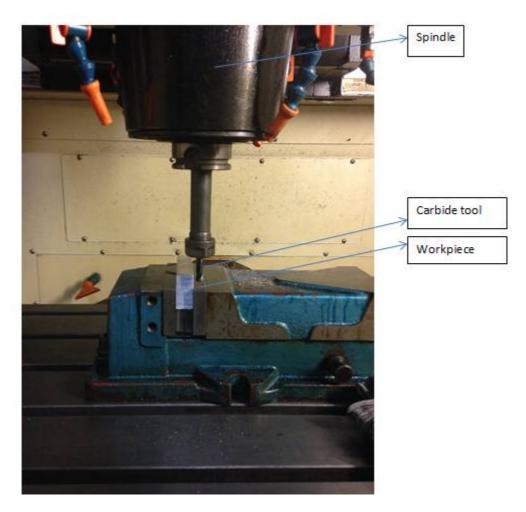


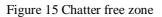
Figure 14 Milling of Aluminum block

In milling 5mm axial depth of cut is provided and spindle speed is 10000 rpm. The work piece material Al Alloy 7010 T7651 of thickness 40 mm is taken. Figure 15 represents the surface finish obtained for chatter free machining while Figure 16 represents finish obtained when chatter is observed.

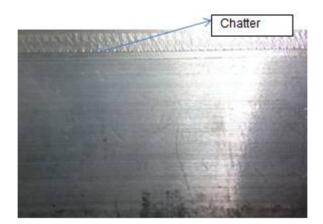


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It is evident from the image that the machined zone is free from chatter marks when the work piece is machined within the permissible depth of cut.



### Figure 16 Chatter zone

It is visually observed that in machining, chatter free zone gives better surface finish than chatter zone. Since the primary aim is to validate the stability of the finishing process where low immersion is used usually in practice, a radial depth of cut (DOC) of 0.5mm is taken for machining.

### VII. CONCLUSION

The major conclusions of the present work are:

The spindle system (32kW and 25,000 rpm) was developed with beam and spring elements. There are four bearings in the spindle shaft. The maximum spindle speed is 25,000 rpm and the corresponding excitation frequency for a two fluted end mill cutter is 800 Hz. The natural frequency of the spindle is found to be 921 Hz by modal analysis.



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Parametric study of factors influencing the dynamic behavior of the spindle was done. An experiment was done to validate the effect of span on the natural frequency of the system. The experiment resulted in a frequency change from 17 Hz to 30 Hz. The effect of design parameters on spindle stiffness and natural frequency was found. There is a significant rise in spindle stiffness from  $8.13 \times 10^6$  N/m to  $15.2 \times 10^6$  N/m on altering the material of the spindle. The vital parameters contributing to natural frequency are bearing preload, material of spindle, spacing between bearings. But certain parameters (spacing between bearings) have negative influence on the dynamic behavior of the spindle. Increasing the value of these parameters led to decrease in natural frequency of the spindle. The cumulative effect of all the design variables resulted in a frequency enhancement of 41.1%. The augmented natural frequency of the spindle allows the usage of three fluted cutter for machining (without resonance). Regression analysis was done to predict the natural frequency of the spindle as a function of the design variables. The parameter that contributes the most to the frequency improvement was found using Pareto chart. Material of the spindle is the highest contributor (16%) to natural frequency of the spindle. Stability lobe diagram is helpful in selecting the proper spindle speeds for chatter free machining. Frequency Response Function (FRF) is the prerequisite to predict the stability lobe. It can be predicted experimentally and numerically. The predicted and experimental FRF show good agreement and the corresponding stability lobe has been predicted and the lobe shows a deviation due to the variation in natural frequency. The predicted stability lobe is verified with real machining process. It is observed that during machining, the predicted chatter free zone leads to better surface finish (0.533 microns) in comparison to the surface finish (2.594 microns) realized when performing machining operation in the unstable region of the stability lobe diagram. The limiting value of axial depth of cut observed from the stability lobe diagram obtained from the results of finite element analysis has a maximum deviation of 10% in comparison to the corresponding depth of cut obtained by experimentation. Parametric (bearing preload, span, material of spindle and diameter of spindle) study of stability lobe was done. The limiting depth of cut improved from 1 mm to 2 mm.

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