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A New Weaker Form of πgb -Continuity

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ABSTRACT : In this paper we have introduced a new class of functions in the light of simple extended topological spaces termed as π generalized b^+ -continuous functions (briefly πgb^+ continuous functions). Further, the concept of almost πgb^+ continuous function and πgb^+ -irresolute functions are discussed and their characterizations are derived.

KEYWORDS: πgb^+ -continuous function, almost πgb^+ -continuous function, πgb^+ -irresolute function.

I. INTRODUCTION

Levine [18] and Andrijevic [1] introduced the concept of generalized open sets and b -open sets respectively in topological spaces. The class of b -open sets is contained in the class of semi pre-open sets and contains the class of semi-open and the class of pre-open sets. Since then several researches were done and the notion of generalized semi-closed, generalized pre-closed and generalized semi pre-open sets were investigated in [20, 3, 31]. In 1968 Zaitsev [33] defined π -closed sets. Later Dontchev and Noiri [7] introduced the notion of πg -closed sets. Park [26] defined πgp -closed sets. Then Aslim, Caksu and Noiri [4] introduced the notion of πgs -closed sets. The idea of πgb -closed sets were introduced by D.Sreeja and S.Janaki [30]. Later the properties and characteristics of πgb -closed sets were introduced by Sinem Caglar and Gulhan Ashim [29]. In 1963, Levine [17] introduced the concept of simple extension of a topology τ as $\tau^+(B) = \{OU(O' \cap B) / O, O' \in \tau, B \notin \tau\}$.

The aim of this paper is to investigate the notion of πgb^+ -continuous functions and its properties. In section 4, πgb^+ -continuous functions and πgb^+ -irresolute functions are discussed. In section 5 we study the basic properties of almost πgb^+ -continuity and some of its related continuities are discussed. Finally, in section 6, πgb^+ -compactness is discussed. In this paper, (X, τ^+) (or simply X) represent a non-empty extended topological space on which no separation axioms are assumed, unless and otherwise mentioned. Let A be a subset of a space X . Then the closure of A is assumed in the extended topology and the interior of A is taken in general topology are denoted by $cl^+(A)$ and $int(A)$ respectively.

II. PRELIMINARIES

Definition 2.1:

A subset A of a topological space X is called a:

- (a) pre-open set [21] if $A \subset int(cl(A))$
- (b) semi-open set [17] if $A \subset cl(int(A))$
- (c) α -open set [23] if $A \subset int(cl(int(A)))$
- (d) semi-pre open set [1] if $A \subset cl(int(cl(A)))$
- (e) regular – open [32] if $A = int(cl(A))$
- (f) b – open [2] or γ – open [9] if $A \subset int(cl(A)) \cup cl(int(A))$

Definition 2.2:

A subset A of a space (X, τ) is called π open [33] if A is a finite union of regular open sets.

Definition 2.3:

A subset A of a topological space X is called:

- (a) g – closed [18] if $cl(A) \subset U$ whenever $A \subset U$ and U is open in X ,
- (b) gp – closed [20] if $pcl(A) \subset U$ whenever $A \subset U$ and U is open in X ,

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- (c) gs - closed [3] if $scl(A) \subset U$ whenever $A \subset U$ and U is open in X ,
- (d) gb - closed [14] if $bcl(A) \subset U$ whenever $A \subset U$ and U is open in X ,
- (e) πg - closed [7] if $cl(A) \subset U$ whenever $A \subset U$ and U is π - open in X ,
- (f) πgp - closed [26] if $pcl(A) \subset U$ whenever $A \subset U$ and U is π - open in X ,
- (g) πgs - closed [4] if $scl(A) \subset U$ whenever $A \subset U$ and U is π - open in X ,
- (h) $\pi g\alpha$ - closed [15] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is π - open in X .
- (i) πgsp - closed [28] if $spcl(A) \subset U$ whenever $A \subset U$ and U is π - open in X .
- (j) πgb - closed [30] if $bcl(A) \subset U$ whenever $A \subset U$ and U is π - open in X .

