

On a Weaker Form of b-open Sets in Extended Topological Spaces

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ABSTRACT: This paper throws light on conceptualizing the weaker form of b-open sets under the ceiling of simple extension topology. Here we intend to introduce a new form of set namely ${}^*+b$ -open set, b^{+**} -open set, ${}^{**+}b$ -open set, t^{+*} -set, $D(c, b^+)$ -set, $D(c, b^{+**})$ -set, $D(c, {}^*+b)$ -set, $D(c, {}^{**+}b)$ -set and its respective continuities are discussed.

KEYWORDS: ${}^*+b$ -open set, ${}^{**+}b$ -open set, t^{+*} -set, $D(c, {}^*+b)$ -set, $D(c, {}^{**+}b)$ -set, ${}^{**+}b$ -continuous, $D(c, {}^{**+}b)$ -continuous.

I. INTRODUCTION

A.S.Mashhour et al. [13] devised the concept of α -open sets. The notion of semi-open sets was introduced by Levine [10] and Mashhour [12] introduced pre-open sets in topological spaces. A new class of generalized open sets in a topological space, called b-open sets was defined by Andrijevic [3]. The class of b-open sets is contained in the class of semi-pre open sets and contains all semi-open and pre-open sets. Abd El-Monsef et al. [1] introduced the concept of β -open set and in [4] these sets were called semi-preopen sets. Palaniappan et.al [15] discussed the concept of regular open and regular closed sets. Tong [16] introduced the concept of t-set and B-set in topological spaces. Indira and Rekha [9] introduced the concept of *b -open set in topological spaces. The class of *b -open set in both semi-open and pre-open sets. Various forms of continuities was introduced and studied by a stream of topologists in [1,2,9,10,12,13,17].

In 1963 Levine [10] introduced the concept of simple extension of a topology τ as $\tau(B) = \{(B \cap O) \cup O' / O, O' \in \tau\}$. Sr.I.Arockiarani and F.Nirmala Irudayam [14] introduced the concept of b^+ -open sets in extended topological spaces. In this paper, we have coined the concept of different weaker forms of b-open sets and its continuity in extended topological spaces. Here we study their characterizations and its applications.

II. PRELIMINARIES

Definition 2.1: A subset A of a space X is said to be:

1. Semi-open [10] if $A \subseteq Cl(Int(A))$
2. Pre-open [12] if $A \subseteq Int(Cl(A))$
3. α -open [13] if $A \subseteq Int(Cl(Int(A)))$
4. β -open [1,4] if $A \subseteq Cl(Int(Cl(A)))$
5. b-open [3] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$
6. *b -open [7] if $A \subseteq Cl(Int(A)) \cap Int(Cl(A))$

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7. b^{**} -open [6] if $A \subseteq \text{Int}(\text{Cl}(\text{Int}(A))) \cup \text{Cl}(\text{Int}(\text{Cl}(A)))$
8. $^{**}b$ -open [7] if $A \subseteq \text{Int}(\text{Cl}(\text{Int}(A))) \cap \text{Cl}(\text{Int}(\text{Cl}(A)))$
9. t -set [16] if $\text{Int}(A) = \text{Int}(\text{Cl}(A))$
10. t^* -set [7] if $\text{Cl}(A) = \text{Cl}(\text{Int}(A))$
11. B -set [16] if $A = U \cap V$, where $U \in \tau^+$ and V is a t -set.
12. B^* -set [7] if $A = U \cap V$, where $U \in \tau^+$ and V is a t^* -set.
13. Regular open [15] if $A = \text{Int}(\text{Cl}(A))$
14. Regular closed [15] if $A = \text{Cl}(\text{Int}(A))$

Definition 2.2: A subset A of a space X is called:

1. Semi $^+$ -open [14] if $A \subseteq \text{Cl}^+(\text{Int}(A))$
2. Pre $^+$ -open [14] if $A \subseteq \text{Int}(\text{Cl}^+(A))$
3. α^+ -open [14] if $A \subseteq \text{Int}(\text{Cl}^+(\text{Int}(A)))$
4. β^+ -open [14] if $A \subseteq \text{Cl}^+(\text{Int}(\text{Cl}^+(A)))$
5. b^+ -open [14] if $A \subseteq \text{Cl}^+(\text{Int}(A)) \cup \text{Int}(\text{Cl}^+(A))$

Definition 2.3: A function $f : X \rightarrow Y$ is called:[1,2,9,10,12,13,17]

