

Different Types of Inequalities Exist Among Human Beings by Using Fuzzy Soft Matrices

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ABSTRACT: In this paper, we put forward certain aspects of inequality existed in all its forms like human beings. The Economic inequality makes the nation developed and undeveloped or developing. The inequality causes and affects the progress of nation. The problem of inequality should be eradicated from the initial level and thereby we extend the field by using fuzzy soft matrices and also we define different types of fuzzy soft matrices so as to study their basic properties. Finally, we define T-product of fuzzy soft matrices and construct decision making problem.

KEYWORDS: Fuzzy soft matrices(FSM), Union, intersection, Complement, Addition, Subtraction and Product of fuzzy soft matrices, T-product of fuzzy soft matrices.

I. INTRODUCTION

In 1965, Zadeh [16] introduced the notion of fuzzy set theory. In 1999, soft set theory was firstly introduced by Molodtsov [5] as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In 2001, Maji et al. [3] studied the theory of soft sets initiated by Molodtsov [5] and developed several basic notions of soft set theory. In 2011, Yong et al. [15] initiated matrix representation of fuzzy soft set and successfully applied the proposed notion of fuzzy soft matrix in certain decision making problems.

In 2012, Borah et al. [4] extended fuzzy soft matrix theory and its application. In 2012, Cagman and Enginoglu [1] defined fuzzy soft matrices and constructed decision making problem. In 2013, Md.Jalilu and T.K.Roy [2] have introduced different types of fuzzy soft matrices and some operators of fuzzy soft matrix on the basis of t-norms. In 2014, P.Rajarajeswari et al. [6] have proposed interval valued fuzzy soft matrix, its types with examples and some new operators on the basis of weights.

In 2014, Dr.N.Sarala and Rajkumari [7] have introduced intuitionistic fuzzy soft matrices in agriculture and issued [8] intuitionistic fuzzy soft matrices in medical diagnosis and also introduced [9] Role model service rendered to orphans by using fuzzy soft matrices and then [10] established characters of fuzzy soft matrices. Further, In 2015, Dr.N.Sarala and Rajkumari [11] proposed drug addiction effect in medical diagnosis by using fuzzy soft matrices and also [12] have presented its essential characteristic features in the field of intuitionistic fuzzy soft matrices and [13] put forward the notion of advantages and characteristic features of human life in villages by using fuzzy soft matrices. In 2016, Dr.N.Sarala and Rajkumari [14] have advocated the problem which is based on interval valued fuzzy soft matrices in food adulteration.

In this paper, we have established different types of properties of fuzzy soft matrices and also advocated decision making in T-Product of fuzzy soft matrices.

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II. PRELIMINARIES

In this section, we recall some basic essential notion of soft set theory and fuzzy soft matrices.

2.1 Soft Set [5]

Let U be an initial universe, Let $P(U)$ be the power set of U , E be the set of all parameters and $A \subseteq E$. A soft set (F_A, E) on the universe U is defined by the set of order pairs $(F_A, E) = \{(e, F_A(e)) : e \in E, F_A(e) \in P(U)\}$, where $F_A: E \rightarrow P(U)$ Such that $F_A(e) = \emptyset$ if $e \notin A$.

Here F_A is called an approximate function of the soft set (F_A, E) . The set $F_A(e)$ is called e-approximate value set or e-approximate set which consist of related objects of the parameter $e \in E$.

2.2 Soft Matrix[1]

Let (F_A, E) be a soft set over U . Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u, e) : e \in A, u \in F_A(e)\}$ which is called relation form of (F_A, E) . The characteristic function of R_A is written by $\chi_{R_A}: U \times E \rightarrow [0,1]$.

$$\chi_{R_A}(u, e) = \begin{cases} 1, & (u, e) \in R_A \\ 0, & (u, e) \notin R_A \end{cases}$$

If $U = \{u_1, u_2, u_3, \dots, u_m\}$, $E = \{e_1, e_2, e_3, \dots, e_n\}$ and $A \subseteq E$, then R_A can be represented by a table in the following form:

R_A	e_1	e_2	...	e_n
u_1	$\chi_{R_A}(u_1, e_1)$	$\chi_{R_A}(u_1, e_2)$...	$\chi_{R_A}(u_1, e_n)$
u_2	$\chi_{R_A}(u_2, e_1)$	$\chi_{R_A}(u_2, e_2)$...	$\chi_{R_A}(u_2, e_n)$
...
u_m	$\chi_{R_A}(u_m, e_1)$	$\chi_{R_A}(u_m, e_2)$...	$\chi_{R_A}(u_m, e_n)$

If $a_{ij} = \chi_{R_A}(u_i, e_j)$, we can define a matrix

$$[a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Which is called an $m \times n$ soft matrix of the soft set (F_A, E) over U .

According to the definition a soft set (F_A, E) is uniquely characterized by the matrix $[a_{ij}]_{m \times n}$. It means that a soft set (F_A, E) is formally equal to its soft matrix $[a_{ij}]_{m \times n}$. Therefore, we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

The set of all $m \times n$ soft matrices over U will be denoted by $SM_{m \times n}$. From now we shall use $[a_{ij}]$ in place of $[a_{ij}]_{m \times n}$, by omitting the subscripts $m \times n$ of $[a_{ij}]_{m \times n}$. Since $[a_{ij}] \in SM_{m \times n}$, it means that $[a_{ij}]$ is an $m \times n$ soft matrix for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.