Definition 2.4:

A function $f: X \rightarrow Y$ is said to be

- (a) regular open [24] if $f(V)$ is regular open in Y for every open set V of X ,
- (b) b - closed [25] if $f(V)$ is b - closed in Y for every b - closed set V of X ,
- (c) m - π - closed [11] if $f(V)$ is π - closed in Y for every π - closed set V of X ,
- (d) π - continuous[7] (resp. πg - continuous[8], πgp - continuous[27], πgs - continuous[4]) if $f^{-1}(V)$ is π - closed (resp. πg - closed, πgp - closed, πgs - closed) in X for every closed set V of Y
- (e) b - continuous [9] (resp. g - continuous[18], gb - continuous[25]) if $f^{-1}(V)$ is b - closed (resp g -closed, gb - closed) in X for every closed set V of Y ,
- (f) Almost b - continuous [16] if $f^{-1}(V)$ is b - closed in X for every regular closed set V of Y ,
- (g) πgb - continuous[30] if $f^{-1}(V)$ is πgb - closed in X for every closed set V of Y ,
- (h) π - irresolute [4] if $f^{-1}(V)$ is π - closed in X for every π - closed set V of Y ,
- (i) b - irresolute [12] if $f^{-1}(V)$ is b - closed in X for every b - closed set V of Y ,
- (j) πgb - irresolute[30] if $f^{-1}(V)$ is πgb - closed in X for every πgb - closed set V of Y .

Definition 2.5:

A subset A of a topological space X is said to be a:

- (a) pre^+ -open set [22] if $A \subset int(cl^+(A))$
- (b) $semi^+$ -open set [22] if $A \subset cl^+(int(A))$
- (c) α^+ -open set [22] if $A \subset int(cl^+(int(A)))$
- (d) $semi$ - pre^+ open set [22] if $A \subset cl^+(int(cl^+(A)))$
- (e) b^+ - open [22] or γ^+ - open if $A \subset int(cl^+(A)) \cup cl^+(int(A))$

III. SIMPLE EXTENSION OF SETS

Definition 3.1:

- (a) A subset A of a topological space X is said to be regular - open in simple extended topology if $A = int(cl^+(A))$.
 - (b) A subset A of a space (X, τ^+) is called π^+ open if A is a finite union of regular⁺ open sets.
- The intersection of all b^+ - closed sets containing A is said to be b^+ - closure of A and is denoted by $b^+cl(A)$.

Definition 3.2:

A subset A of a topological space X is called:

- (a) g^+ - closed if $cl^+(A) \subset U$ whenever $A \subset U$ and U is open in X ,
- (b) gb^+ - closed if $b^+cl(A) \subset U$ whenever $A \subset U$ and U is open in X ,
- (c) πg^+ - closed if $cl^+(A) \subset U$ whenever $A \subset U$ and U is π^+ - open in X ,

Definition 3.3:

A function $f: X \rightarrow Y$ is said to be

- (a) regular⁺ open if $f(V)$ is regular⁺ open in Y for every open set V of X ,
- (b) b^+ - closed if $f(V)$ is b^+ - closed in Y for every b^+ - closed set V of X ,
- (c) m - π^+ - closed if $f(V)$ is π^+ - closed in Y for every π^+ - closed set V of X ,

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- (d) π^+ - continuous (resp. πg^+ - continuous, $\pi g p^+$ - continuous, $\pi g s^+$ - continuous) if $f^{-1}(V)$ is π^+ - closed (resp. πg^+ - closed, $\pi g p^+$ - closed, $\pi g s^+$ - closed) in X for every closed set V of Y
- (e) b^+ - continuous (resp. g^+ - continuous, $g b^+$ - continuous) if $f^{-1}(V)$ is b^+ - closed (resp. g^+ - closed, $g b^+$ - closed) in X for every closed set V of Y ,
- (f) Almost b^+ - continuous if $f^{-1}(V)$ is b^+ - closed in X for every regular closed set V of Y ,
- (g) $\pi g b^+$ - continuous if $f^{-1}(V)$ is $\pi g b^+$ - closed in X for every closed set V of Y ,
- (h) π^+ - irresolute if $f^{-1}(V)$ is π^+ - closed in X for every π^+ - closed set V of Y ,
- (i) b^+ - irresolute if $f^{-1}(V)$ is b^+ - closed in X for every b^+ - closed set V of Y ,
- (j) $\pi g b^+$ - irresolute if $f^{-1}(V)$ is $\pi g b^+$ - closed in X for every $\pi g b^+$ - closed set V of Y .

Definition 3.4:

A subset A of (X, τ^+) is called $\pi g b^+$ - closed if $b^+cl(A) \subset U$ whenever $A \subset U$ and U is π^+ - open in (X, τ^+) .

