

The Position Vectors of Space Curves with constant Curvatures According to Type-1 Bishop Frame in Euclidean 3- Space E^3

Dr. Fathi El-Zaki

Department of Mathematics, College of Science and Technology, Omdurman Islamic University, SUDAN

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Department of Mathematics, College of Sciences and Arts, Ranyah Branch, Taif University, KSA

ABSTRACT: In this article we denote the type-1 Bishop frame by $\{T, N_1, N_2\}$. And we consider a unit speed curve α in Euclidean three-dimensional space. Then we study the Position Vectors of Space Curves with constant curvatures in an arbitrary space curves in term of type-1 Bishop Frame in Euclidean 3- Space E^3 .

KEYWORDS: Euclidean 3-space; type-1 Bishop Frame; position vectors.

I. INTRODUCTION

In the differential geometry L.R. Bishop was introduced Bishop Frame since 1975, by means of parallel vector fields. The Bishop frame has many applications such as computer graphics and the biology areas and it may be so possible to compute information about the shape of sequences of DNA using a curve defined by it. Although it is desirable to have the camera gaze direction pointing forward along a space curve throughout the motion [8]. Recently, research papers which have something to do with this concept have been examined in Minkowski space, (see [2]). Since the 3-time continuously differentiable non-degenerate curves can be the Frenet frame. And we can take an alternative frame in E^3 because curvature may vanish so that the second derivative of the curve may be zero at some points. As in [2]. Since a new frame for a curve which has vanishing second derivative in Euclidean 3-space defined as well by Bishop. The goodness of Bishop Frame and also in comparable Bishop Frame according to the Frenet frame in Euclidean 3-space was given by Bishop [2] and Hanson [7]. The problem of the determination of position vector of an arbitrary space curve according to the type-1 Bishop frame is still open in the Euclidean 3- space our main problem is establish a system of differential equations whose solution gives the components of the position vector of a curve on the Bishop axis.

II. THE POSITION VECTOR OF A SPACE CURVE

For the importance of Position Vectors of Space Curves in mathematics field it appears in many work, such as in the local differential geometry as to describe the general helices, admissible curves, space like W-curves and so on. We think of curves as a geometric set of points, or locus. Intuitively, we are thinking of a curve as the path traced out by a particle moving in E^3 . So, investigating position vectors of the curves is a classical aim to determine behaviour of the particle. Ahmad T. Ali in [1] take the Position vectors of slant helices in Euclidean 3-space. In [5] Fathi study the position vector of space curve with constant curvatures in Euclidean 4- space with respect to Frenet frame. In [10] the authors establish a system of differential equations whose solution gives the components of the position vector of a curve on the Frenet axis with some special solutions. Kazm. Ilarslan and Özgürin [9] are study the position vectors of a Spacelike curve with constant curvatures, with Spacelike, time like and null principal normal in the Minkowski 3-space E_1^3 . The problem of the determination of the position vector of an arbitrary space curve according to type-1 Bishop Frame is still open in the Euclidean 3-space.

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III. PRELIMINARIES

In view to what is required in the next section, the following are the basic elements, of the theory of curves in the Euclidean space. Although these elements are briefly presented, the generalizations which give their overall idea are illustrated, with an inner product,

$$\langle x, y \rangle = \|x\| \|y\| \cos\theta \quad (2.1)$$

And a cross product,

$$x \times y = \|x\| \|y\| \sin\theta \hat{n} \quad (2.2)$$

If $x, y \in E^3$, and $\theta \in [0, \pi]$ is the angle between x and y and \hat{n} is the unit vector which is positively oriented perpendicular to the plane and spanned by x and y . In general,

$$x \perp y \Leftrightarrow \langle x, y \rangle = 0 \quad \text{and} \quad x \parallel y \Leftrightarrow x \times y = 0 \quad (2.3)$$

The standard flat metric was considered in Euclidean 3-space E^3 which is given by:

$$\langle \cdot, \cdot \rangle = dx_1^2 + dx_2^2 + dx_3^2 \quad (2.4)$$

And we can define the norm of a vector as $\|x\| = \sqrt{\langle x, x \rangle}$

If we take an arbitrary curve $\alpha: I \subset \mathbb{R} \rightarrow E^3$ in E^3 . Then the curve α is said to be of unit speed (or parameterized by arc length function s) if $\langle \frac{d\alpha}{ds}, \frac{d\alpha}{ds} \rangle = 1$.

We can re-parameterize $\alpha(s)$ as $\psi(\theta(s)) \neq 0$ if $\alpha(s) \neq 0$ for any s . So that ψ is parameterized by the arc-length.

The Frenet formulas are:

$$\begin{pmatrix} T \\ N \\ B \end{pmatrix}' = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} T \\ N \\ B \end{pmatrix} \quad (2.5)$$

Let $\{T, N, B\}$ be the Frenet frame of the curve α which have curvature κ and torsion τ . It is utterly recognized that unit speed curve no less than four continuous derivatives, one can combined three together orthogonal unit vector fields T, N_1 and N_2 are respectively, the tangent, the principal normal and then binormal vector fields [5].