Example 2.2: Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of all parameters. If $A = \{e_1, e_2, e_3\}$ and $F_A(e_1) = \{u_1, u_3\}$, $F_A(e_2) = \emptyset$, $F_A(e_3) = U$, then we write a soft set $(F_A, E) = \{(e_1, \{u_1, u_3\}), (e_3, U)\}$ and then the relation form of (F_A, E) is written by $R_A = \{(u_1, e_1), (u_3, e_1), (u_1, e_3), (u_2, e_3), (u_3, e_3), (u_4, e_3), (u_5, e_3)\}$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Hence the soft matrix $[a_{ij}]$ is written as $[a_{ij}] = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

2.3 Fuzzy Soft Set [3]

Let U be an initial universe set and E be a set of parameters. Let $A \subseteq E$. A pair (\tilde{F}_A, E) is called a fuzzy soft set (FSS) over U , where \tilde{F}_A is a mapping given by, $\tilde{F}_A: E \rightarrow I^U$, where I^U denotes the collection of all fuzzy subsets of U .

2.4 Fuzzy Soft Matrices(FSM) [1]

Let (\tilde{F}_A, E) be a fuzzy soft set over U . Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u, e) : e \in A, u \in \tilde{F}_A(e)\}$ which is called relation form of (\tilde{F}_A, E) . The characteristic function of R_A is written by $\mu_{R_A}: U \times E \rightarrow [0,1]$, where $\mu_{R_A}(u, e) \in [0,1]$ is the membership value of $u \in U$ for each $e \in E$.

If $\mu_{ij} = \mu_{R_A}(u_i, e_j)$, we can define a matrix

$$[\mu_{ij}]_{m \times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{m1} & \mu_{m2} & \dots & \mu_{mn} \end{bmatrix}$$

Which is called an $m \times n$ soft matrix of the soft set (\tilde{F}_A, E) over U .

Therefore we can say that a fuzzy soft set (\tilde{F}_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m \times n}$ and both concepts are interchangeable.

The set of all $m \times n$ Fuzzy soft matrices over U will be denoted by $FSM_{m \times n}$.

Example 2.4: Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of all parameters.

If $A \subseteq E = \{e_1, e_3, e_4\}$ and

$$\tilde{F}_A(e_1) = \{(u_1, 0.5), (u_2, 0.6), (u_3, 0.3), (u_4, 0.4), (u_5, 0.7)\}$$

$$\tilde{F}_A(e_3) = \{(u_1, 0.2), (u_2, 0.8), (u_3, 0.1), (u_4, 0.5), (u_5, 0.9)\}$$

$$\tilde{F}_A(e_4) = \{(u_1, 0.6), (u_2, 0.3), (u_3, 0.7), (u_4, 0.1), (u_5, 0.4)\}$$

Then the fuzzy soft set (\tilde{F}_A, E) is a parameterized family $\{\tilde{F}_A(e_1), \tilde{F}_A(e_3), \tilde{F}_A(e_4)\}$ of all fuzzy set over U .

Hence the fuzzy soft matrix $[\mu_{ij}]$ can be written as

$$[\mu_{ij}] = \begin{bmatrix} 0.5 & 0.0 & 0.2 & 0.6 \\ 0.6 & 0.0 & 0.8 & 0.3 \\ 0.3 & 0.0 & 0.1 & 0.7 \\ 0.4 & 0.0 & 0.5 & 0.1 \\ 0.7 & 0.0 & 0.9 & 0.4 \end{bmatrix}$$

2.5 Zero fuzzy soft matrix[1]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then \tilde{A} is called a zero fuzzy soft matrix denoted by $[\tilde{0}]$, if $a_{ij}^{\tilde{A}} = 0$ for all i and j .

2.6 Universal fuzzy soft matrix[1]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then \tilde{A} is called a universal fuzzy soft matrix denoted by $[\tilde{1}]$, if $a_{ij}^{\tilde{A}} = 1$ for all i and j .

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

2.7 Union of Fuzzy Soft Matrices[1]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$. Then union of \tilde{A} and \tilde{B} , denoted by $\tilde{A} \cup \tilde{B}$ is defined as $\tilde{A} \cup \tilde{B} = \max \{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}$ for all i and j .

2.8 Intersection of Fuzzy Soft Matrices[1]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$. Then intersection of \tilde{A} and \tilde{B} , denoted by $\tilde{A} \cap \tilde{B}$ is defined as $\tilde{A} \cap \tilde{B} = \min \{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}$ for all i and j .

2.9 Complement Fuzzy Soft Matrix[1]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then complement of fuzzy soft matrix \tilde{A} denoted by \tilde{A}^c is defined as $\tilde{A}^c = [a_{ij}^{\tilde{A}^c}] = [1 - a_{ij}^{\tilde{A}}]$ for all i and j .

2.10 Fuzzy soft Rectangular matrix[4]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then \tilde{A} is said to be a fuzzy soft rectangular matrix if $m \neq n$.

2.11 Fuzzy soft Square matrix[4]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then \tilde{A} is said to be a fuzzy soft square matrix if $m = n$.

2.12 Fuzzy soft Row matrix[4]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then \tilde{A} is said to be a fuzzy soft Row matrix if $m = 1$.

2.13 Fuzzy soft Colum matrix[4]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then \tilde{A} is said to be a fuzzy soft Colum matrix if $n = 1$.

2.14 Fuzzy soft sub matrix[1]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$. Then \tilde{A} is said to be a fuzzy soft sub matrix of \tilde{B} denoted by $\tilde{A} \subseteq \tilde{B}$, if $a_{ij}^{\tilde{A}} \leq b_{ij}^{\tilde{B}}$ for all i, j .