1. semi continuous if $f^{-1}(V)$ is semi open in X for each open set V of Y .
2. pre continuous if $f^{-1}(V)$ is pre open in X for each open set V of Y .
3. *b -continuous if $f^{-1}(V)$ is *b -open in X for each open set V of Y .
4. $^{**}b$ -continuous if $f^{-1}(V)$ is $^{**}b$ -open in X for each open set V of Y .
5. t^* -continuous if $f^{-1}(V)$ is t^* -open in X for each open set V of Y .
6. B^* -continuous if $f^{-1}(V)$ is B^* -open in X for each open set V of Y .
7. locally *b -closed continuous if $f^{-1}(V)$ is locally *b -closed in X for each open set V of Y .
8. locally $^{**}b$ -closed continuous if $f^{-1}(V)$ is locally $^{**}b$ -closed in X for each open set V of Y .
9. $D(c, ^*b)$ -continuous if $f^{-1}(V)$ is $D(c, ^*b)$ -set in X for each open set V of Y .
10. $D(c, ^{**}b)$ -continuous if $f^{-1}(V)$ is $D(c, ^{**}b)$ -open in X for each open set V of Y .
11. completely continuous if $f^{-1}(V)$ is regular open in X for each open set V of Y .

III. CHARACTERIZATION OF $^{**}b$ -OPEN AND $^{**+}b$ -OPEN SETS

Definition 3.1: A subset A of a space X is said to be:

1. $^{**+}b$ -open if $A \subseteq \text{Cl}^+(\text{Int}(A)) \cap \text{Int}(\text{Cl}^+(A))$
2. b^{+**} -open if $A \subseteq \text{Int}(\text{Cl}^+(\text{Int}(A))) \cup \text{Cl}^+(\text{Int}(\text{Cl}^+(A)))$
3. $^{**+}b$ -open if $A \subseteq \text{Int}(\text{Cl}^+(\text{Int}(A))) \cap \text{Cl}^+(\text{Int}(\text{Cl}^+(A)))$
4. Regular $^+$ open if $A = \text{Int}(\text{Cl}^+(A))$
5. Regular $^+$ closed if $A = \text{Cl}^+(\text{Int}(A))$
6. t^+ -set if $\text{Int}(A) = \text{Int}(\text{Cl}^+(A))$
7. t^{+*} -set if $\text{Cl}^+(A) = \text{Cl}^+(\text{Int}(A))$
8. B^+ -set if $A = U \cap V$, where $U \in \tau^+$ and V is a t^+ -set.
9. B^{+*} -set if $A = U \cap V$, where $U \in \tau^+$ and V is a t^{+*} -set.
10. locally closed if $A = U \cap V$, where $U \in \tau^+$ and V is a closed set.

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11. locally b^+ -closed if $A = U \cap V$, where $U \in \tau^+$ and V is a b^+ -closed set.
12. locally $^{*+}b$ -closed if $A = U \cap V$, where $U \in \tau^+$ and V is a $^{*+}b$ -closed set.
13. locally b^{+**} -closed if $A = U \cap V$, where $U \in \tau^+$ and V is a b^{+**} -closed set.
14. locally $^{**+}b$ -closed if $A = U \cap V$, where $U \in \tau^+$ and V is a $^{**+}b$ -closed set.
15. $D(c, b^+)$ -set if $\text{Int}(A) = b^+ \text{Int}(A)$
16. $D(c, b^{+**})$ -set if $\text{Int}(A) = b^{+**} \text{Int}(A)$
17. $D(c, ^{*+}b)$ -set if $\text{Int}(A) = ^{*+}b \text{Int}(A)$
18. $D(c, ^{**+}b)$ -set if $\text{Int}(A) = ^{**+}b \text{Int}(A)$

Proposition 3.2:

1. Every regular $^+$ open set is a t^+ -set.
2. Every regular $^+$ closed set is a t^{+*} -set.
3. Every locally closed set is a B^+ -set.

Theorem 3.3:

Let A be a subset of (X, τ^+) . Then we have the following:

1. A is a t^+ -set iff it is semi $^+$ closed.
2. A is a t^{+*} -set iff it is semi $^+$ open.

Proof:

1) Let A be a t^+ -set. Then $\text{Int}(A) = \text{Int}(Cl^+(A))$. Therefore $\text{Int}(Cl^+(A)) \subseteq A \Rightarrow A$ is semi $^+$ closed. Conversely, assume that A is semi $^+$ closed. Then $\text{Int}(Cl^+(A)) \subseteq A \Rightarrow \text{Int}(\text{Int}(Cl^+(A))) \subseteq \text{Int}(A)$. $\text{Int}(Cl^+(A)) \subseteq \text{Int}(A) \dots (1)$. Since $\text{Int}(A) \subseteq \text{Int}(Cl^+(A)) \dots (2)$. From (1) and (2), $\text{Int}(A) = \text{Int}(Cl^+(A)) \Rightarrow A$ is a t^+ -set.