Denote by $\{T, N_1, N_2\}$ the type-1 Bishop moving frame along the unit Speed curve α . Where the vectors T, N_1 and N_2 are mutually orthogonal vectors satisfying $\langle T, T \rangle = \langle N_1, N_1 \rangle = \langle N_2, N_2 \rangle = 1$. Then the type-1 Bishop formulas. And satisfying α are:

$$\begin{pmatrix} T \\ N_1 \\ N_2 \end{pmatrix}' = \begin{pmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & 0 \\ -k_2 & 0 & 0 \end{pmatrix} \begin{pmatrix} T \\ N_1 \\ N_2 \end{pmatrix} \quad (2.6)$$

Where k_1 and k_2 are the first and second Bishop curvature.

We can find that the Frenet frame and the type-1 Bishop Frame are satisfying the follows

$$\kappa(s) = \sqrt{k_1^2 + k_2^2}, \quad \theta(s) = \arctan\left(\frac{k_2}{k_1}\right) \text{ where } k_1 \neq 0, \quad \tau(s) = \frac{d\theta(s)}{ds}$$

Since k_1 and k_2 are effectively correspond to a Rectangular coordinate system for the polar coordinates, with $\theta = \int \tau(s) ds$. The tendency of the parallel transport frame take in the arbitrary choice of integration constant θ_0 , which vanish from τ (and hence from the Frenet frame) due to the differentiation.

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IV. MAIN RESULTS

Theorem 3.1. The position vector $\alpha(s)$ of space curve with constant curvatures in Euclidean 3- space with respect to the type-1 Bishop frame is given by:

$$\psi(s) = \lambda(s)T(s) + \left(\frac{(k_2 - c)}{m} + c_1 e^{\frac{1}{2}[-m - \sqrt{4+m^2}] \int \tau(s) ds} + c_2 e^{\frac{1}{2}[-m - \sqrt{4+m^2}] \int \tau(s) ds} \right) N_1(s) + cN_2(s) \quad (3.1)$$

Where

$$\lambda(s) = \left(\frac{c_1}{2m} [-m - \sqrt{4 + m^2}] e^{\frac{1}{2}[-m - \sqrt{4+m^2}] \int \tau(s) ds} + \frac{c_2}{2m} [-m + \sqrt{4 + m^2}] e^{\frac{1}{2}[-m + \sqrt{4+m^2}] \int \tau(s) ds} \right) \quad (3.2)$$

While c, c_1, c_2 are arbitrary constants and $m = f(\theta)$.

Proof. If we take $\psi(s)$ as an arbitrary curve in Euclidean space E^3 , then we can conclude that its position vector as follows:

$$\psi(s) = \lambda(s)T(s) + \mu(s)N_1(s) + \nu(s)N_2(s) \quad (3.3)$$

Where $\lambda(s), \mu(s)$ and $\nu(s)$ are differentiable functions of $s \in I \subset R$. Differentiating the above equation with respect to s , we get the following:

$$\psi' = \lambda'T + \lambda T' + \mu'N_1 + \mu N_1' + \nu'N_2 + \nu N_2' \quad (3.4)$$

And using the type-1 Bishop equations, we get the following:

$$\begin{cases} \lambda k_1 + \lambda' - \mu k_1 - \nu k_2 = 1 \\ \lambda k_2 + \mu' = 0 \\ \nu' = 0 \end{cases} \quad (3.5)$$

The last equation of (3.5) leads to:

$$\nu = c \quad (3.6)$$

Where c is constant

By change of variables as in [8]. $\theta = \int \tau(s) ds$. The second equation of (3.5) becomes:

$$\lambda = \frac{k_1 \dot{\mu}}{k_2} = f(\theta) \dot{\mu} \quad (3.7)$$

Where $\frac{k_1}{k_2} = f(\theta)$ and the dot denote the derivative with respect to θ

If we substitute the equations (3.6), and (3.7) into the first equation of (3.5) we will get the next equation:

$$\ddot{\mu} - \dot{\mu} f(\theta) + \dot{\mu} f(\theta) - \mu = \frac{(k_2 - c)}{f(\theta)} \quad (3.8)$$

By solving the equation(3.8), we obtain the position vector of an arbitrary curve in the type-1 Bishop frame. Here, we take a special case when $f(\theta) = m$. The equation above becomes:

$$\ddot{\mu} + m\dot{\mu} - \mu = \frac{(k_2 - c)}{m} \quad (3.9)$$

The general solution of equation (3.9) is:

$$\mu(\theta) = \frac{(k_2 - c)}{m} + c_1 e^{\frac{1}{2}[-m - \sqrt{4+m^2}] \theta} + c_2 e^{\frac{1}{2}[-m + \sqrt{4+m^2}] \theta} \quad (3.10)$$

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From (3.7), the function $\lambda(\theta)$ is given by:

$$\lambda(\theta) = \frac{c_1}{2m} \left[-m - \sqrt{4 + m^2} \right] e^{\frac{1}{2}[-m - \sqrt{4 + m^2}]\theta} + \frac{c_2}{2m} \left[-m + \sqrt{4 + m^2} \right] e^{\frac{1}{2}[-m + \sqrt{4 + m^2}]\theta} \quad (3.11)$$

By substituting equations (3.9), (3.10) and (3.11) in to equation (3.3) we arrive to equation (3.1), so we complete the proof of the theorem.

V. CONCLUSION

This paper presented the problem of the determination of position vector of an arbitrary space curve according to the type-1 Bishop frame in the Euclidean 3- space our main problem is establish a system of differential equations whose solution gives the components of the position vector of a curve on the Bishop axis.

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