2.15 Fuzzy soft Transpose matrix[15]

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then we define $\tilde{A}^T = [a_{ij}^{\tilde{A}^T}]_{n \times m}$. Where $[a_{ij}^{\tilde{A}^T}]^T = a_{ji}$.

2.16 Addition of fuzzy soft matrices[6]

If $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$, $\tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$, then we define $\tilde{A} + \tilde{B}$, addition of \tilde{A} and \tilde{B} as $\tilde{A} + \tilde{B} = [c_{ij}^{\tilde{C}}]_{m \times n} = [\max(a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}})]$ for all i and j .

Example 2.16: Consider $\tilde{A} = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.3 \end{bmatrix}_{2 \times 2}$ and $\tilde{B} = \begin{bmatrix} 0.2 & 0.8 \\ 0.7 & 0.1 \end{bmatrix}_{2 \times 2}$ are two fuzzy soft matrices then sum of these

two is $\tilde{A} + \tilde{B} = \begin{bmatrix} 0.4 & 0.8 \\ 0.7 & 0.3 \end{bmatrix}_{2 \times 2}$.

2.17 Subtraction of fuzzy soft matrices[6]

If $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$, $\tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$, then we define $\tilde{A} - \tilde{B}$, subtraction of \tilde{A} and \tilde{B} as $\tilde{A} - \tilde{B} = [c_{ij}^{\tilde{C}}]_{m \times n} = [\min(a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}})]$ for all i and j .

Example 2.17: Consider $\tilde{A} = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.3 \end{bmatrix}_{2 \times 2}$ and $\tilde{B} = \begin{bmatrix} 0.2 & 0.8 \\ 0.7 & 0.1 \end{bmatrix}_{2 \times 2}$ are two fuzzy soft matrices then subtraction

of these two is $\tilde{A} - \tilde{B} = \begin{bmatrix} 0.2 & 0.6 \\ 0.5 & 0.1 \end{bmatrix}_{2 \times 2}$.

2.18 Product of fuzzy soft matrices[6]

If $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$, $\tilde{B} = [b_{jk}^{\tilde{B}}] \in FSM_{n \times p}$, then we define $\tilde{A} * \tilde{B}$, multiplication of \tilde{A} and \tilde{B} as $\tilde{A} * \tilde{B} = [c_{ik}^{\tilde{C}}]_{m \times p} = [\maxmin(a_{ij}^{\tilde{A}}, b_{jk}^{\tilde{B}})]$ for all i, j and k .

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Example 2.18: Consider $\tilde{A} = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.3 \end{bmatrix}_{2 \times 2}$ and $\tilde{B} = \begin{bmatrix} 0.2 & 0.8 \\ 0.7 & 0.1 \end{bmatrix}_{2 \times 2}$ are two fuzzy soft matrices then product of these two is $\tilde{A} * \tilde{B} = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.5 \end{bmatrix}_{2 \times 2}$.

III.IMPORTANT CHARACTERS OF FUZZY SOFT MATRICES

In this section, we put forward certain important characters of fuzzy soft matrices with some examples.

Proposition 3.1:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}], \tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m \times n}$. Then (i) $\tilde{A} \tilde{\cup} \tilde{A} = \tilde{A}$, (ii) $\tilde{A} \tilde{\cup} [\tilde{0}] = \tilde{A}$, (iii) $\tilde{A} \tilde{\cup} [\tilde{1}] = [\tilde{1}]$, (iv) $\tilde{A} \tilde{\cup} \tilde{B} = \tilde{B} \tilde{\cup} \tilde{A}$, (v) $(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} \tilde{C} = \tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cup} \tilde{C})$, (vi) $(\tilde{A} \tilde{\cup} \tilde{B})^T = \tilde{A}^T \tilde{\cup} \tilde{B}^T$.

Proof: For all i and j,

$$(i) \tilde{A} \tilde{\cup} \tilde{A} = \max\{a_{ij}^{\tilde{A}}, a_{ij}^{\tilde{A}}\} = \tilde{A}$$

$$(ii) \tilde{A} \tilde{\cup} [\tilde{0}] = \max\{a_{ij}^{\tilde{A}}, 0\} = [a_{ij}^{\tilde{A}}] = \tilde{A}$$

$$(iii) \tilde{A} \tilde{\cup} [\tilde{1}] = \max\{a_{ij}^{\tilde{A}}, 1\} = [1] = [\tilde{1}]$$

$$(iv) \tilde{A} \tilde{\cup} \tilde{B} = \max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\} = \max\{b_{ij}^{\tilde{B}}, a_{ij}^{\tilde{A}}\} = \tilde{B} \tilde{\cup} \tilde{A}$$

$$(v) (\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} \tilde{C} = [\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] \tilde{\cup} [c_{ij}^{\tilde{C}}] \\ = [(\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})] \\ = [(\max\{a_{ij}^{\tilde{A}}, (b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}})\})] \\ = [a_{ij}^{\tilde{A}}] \tilde{\cup} [\max\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\}] \\ = \tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cup} \tilde{C})$$

$$(vi) (\tilde{A} \tilde{\cup} \tilde{B})^T = [\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}]^T = [\max\{a_{ji}^{\tilde{A}}, b_{ji}^{\tilde{B}}\}] = [a_{ji}^{\tilde{A}}] \tilde{\cup} [b_{ji}^{\tilde{B}}] = \tilde{A}^T \tilde{\cup} \tilde{B}^T$$

Example 3.1:

Let $\tilde{A}, \tilde{B}, \tilde{C} \in FSM_{2 \times 2}$, where $\tilde{A} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0.1 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}, \tilde{C} = \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.5 \end{bmatrix}$

$$(i) \tilde{A} \tilde{\cup} \tilde{A} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} = \tilde{A}$$

Hence $\tilde{A} \tilde{\cup} \tilde{A} = \tilde{A}$

$$(ii) \tilde{A} \tilde{\cup} [\tilde{0}] = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} = \tilde{A}$$

Hence $\tilde{A} \tilde{\cup} [\tilde{0}] = \tilde{A}$

$$(iii) \tilde{A} \tilde{\cup} [\tilde{1}] = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 1.0 & 1.0 \\ 1.0 & 1.0 \end{bmatrix} = \begin{bmatrix} 1.0 & 1.0 \\ 1.0 & 1.0 \end{bmatrix} = [\tilde{1}]$$

Hence $\tilde{A} \tilde{\cup} [\tilde{1}] = [\tilde{1}]$

$$(iv) \tilde{A} \tilde{\cup} \tilde{B} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 0.1 & 0.7 \\ 0.4 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

$$\tilde{B} \tilde{\cup} \tilde{A} = \begin{bmatrix} 0.1 & 0.7 \\ 0.4 & 0.3 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

Hence $\tilde{A} \tilde{\cup} \tilde{B} = \tilde{B} \tilde{\cup} \tilde{A}$.

$$(v) (\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} \tilde{C} = \begin{bmatrix} 0.5 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

$$\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cup} \tilde{C}) = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

Hence $(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} \tilde{C} = \tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cup} \tilde{C})$

$$(vi) (\tilde{A} \tilde{\cup} \tilde{B})^T = \begin{bmatrix} 0.5 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}^T = \begin{bmatrix} 0.5 & 0.4 \\ 0.7 & 0.6 \end{bmatrix}$$

$$\tilde{A}^T \tilde{\cup} \tilde{B}^T = \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & 0.6 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 0.1 & 0.4 \\ 0.7 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.4 \\ 0.7 & 0.6 \end{bmatrix}$$

Hence $(\tilde{A} \tilde{\cup} \tilde{B})^T = \tilde{A}^T \tilde{\cup} \tilde{B}^T$

Proposition 3.2:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}], \tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m \times n}$. Then (i) $\tilde{A} \tilde{\cap} \tilde{A} = \tilde{A}$, (ii) $\tilde{A} \tilde{\cap} [\tilde{0}] = [\tilde{0}]$, (iii) $\tilde{A} \tilde{\cap} [\tilde{1}] = [\tilde{1}]$, (iv) $\tilde{A} \tilde{\cap} \tilde{B} = \tilde{B} \tilde{\cap} \tilde{A}$, (v) $(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cap} \tilde{C} = \tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cap} \tilde{C})$, (vi) $(\tilde{A} \tilde{\cap} \tilde{B})^T = \tilde{A}^T \tilde{\cap} \tilde{B}^T$.

Proof: For all i and j,

$$(i) \tilde{A} \tilde{\cap} \tilde{A} = \min\{a_{ij}^{\tilde{A}}, a_{ij}^{\tilde{A}}\} = \tilde{A}$$

$$(ii) \tilde{A} \tilde{\cap} [\tilde{0}] = \min\{a_{ij}^{\tilde{A}}, 0\} = [\tilde{0}]$$

$$(iii) \tilde{A} \tilde{\cap} [\tilde{1}] = \min\{a_{ij}^{\tilde{A}}, 1\} = [\tilde{1}]$$

$$(iv) \tilde{A} \tilde{\cap} \tilde{B} = \min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\} = \min\{b_{ij}^{\tilde{B}}, a_{ij}^{\tilde{A}}\} = \tilde{B} \tilde{\cap} \tilde{A}$$

$$\begin{aligned} (v) (\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cap} \tilde{C} &= [\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] \tilde{\cap} [c_{ij}^{\tilde{C}}] \\ &= [(\min\{(a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}), c_{ij}^{\tilde{C}}\})] \\ &= [(\min\{a_{ij}^{\tilde{A}}, (b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}})\})] \\ &= [a_{ij}^{\tilde{A}}] \tilde{\cap} [\min\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\}] \\ &= \tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cap} \tilde{C}) \end{aligned}$$

$$(vi) (\tilde{A} \tilde{\cap} \tilde{B})^T = [\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}]^T = [\min\{a_{ji}^{\tilde{A}}, b_{ji}^{\tilde{B}}\}] = [a_{ji}^{\tilde{A}}] \tilde{\cap} [b_{ji}^{\tilde{B}}] = \tilde{A}^T \tilde{\cap} \tilde{B}^T$$

Example 3.2: Let $\tilde{A}, \tilde{B}, \tilde{C} \in FSM_{2 \times 2}$ as in **Example 3.1**,

$$(i) \tilde{A} \tilde{\cap} \tilde{A} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} = \tilde{A}$$

Hence $\tilde{A} \tilde{\cap} \tilde{A} = \tilde{A}$

$$(ii) \tilde{A} \tilde{\cap} [\tilde{0}] = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} = [\tilde{0}]$$

Hence $\tilde{A} \tilde{\cap} [\tilde{0}] = \tilde{A}$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

$$(iii) \tilde{A} \tilde{\cap} [\tilde{1}] = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 1.0 & 1.0 \\ 1.0 & 1.0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} = \tilde{A}$$