IV . $\pi g b^+$ - CONTINUOUS AND $\pi g b^+$ - IRRESOLUTE FUNCTIONS

Definition 4.1:

A function $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called $\pi g b^+$ - continuous if every $f^{-1}(V)$ is $\pi g b^+$ - closed in (X, τ^+) for every closed set V of (Y, σ^+) .

Definition 4.2:

A function $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called $\pi g b^+$ - irresolute if $f^{-1}(V)$ is $\pi g b^+$ - closed in (X, τ^+) for every $\pi g b^+$ - closed set V in (Y, σ^+) .

Definition 4.3:

A function $f: X \rightarrow Y$ is said to be pre b^+ -closed if $f(U)$ is b^+ -closed in Y for each b^+ -closed set in X .

Proposition 4.4:

Let $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ be π^+ - irresolute and pre b^+ -closed map. Then $f(A)$ is $\pi g b^+$ - closed in Y for every $\pi g b^+$ - closed set A of X .

Definition 4.5:

A topological space X is a $\pi g b^+$ - space if every $\pi g b^+$ - closed set is closed.

Proposition 4.6:

Every $\pi g b^+$ -space is $\pi g b^+$ - $T_{1/2}$ space.

Theorem 4.7:

Let $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ be a function.

- (1) If f is $\pi g b^+$ - irresolute and X is $\pi g b^+$ - $T_{1/2}$ space, then f is b^+ -irresolute.
- (2) If f is $\pi g b^+$ - continuous and X is $\pi g b^+$ - $T_{1/2}$ space, then f is b^+ -continuous

Proof:

(1) Let V be b^+ -closed in Y . Since f is $\pi g b^+$ -irresolute, $f^{-1}(V)$ is $\pi g b^+$ -closed in X . Since X is $\pi g b^+$ - $T_{1/2}$ space, $f^{-1}(V)$ is b^+ -closed in X . Hence f is b^+ -irresolute.

(2) Let V be closed in Y . Since f is $\pi g b^+$ -continuous, $f^{-1}(V)$ is $\pi g b^+$ - closed in X . By assumption, it is b^+ -closed. Therefore f is b^+ -continuous.

Definition 4.8:

A function $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is π^+ -open map if $f(F)$ is π^+ -open map in Y for every π^+ -open in X .

Theorem 4.9:

If the bijective $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is b^+ -irresolute and π^+ -open map, then f is $\pi g b^+$ - irresolute.

Proof:

Let V be $\pi g b^+$ -closed in Y . Let $f^{-1}(V) \subset U$ where U is π^+ - open in X . hence $V \subset f(U)$ and $f(U)$ is π^+ -open implies $b^+cl(V) \subset f(U)$. Since f is b^+ -irresolute, $(f^{-1}(b^+cl(V)))$ is b^+ -closed. Hence $b^+cl(f^{-1}(V)) \subset b^+cl(f^{-1}(b^+cl(V))) = f^{-1}(b^+cl(V)) \subset U$. Therefore, f is $\pi g b^+$ - irresolute.

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Theorem 4.10:

If $f: X \rightarrow Y$ is π^+ -open, b^+ -irresolute, pre b^+ -closed surjective function. If X is $\pi gb^+-T_{1/2}$ space, then Y is $\pi gb^+-T_{1/2}$ space.

Proof:

Let F be a πgb^+ -closed set in Y . Let $f^{-1}(F) \subset U$ where U is π^+ -open in X . Then $F \subset f(U)$ and F is a πgb^+ -closed set in Y implies $b^+cl(F) \subset f(U)$. Since f is b^+ -irresolute, $b^+cl(f^{-1}(F)) \subset b^+cl(f^{-1}(b^+cl(F))) = f^{-1}(b^+cl(F)) \subset U$. Therefore $f^{-1}(F)$ is πgb^+ -closed in X . Since X is $\pi gb^+-T_{1/2}$ space, $f^{-1}(F)$ is b^+ -closed in X . Since f is pre b^+ -closed, $f(f^{-1}(F)) = F$ is b^+ -closed in Y . Hence Y is $\pi gb^+-T_{1/2}$ space.

V. ALMOST πgb^+ - CONTINUITY AND RELATED SOME CONTINUITIES

Definition 5.1:

A function $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called almost πgb^+ - continuous if every $f^{-1}(V)$ is πgb^+ - closed in (X, τ^+) for every regular closed set V of (Y, σ^+) .