Hence $\tilde{A} \tilde{\cap} [\tilde{1}] = [\tilde{1}]$

$$(iv) \tilde{A} \tilde{\cap} \tilde{B} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 0.1 & 0.7 \\ 0.4 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{bmatrix}$$

$$\tilde{B} \tilde{\cap} \tilde{A} = \begin{bmatrix} 0.1 & 0.7 \\ 0.4 & 0.3 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{bmatrix}$$

Hence $\tilde{A} \tilde{\cap} \tilde{B} = \tilde{B} \tilde{\cap} \tilde{A}$.

$$(v) (\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cap} \tilde{C} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}$$

$$\tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cap} \tilde{C}) = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}$$

Hence $(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cap} \tilde{C} = \tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cap} \tilde{C})$

$$(vi) (\tilde{A} \tilde{\cap} \tilde{B})^T = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{bmatrix}^T = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.3 \end{bmatrix}$$

$$\tilde{A}^T \tilde{\cap} \tilde{B}^T = \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & 0.6 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 0.1 & 0.4 \\ 0.7 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.3 \end{bmatrix}$$

Hence $(\tilde{A} \tilde{\cap} \tilde{B})^T = \tilde{A}^T \tilde{\cap} \tilde{B}^T$

Proposition 3.3:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$. Then De Morgan's type results are true (i) $(\tilde{A} \tilde{\cup} \tilde{B})^\circ = \tilde{A}^\circ \tilde{\cap} \tilde{B}^\circ$, (ii) $(\tilde{A} \tilde{\cap} \tilde{B})^\circ = \tilde{A}^\circ \tilde{\cup} \tilde{B}^\circ$.

Proof: For all i and j,

$$(i) (\tilde{A} \tilde{\cup} \tilde{B})^\circ = [\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}]^\circ = [1 - \max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] = [\min\{1 - a_{ij}^{\tilde{A}}, 1 - b_{ij}^{\tilde{B}}\}] = \tilde{A}^\circ \tilde{\cap} \tilde{B}^\circ$$

$$(ii) (\tilde{A} \tilde{\cap} \tilde{B})^\circ = [\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}]^\circ = [1 - \min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] = [\max\{1 - a_{ij}^{\tilde{A}}, 1 - b_{ij}^{\tilde{B}}\}] = \tilde{A}^\circ \tilde{\cup} \tilde{B}^\circ$$

Example 3.3:

Let $\tilde{A}, \tilde{B} \in FSM_{2 \times 2}$, where $\tilde{A} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.4 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.5 \end{bmatrix}$

$$(i) (\tilde{A} \tilde{\cup} \tilde{B})^\circ = \left(\begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.4 \end{bmatrix} \tilde{\cup} \begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.5 \end{bmatrix} \right)^\circ = \left(\begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.5 \end{bmatrix} \right)^\circ = \begin{bmatrix} 0.4 & 0.7 \\ 0.6 & 0.5 \end{bmatrix}$$

$$\tilde{A}^\circ \tilde{\cap} \tilde{B}^\circ = \begin{bmatrix} 0.5 & 0.7 \\ 0.8 & 0.6 \end{bmatrix} \tilde{\cap} \begin{bmatrix} 0.4 & 0.9 \\ 0.6 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.7 \\ 0.6 & 0.5 \end{bmatrix}$$

Hence $(\tilde{A} \tilde{\cup} \tilde{B})^\circ = \tilde{A}^\circ \tilde{\cap} \tilde{B}^\circ$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

$$(ii) (\tilde{A} \tilde{\cap} \tilde{B})^\circ = \left(\left[\begin{array}{cc} 0.5 & 0.3 \\ 0.2 & 0.4 \end{array} \right] \tilde{\cap} \left[\begin{array}{cc} 0.6 & 0.1 \\ 0.4 & 0.5 \end{array} \right] \right)^\circ = \left(\left[\begin{array}{cc} 0.5 & 0.1 \\ 0.2 & 0.4 \end{array} \right] \right)^\circ = \left[\begin{array}{cc} 0.5 & 0.9 \\ 0.8 & 0.6 \end{array} \right]$$

$$\tilde{A}^\circ \tilde{\cup} \tilde{B}^\circ = \left[\begin{array}{cc} 0.5 & 0.7 \\ 0.8 & 0.6 \end{array} \right] \tilde{\cup} \left[\begin{array}{cc} 0.4 & 0.9 \\ 0.6 & 0.5 \end{array} \right] = \left[\begin{array}{cc} 0.5 & 0.9 \\ 0.8 & 0.6 \end{array} \right]$$

$$\text{Hence } (\tilde{A} \tilde{\cap} \tilde{B})^\circ = \tilde{A}^\circ \tilde{\cup} \tilde{B}^\circ$$

Proposition 3.4:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$. Then (i) $(\tilde{A}^T)^T = \tilde{A}$ (ii) $(\tilde{A}^\circ)^T = (\tilde{A}^T)^\circ$

Proof: For all i and j,

(i) Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$

Here $\tilde{A}^T = [a_{ij}^{\tilde{A}}]^T = [a_{ji}^{\tilde{A}}]$

$(\tilde{A}^T)^T = [a_{ji}^{\tilde{A}}]^T = [a_{ij}^{\tilde{A}}] = \tilde{A}$

Hence $(\tilde{A}^T)^T = \tilde{A}$

(ii) Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, then

$\tilde{A}^\circ = [a_{ij}^{\tilde{A}}]^\circ = [1 - a_{ij}^{\tilde{A}}]$

$(\tilde{A}^\circ)^T = ([1 - a_{ij}^{\tilde{A}}])^T = [1 - a_{ji}^{\tilde{A}}]$

For $\tilde{A}^T = [a_{ji}^{\tilde{A}}]$

We have $(\tilde{A}^T)^\circ = [a_{ji}^{\tilde{A}}]^\circ = [1 - a_{ji}^{\tilde{A}}]$

Hence $(\tilde{A}^\circ)^T = (\tilde{A}^T)^\circ$

Proposition 3.5:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}], \tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m \times n}$. Then (i) $\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$, (ii) $(\tilde{A} + \tilde{B}) + \tilde{C} = \tilde{A} + (\tilde{B} + \tilde{C})$, (iii) $(\tilde{A} + \tilde{B})^T = \tilde{A}^T + \tilde{B}^T$.

Proof:

(i) Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$

Now $\tilde{A} + \tilde{B} = [a_{ij}^{\tilde{A}}] + [b_{ij}^{\tilde{B}}] = [\max(a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}})] = \max([b_{ij}^{\tilde{B}}, a_{ij}^{\tilde{A}}]) = \tilde{B} + \tilde{A}$

Hence $\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$

(ii) Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}], \tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m \times n}$,

Now $(\tilde{A} + \tilde{B}) + \tilde{C} = [\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] + [c_{ij}^{\tilde{C}}]$

$= [\max((a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}), c_{ij}^{\tilde{C}})]$

$= [\max(a_{ij}^{\tilde{A}}, (b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}))]$

$= \tilde{A} + (\tilde{B} + \tilde{C})$

Hence $(\tilde{A} + \tilde{B}) + \tilde{C} = \tilde{A} + (\tilde{B} + \tilde{C})$

(iii) Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$,

Now $\tilde{A} + \tilde{B} = [\max(a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}})]$

$(\tilde{A} + \tilde{B})^T = [\max(a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}})]^T$

$= [\max(a_{ji}^{\tilde{A}}, b_{ji}^{\tilde{B}})]$

$= [a_{ji}^{\tilde{A}}] + [b_{ji}^{\tilde{B}}]$

$= [a_{ij}^{\tilde{A}}]^T + [b_{ij}^{\tilde{B}}]^T$

$= \tilde{A}^T + \tilde{B}^T$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Hence $(\tilde{A} + \tilde{B})^T = \tilde{A}^T + \tilde{B}^T$

Example 3.5:

Let $\tilde{A}, \tilde{B}, \tilde{C} \in FSM_{2 \times 2}$, where $\tilde{A} = \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.2 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.1 \end{bmatrix}, \tilde{C} = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}$

$$(i) \quad \tilde{A} + \tilde{B} = \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.2 \end{bmatrix} + \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.7 & 1.0 \\ 1.1 & 0.3 \end{bmatrix}$$

$$\tilde{B} + \tilde{A} = \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.1 \end{bmatrix} + \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.7 & 1.0 \\ 1.1 & 0.3 \end{bmatrix}$$

Hence $\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$

$$(ii) \quad (\tilde{A} + \tilde{B}) + \tilde{C} = \begin{bmatrix} 0.7 & 1.0 \\ 1.1 & 0.3 \end{bmatrix} + \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} = \begin{bmatrix} 1.4 & 1.2 \\ 1.4 & 0.7 \end{bmatrix}$$

$$\tilde{A} + (\tilde{B} + \tilde{C}) = \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.2 \end{bmatrix} + \begin{bmatrix} 1.0 & 0.5 \\ 0.9 & 0.5 \end{bmatrix} = \begin{bmatrix} 1.3 & 1.2 \\ 1.4 & 0.7 \end{bmatrix}$$

Hence $(\tilde{A} + \tilde{B}) + \tilde{C} = \tilde{A} + (\tilde{B} + \tilde{C})$

$$(iii) \quad (\tilde{A} + \tilde{B})^T = \left(\begin{bmatrix} 0.7 & 1.0 \\ 1.1 & 0.3 \end{bmatrix} \right)^T = \begin{bmatrix} 0.7 & 1.1 \\ 1.0 & 0.3 \end{bmatrix}$$

$$\tilde{A}^T = \begin{bmatrix} 0.3 & 0.5 \\ 0.7 & 0.2 \end{bmatrix}; \tilde{B}^T = \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.1 \end{bmatrix}$$

$$\tilde{A}^T + \tilde{B}^T = \begin{bmatrix} 0.3 & 0.5 \\ 0.7 & 0.2 \end{bmatrix} + \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.7 & 1.1 \\ 1.0 & 0.3 \end{bmatrix}$$

Hence $(\tilde{A} + \tilde{B})^T = \tilde{A}^T + \tilde{B}^T$

Proposition 3.6:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}], \tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m \times n}$. Then (i) $\tilde{A} - \tilde{B} = \tilde{B} - \tilde{A}$, (ii) $(\tilde{A} - \tilde{B}) - \tilde{C} = \tilde{A} - (\tilde{B} + \tilde{C})$, (iii) $(\tilde{A} + \tilde{B}) - \tilde{C} = (\tilde{A} - \tilde{C}) + (\tilde{B} - \tilde{C})$ (iv) $\tilde{A} - (\tilde{B} + \tilde{C}) = (\tilde{A} - \tilde{B}) + (\tilde{A} - \tilde{C})$

Proof:

It follows from the definition.

Proposition 3.7:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$. Then (i) $\tilde{A} * \tilde{B} \neq \tilde{B} * \tilde{A}$.

Proof:

Let the following example may be taken into account for proof.