Remark 5.2:

1. Every continuous function is πgb^+ -continuous.
2. Every g^+ - continuous function is πgb^+ - continuous.
3. Every α^+ - continuous function is πgb^+ - continuous.
4. Every pre $^+$ - continuous function is πgb^+ - continuous.
5. Every gb^+ - continuous function is πgb^+ - continuous.
6. Every πg^+ - continuous function is πgb^+ - continuous.
7. Every πgp^+ - continuous function is πgb^+ - continuous.
8. Every $\pi g\alpha^+$ - continuous function is πgb^+ - continuous.
9. Every πgs^+ - continuous function is πgb^+ - continuous.
10. Every πgb^+ - continuous function is πgsp^+ - continuous.

Proof:

Straight forward.

Converse of the above need not be true.

Theorem 5.3:

Let $f: X \rightarrow Y$ be a function. Then the following statements are equivalent:

- (1) f is πgb^+ -continuous;
- (2) The inverse image of every open set in Y is πgb^+ -open in X .

Proof:

(1) \Rightarrow (2) Let U be a open subset of X . Then $Y \setminus U$ is closed. Since f is πgb^+ -continuous,

$f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is πgb^+ -closed in X . Hence $f^{-1}(U)$ is πgb^+ -open in X .

(2) \Rightarrow (1) Let V be a closed subset of Y . Then $Y \setminus V$ is open and by the hypothesis (2) $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$

is πgb^+ -open in X . So $f^{-1}(V)$ is πgb^+ -closed. Therefore, f is πgb^+ -continuous.

Theorem 5.4:

If $f: X \rightarrow Y$ is πgb^+ -continuous then $f(\pi gb^+-cl(A)) \subset cl^+(f(A))$ for every subset A of X .

Proof:

Let A be a subset of X . Since f is πgb^+ -continuous and $A \subset f^{-1}(cl^+(f(A)))$, we obtain $\pi gb^+-cl(A) \subset f^{-1}(cl^+(f(A)))$ and then $f(\pi gb^+-cl(A)) \subset cl^+(f(A))$.

Theorem 5.5:

Let X be an extremely disconnected space and $f: X \rightarrow Y$ be a function. If f is πgb^+ -continuous and $m-\pi^+$ -closed then f is πgb^+ -irresolute.

Proof:

Let A be a πgb^+ -closed subset of Y . Then $f^{-1}(A) \subset U$, where U is π^+ -open in X . So $X \setminus U \subset f^{-1}(Y \setminus A)$. Hence $f(X \setminus U) \subset Y \setminus A$. Since f is $m-\pi^+$ -closed, $f(X \setminus U)$ is π^+ -closed. Since $Y \setminus A$ is πgb^+ - open then $f(X \setminus U) \subset b^+int(Y \setminus A) = Y \setminus b^+cl(A)$. Thus $f^{-1}(b^+cl(A)) \subset U$. Since f is πgb^+ -continuous and X is extremely disconnected, $f^{-1}(cl^+(A))$ is πgb^+ -closed. Therefore, $b^+cl(f^{-1}(b^+cl(A))) \subset U$ and hence $b^+cl(f^{-1}(A)) \subset b^+cl(f^{-1}(b^+cl(A))) \subset U$. It follows that $f^{-1}(A)$ is πgb^+ -closed. This shows that f is πgb^+ -irresolute.

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Theorem 5.6:

For a function $f : X \rightarrow Y$, the following statements are equivalent:

- (1) f is almost πgb^+ -continuous;
- (2) $f^{-1}(V)$ is πgb^+ -open in X for every regular open set V of Y ;
- (3) $f^{-1}(\text{int}(\text{cl}^+(V)))$ is πgb^+ -open in X for every open set V of Y ;
- (4) $f^{-1}(\text{cl}^+(\text{int}(V)))$ is πgb^+ -closed in X for every closed set V of Y .