Example 3.7:

$$\text{Let } \tilde{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.2 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0.6 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

are two fuzzy soft matrices, then the product of these two matrices is

$$\tilde{A} * \tilde{B} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.2 \end{bmatrix}; \tilde{B} * \tilde{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.3 \end{bmatrix}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Hence $\tilde{A} * \tilde{B} \neq \tilde{B} * \tilde{A}$.

Proposition 3.8:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$. Then the following result hold:

(i) $(\tilde{A}^\circ) = \tilde{A}$ (ii) $[\tilde{0}] = [\tilde{1}]$ (iii) $(\tilde{A} + \tilde{B})^\circ = \tilde{A}^\circ - \tilde{B}^\circ$ (iv) $(\tilde{A} - \tilde{B})^\circ = \tilde{A}^\circ + \tilde{B}^\circ$.

Proof:

(i) Let $\tilde{A} = [a_{ij}^{\tilde{A}}] \in FSM_{m \times n}$,

Here $\tilde{A}^\circ = [1 - a_{ij}^{\tilde{A}}]$

$$\begin{aligned} (\tilde{A}^\circ)^\circ &= ([1 - a_{ij}^{\tilde{A}}])^\circ \\ &= [1 - (1 - a_{ij}^{\tilde{A}})] \\ &= [a_{ij}^{\tilde{A}}] = \tilde{A} \end{aligned}$$

Hence $(\tilde{A}^\circ)^\circ = \tilde{A}$

(ii) $[\tilde{0}]^\circ = [1 - 0] = [1] = [\tilde{1}]$

Hence $[\tilde{0}]^\circ = [\tilde{1}]$

(iii) Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$.

$$\begin{aligned} \text{Then } (\tilde{A} + \tilde{B})^\circ &= ([\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}])^\circ \\ &= [1 - \max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] \\ &= [\min\{1 - a_{ij}^{\tilde{A}}, 1 - b_{ij}^{\tilde{B}}\}] \\ &= \tilde{A}^\circ - \tilde{B}^\circ \end{aligned}$$

Hence $(\tilde{A} + \tilde{B})^\circ = \tilde{A}^\circ - \tilde{B}^\circ$

(iv) Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m \times n}$.

$$\begin{aligned} \text{Then } (\tilde{A} - \tilde{B})^\circ &= ([\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}])^\circ \\ &= [1 - \min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] \\ &= [\max\{1 - a_{ij}^{\tilde{A}}, 1 - b_{ij}^{\tilde{B}}\}] \\ &= \tilde{A}^\circ + \tilde{B}^\circ \end{aligned}$$

Hence $(\tilde{A} - \tilde{B})^\circ = \tilde{A}^\circ + \tilde{B}^\circ$

IV. T-PRODUCT OF FUZZY SOFT MATRIX THEORY

In this section, we exercise the work and gave the definition of T-product of fuzzy soft matrices and some properties.

4.1 T-product of fuzzy soft matrices:

Let $\tilde{A}_k = [a_{ij}^{\tilde{A}_k}] \in FSM_{m \times n}, k = 1, 2, \dots, l$. Then the T-product of fuzzy soft matrices, denoted by $\prod_{k=1}^l \tilde{A}_k =$

$\tilde{A}_1 \times \tilde{A}_2 \times \dots \times \tilde{A}_l$, is defined by $\prod_{k=1}^l \tilde{A}_k = [c_i]_{m \times 1}$, where $c_i = \frac{1}{n} \sum_{j=1}^n \prod_{k=1}^l a_{ij}^{\tilde{A}_k}, i = 1, 2, \dots, m$.

In this paper, we will take $T = \wedge$ or $T = \vee$ according to the type of the problems.

Example 4.1: Assume that $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3 \in FSM_{3 \times 2}$ are given as follows

$$\tilde{A}_1 = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.4 \\ 0.6 & 0.1 \end{bmatrix}, \tilde{A}_2 = \begin{bmatrix} 0.2 & 0.5 \\ 0.8 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}, \tilde{A}_3 = \begin{bmatrix} 0.4 & 0.6 \\ 0.1 & 0.9 \\ 0.7 & 0.2 \end{bmatrix}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Then the \wedge -product is

$$\prod_{k=1}^3 \tilde{A}_k = \tilde{A}_1 \times \tilde{A}_2 \times \tilde{A}_3 = \frac{1}{2} \begin{bmatrix} 0.5 \wedge 0.2 \wedge 0.4 + 0.3 \wedge 0.5 \wedge 0.6 \\ 0.2 \wedge 0.8 \wedge 0.1 + 0.4 \wedge 0.1 \wedge 0.9 \\ 0.6 \wedge 0.3 \wedge 0.7 + 0.1 \wedge 0.7 \wedge 0.2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0.5 \\ 0.2 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.10 \\ 0.20 \end{bmatrix}$$

Proposition 4.2:

Let $\tilde{A}, \tilde{B}, \tilde{C} \in FSM_{m \times n}$. Then (i) $\tilde{A} \times \tilde{B} = \tilde{B} \times \tilde{A}$ (ii) $(\tilde{A} \times \tilde{B}) \times \tilde{C} = \tilde{A} \times (\tilde{B} \times \tilde{C})$

Proof:

(i) Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}]$ be two fuzzy soft matrices. Then $\tilde{A} \times \tilde{B} = [c_i] = \left[\frac{1}{n} \sum_{j=1}^n a_{ij}^{\tilde{A}} T b_{ij}^{\tilde{B}} \right]; \tilde{B} \times \tilde{A} = [d_i] = \left[\frac{1}{n} \sum_{j=1}^n b_{ij}^{\tilde{B}} T a_{ij}^{\tilde{A}} \right]$
Since $a_{ij}^{\tilde{A}} T b_{ij}^{\tilde{B}} = b_{ij}^{\tilde{B}} T a_{ij}^{\tilde{A}}$, we have $c_i = d_i$
Hence $\tilde{A} \times \tilde{B} = \tilde{B} \times \tilde{A}$.

(ii) The proof is similar to that of (i).

Proposition 4.3:

Let $\tilde{A}, \tilde{B}, \tilde{C} \in FSM_{m \times n}, \tilde{B} \subseteq \tilde{A}$. Then $\tilde{A} \times \tilde{B} \subseteq \tilde{A} \times \tilde{C}$.

Proof:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}], \tilde{B} = [b_{ij}^{\tilde{B}}], \tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m \times n}$. Then
 $\tilde{A} \times \tilde{B} = [d_i] = \left[\frac{1}{n} \sum_{j=1}^n a_{ij}^{\tilde{A}} T b_{ij}^{\tilde{B}} \right]; \tilde{A} \times \tilde{C} = [e_i] = \left[\frac{1}{n} \sum_{j=1}^n a_{ij}^{\tilde{A}} T c_{ij}^{\tilde{C}} \right]$
Since $b_{ij}^{\tilde{B}} \leq c_{ij}^{\tilde{C}}$, we have $a_{ij}^{\tilde{A}} T b_{ij}^{\tilde{B}} \leq a_{ij}^{\tilde{A}} T c_{ij}^{\tilde{C}}$, thus $d_i \leq e_i$.
Hence $\tilde{A} \times \tilde{B} \subseteq \tilde{A} \times \tilde{C}$.

Definition 4.4:

Let $\tilde{A}_k \in FSM_{m \times n}, k = 1, 2, \dots, l$, their product is $\prod_{k=1}^l \tilde{A}_k = [c_i]_{m \times 1}$. Then the set $O_s = \{j: c_j = \max\{c_i: i = 1, 2, \dots, m\}\}$ is called the optimum subscript set, and the set $O_d = \{u_j: u_j \in U \text{ and } j \in O_s\}$ is called the optimum decision set of U .

V. APPLICATION OF FUZZY SOFT MATRICES IN DECISION MAKING PROBLEM

In this section, we elucidated the problem which is based on T-product of fuzzy soft matrices.

5.1 Algorithm:

Input: Fuzzy soft set with m objects, each of which has n parameters.

Output: An optimum set.

Step 1: Write out the fuzzy soft matrices according to the given fuzzy soft sets.

Step 2: Calculate the T-product of the fuzzy soft matrices.

Step 3: Find the optimum subscript set O_s .

Step 4: Find the optimum decision set O_d .

5.2 Case study:

Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of inequalities of human beings. Let $E = \{e_1, e_2, e_3\}$ be the set of living status of the people, Where $e_1 =$ “ Represent inequalities like rich and poor ”, $e_2 =$ “ Hierarchical patriarchal between men and women”, $e_3 =$ “ Poor status of children lead to child labour”. Suppose that an expert **Mr.X** and **Mr.Y** come to rank the inequalities and the following fuzzy soft matrices are constructed according to their parameters.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

$$\tilde{A} = \begin{bmatrix} 0.4 & 0.7 & 0.5 \\ 0.2 & 0.6 & 0.4 \\ 0.3 & 0.5 & 0.8 \\ 0.8 & 0.7 & 0.6 \end{bmatrix}; \tilde{B} = \begin{bmatrix} 0.8 & 0.2 & 0.7 \\ 0.4 & 0.3 & 0.1 \\ 0.2 & 0.8 & 0.7 \\ 0.5 & 0.6 & 0.3 \end{bmatrix}; \tilde{C} = \begin{bmatrix} 0.5 & 0.9 & 0.2 \\ 0.3 & 0.2 & 0.6 \\ 1.0 & 0.3 & 0.4 \\ 0.7 & 0.5 & 0.8 \end{bmatrix}$$

$$\tilde{A} \times \tilde{B} \times \tilde{C} = \frac{1}{3} \begin{bmatrix} 0.4 \wedge 0.8 \wedge 0.5 + 0.7 \wedge 0.2 \wedge 0.9 + 0.5 \wedge 0.7 \wedge 0.2 \\ 0.2 \wedge 0.4 \wedge 0.3 + 0.6 \wedge 0.3 \wedge 0.2 + 0.4 \wedge 0.1 \wedge 0.6 \\ 0.3 \wedge 0.2 \wedge 1.0 + 0.5 \wedge 0.8 \wedge 0.3 + 0.8 \wedge 0.7 \wedge 0.4 \\ 0.8 \wedge 0.5 \wedge 0.7 + 0.7 \wedge 0.6 \wedge 0.5 + 0.6 \wedge 0.3 \wedge 0.8 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0.4 + 0.2 + 0.2 \\ 0.2 + 0.2 + 0.1 \\ 0.2 + 0.3 + 0.4 \\ 0.5 + 0.5 + 0.3 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0.8 \\ 0.5 \\ 0.9 \\ 1.3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.27 \\ 0.17 \\ 0.30 \\ 0.43 \end{bmatrix}$$

It is clear that the maximum score is 0.43, scored by u_4 and the decision is selecting the inequality higher rank found in u_4 .

VI. CONCLUSION

In this paper, we define the properties of fuzzy soft matrices and T-product of fuzzy soft matrices. We create awareness to eradicate inequalities and balanced stage to be existed in all kinds. We put forward that our work would enhance the scope and specific features among human life by using fuzzy soft matrices.

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