Proof:

- (1) \Rightarrow (2) Let V be a regular open subset of Y . Since $Y \setminus V$ is regular closed and f is almost πgb^+ -continuous then $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is πgb^+ -closed in X . Thus $f^{-1}(V)$ is πgb^+ -open in X .
- (2) \Rightarrow (1) Let V be a regular closed subset of Y . Then $Y \setminus V$ is regular open. By the hypothesis, $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is πgb^+ -open in X . Hence $f^{-1}(V)$ is πgb^+ -closed. This shows that f is πgb^+ -continuous.
- (2) \Rightarrow (3) Let V be an open subset of Y . Then $\text{int}(\text{cl}^+(V))$ is regular open. By the hypothesis, $f^{-1}(\text{int}(\text{cl}^+(V)))$ is πgb^+ -open in X .
- (3) \Rightarrow (2) Let V be a regular open subset of Y . Since $V = \text{int}(\text{cl}^+(V))$ and every regular open set is open then $f^{-1}(V)$ is πgb^+ -open in X .
- (3) \Rightarrow (4) Let V be a closed subset of Y . Then $Y \setminus V$ is open. By the hypothesis, $f^{-1}(\text{int}(\text{cl}^+(Y \setminus V))) = f^{-1}(\text{int}(\text{cl}^+(Y \setminus V))) = f^{-1}(Y \setminus \text{cl}^+(\text{int}(V))) = X \setminus f^{-1}(\text{cl}^+(\text{int}(V)))$ is πgb^+ -open in X . Therefore, $f^{-1}(\text{cl}^+(\text{int}(V)))$ is πgb^+ -closed in X .
- (4) \Rightarrow (3) Let V be an open subset of Y . Then $Y \setminus V$ is closed. By the hypothesis, $f^{-1}(\text{cl}^+(\text{int}(Y \setminus V))) = f^{-1}(Y \setminus \text{int}(\text{cl}^+(V))) = X \setminus f^{-1}(\text{int}(\text{cl}^+(V)))$ is πgb^+ -closed in X . Hence $f^{-1}(\text{int}(\text{cl}^+(V)))$ is πgb^+ -open in X .

Remark 5.7:

- (a) Every πgb^+ -continuous function is almost πgb^+ -continuous,
- (b) Every almost b^+ -continuous function is almost πgb^+ -continuous.

VI. πgb^+ -COMPACTNESS

Definition 6.1:

A collection $\{G_i : i \in \Lambda\}$ of πgb^+ -open sets in a topological space X is called a πgb^+ -open cover of a subset A of X if $A \subset \bigcup \{G_i : i \in \Lambda\}$ holds.

Definition 6.2:

A topological space X is πgb^+ -compact if every πgb^+ -open cover of X has a finite subcover.

Definition 6.3:

A subset A of a topological space X is said to be πgb^+ -compact relative to X if, for every collection $\{U_i : i \in I\}$ of πgb^+ -open subsets of X such that $A \subset \bigcup \{U_i : i \in I\}$ there exists a finite subset I_0 of I such that $A \subset \bigcup \{U_i : i \in I_0\}$.

Definition 6.4:

A subset A of a topological space X is said to be πgb^+ -compact if A is πgb^+ -compact as a subspace of X .

Theorem 6.5:

Every πgb^+ -closed subset of a πgb^+ -compact space is πgb^+ -compact space relative to X .

Proof:

Let A be a πgb^+ -closed subset of a πgb^+ -compact space X . Let $\{U_i : i \in I\}$ be a πgb^+ -open cover of X . So $A \subset \bigcup_{i \in I} U_i$ and then $(X \setminus A) \cup (\bigcup_{i \in I} U_i) = X$. Since X is πgb^+ -compact, there exists a finite subset I_0 of I such that $(X \setminus A) \cup (\bigcup_{i \in I_0} U_i) = X$. Then $A \subset \bigcup_{i \in I_0} U_i$ and hence A is πgb^+ -compact relative to X .

Definition 6.6:

A space X is said to be

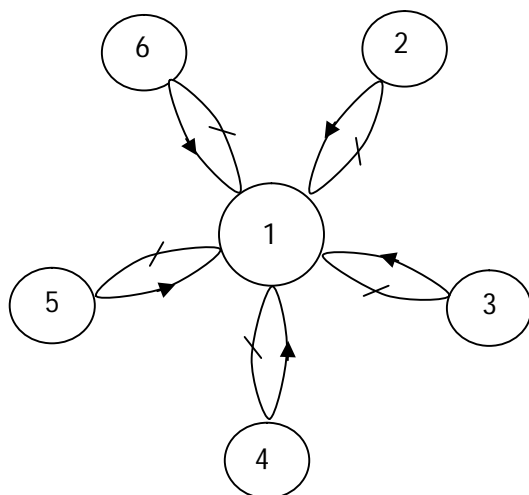
- (1) πgb^+ -compact if every πgb^+ -open cover of X has a finite subcover.
- (2) gb^+ -compact if every gb^+ -open cover of X has a finite subcover.
- (3) b^+ -compact if every b^+ -open cover of X has a finite subcover.

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Remark 6.7:



1. πgb^+ -compact
2. gb^+ -compact
3. $b3^+$ -compact
4. compact
5. nearly-compact
6. πgp^+ -compact

VI. QUASI b^+ -NORMAL SPACES

Definition 6.8:

A space X is said to be quasi- b^+ -normal if for every pair of disjoint π^+ -closed subsets A, B of X , there exist disjoint b^+ -open subsets U, V of X such that $A \subset U$ and $B \subset V$.

Definition 6.9:

A space X is said to be quasi $^+$ -normal (resp. quasi- s^+ -normal) if for every pair of disjoint π^+ -closed subsets A, B of X , there exist disjoint open (resp. semi $^+$ -open) subsets U, V of X such that $A \subset U$ and $B \subset V$.

Definition 6.10:

A space X is said to be b^+ -normal (or γ^+ -normal) if for every pair of disjoint closed subsets A, B of X , there exist disjoint b^+ -open subsets U, V of X such that $A \subset U$ and $B \subset V$.

Theorem 6.11:

The following statements are equivalent for a space X ;

- (a) X is quasi- b^+ -normal;
- (b) For any disjoint π^+ -closed sets A and B , there exist disjoint gb^+ -open subsets U, V of X such that $A \subset U$ and $B \subset V$;
- (c) For any closed set A and any π^+ -open set B containing A , there exists a gb^+ -open set U such that $A \subset U \subset b^+cl(U) \subset B$;
- (d) For any disjoint π^+ -closed sets A and B , there exist disjoint πgb^+ -open subsets U, V of X such that $A \subset U$ and $B \subset V$;
- (e) For any π^+ -closed set A and any π^+ -open set B containing A , there exists a πgb^+ -open set U such that $A \subset U \subset b^+cl(U) \subset B$.

Proof:

(a) \Rightarrow (b) The proof is obvious.

(b) \Rightarrow (c) Let A be any π^+ -closed subset of X and B any π^+ -open subset of X such that $A \subset B$. Then A and $X \setminus B$ disjoint π^+ -closed subset of X . Therefore, there exist disjoint gb^+ -open sets U and V such that $A \subset U$ and $X \setminus B \subset V$.

By the definition of gb^+ -open sets, we have $X \setminus B \subset b^+int(V)$ and $U \cap b^+int(V) = \emptyset$. Therefore, we obtain $b^+cl(U) \cap b^+int(V) = \emptyset$ and hence $A \subset U \subset b^+cl(U) \subset B$.

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(c) \Rightarrow (d) Let A and B be any disjoint π^+ -closed subsets of X . Then $A \subset X \setminus B$ and $X \setminus B$ is π^+ -open and hence there exists a gb^+ -open subset G of X such that $A \subset G \subset b^+cl(G) \subset X \setminus B$. Since every gb^+ -open set is πgb^+ -open, G is πgb^+ -open and $X \setminus b^+cl(G)$ is b^+ -open so $X \setminus b^+cl(G)$ is πgb^+ -open. Now put $V = X \setminus b^+cl(G)$. Then G and V are disjoint πgb^+ -open subsets of X such that $A \subset G$ and $B \subset V$.

(d) \Rightarrow (e) The proof is similar to that of (b) \Rightarrow (c)

(e) \Rightarrow (a) Let A and B be any disjoint π^+ -closed subsets of X . Then $A \subset X \setminus B$ and $X \setminus B$ is π^+ -open and hence there exists a πgb^+ -open subset G of X such that $A \subset G \subset b^+cl(G) \subset X \setminus B$. Put $U = b^+int(G)$ and $V = X \setminus b^+cl(G)$. Then U and V are disjoint b^+ -open subsets of X such that $A \subset U$ and $B \subset V$. Therefore, X is quasi b^+ -normal.

VII. CONCLUSION

This paper have explored the relationship between πgb^+ -closed sets using different types of continuities and irresoluteness. Further new spaces such as quasi b^+ normal and πgb^+ compact spaces have been introduced and its properties are studied.

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