2) Let A be a t^{+*} -set. Then $Cl^+(A) = Cl^+(\text{Int}(A))$. Therefore $A \subseteq Cl^+(\text{Int}(A)) \Rightarrow A$ is semi $^+$ open. Conversely, assume that A is semi $^+$ open. Then $A \subseteq Cl^+(\text{Int}(A)) \Rightarrow Cl^+(A) \subseteq Cl^+(Cl^+(\text{Int}(A)))$. $Cl^+(A) \subseteq Cl^+(\text{Int}(A)) \dots (1)$. Since $Cl^+(\text{Int}(A)) \subseteq Cl^+(A) \dots (2)$. From (1) and (2), $Cl^+(A) = Cl^+(\text{Int}(A)) \Rightarrow A$ is a t^{+*} -set.

Theorem 3.4:

1. If A and B are t^+ -sets, then $A \cap B$ is a t^+ -set.
2. If A and B are t^{+*} -sets, then $A \cup B$ is a t^{+*} -set.

Proof:

1) Let A and B be t^+ -set. Then we have $\text{Int}(A) = \text{Int}(Cl^+(A))$; $\text{Int}(B) = \text{Int}(Cl^+(B))$. Since $\text{Int}(A \cap B) \subseteq \text{Int}(Cl^+(A \cap B)) \subseteq \text{Int}(Cl^+(A) \cap Cl^+(B)) = \text{Int}(Cl^+(A)) \cap \text{Int}(Cl^+(B)) = \text{Int}(A) \cap \text{Int}(B) = \text{Int}(A \cap B) \Rightarrow \text{Int}(A \cap B) \subseteq \text{Int}(Cl^+(A \cap B)) \subseteq \text{Int}(A \cap B)$. Therefore, $\text{Int}(A \cap B) = \text{Int}(Cl^+(A \cap B)) \Rightarrow A \cap B$ is a t^+ -set.

2) Let A and B be t^{+*} -set. Then we have $Cl^+(A) = Cl^+(\text{Int}(A))$; $Cl^+(B) = Cl^+(\text{Int}(B))$. Since $Cl^+(A \cup B) = Cl^+(A) \cup Cl^+(B) = Cl^+(\text{Int}(A)) \cup Cl^+(\text{Int}(B)) = Cl^+(\text{Int}(A) \cup \text{Int}(B)) \subset Cl^+(\text{Int}(A \cup B)) \Rightarrow Cl^+(A \cup B) \subset Cl^+(\text{Int}(A \cup B)) \dots (1)$. Since $\text{Int}(A \cup B) \subset A \cup B$. $Cl^+(\text{Int}(A \cup B)) \subset Cl^+(A \cup B) \dots (2)$. From (1) and (2), $Cl^+(A \cup B) = Cl^+(\text{Int}(A \cup B)) \Rightarrow A \cup B$ is a t^{+*} -set.

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Theorem 3.5:

A set A is a t^+ -set iff complement is a t^{+*} -set.

Proof:

Let A be a t^+ -set. Then $\text{Int}(A) = \text{Int}(Cl^+(A)) \Leftrightarrow X - \text{Int}(A) = X - \text{Int}(Cl^+(A)) \Leftrightarrow Cl^+(X - A) = Cl^+(\text{Int}(X - A)) \Leftrightarrow Cl^+(A^c) = Cl^+(\text{Int}(A^c)) \Leftrightarrow A^c$ is a t^{+*} -set.

Theorem 3.6:

For a subset A of a space (X, τ^+) , the following are equivalent:

1. A is open.
2. A is pre^+ open and a B^+ -set.

Proof:

To prove: (1) \Rightarrow (2): Let A be open. Then $A = \text{Int}(A) \Rightarrow \text{Int}(A) \subseteq \text{Int}(Cl^+(A)) \Rightarrow A \subseteq \text{Int}(Cl^+(A)) \Rightarrow A$ is pre^+ open.

Let $U = A \in \tau^+$ and $V = X$ be a t^+ -set containing $A \Rightarrow A = U \cap V \Rightarrow A$ is a B^+ -set. Hence A is pre^+ open and a B^+ -set.

To prove: (2) \Rightarrow (1): Let A be pre^+ open and a B^+ -set. Since A is a B^+ -set. We have $A = U \cap V$ where U is open and V is a t^+ -set. $\Rightarrow \text{Int}(V) = \text{Int}(Cl^+(V))$. Since A is pre^+ open. $\Rightarrow A \subseteq \text{Int}(Cl^+(A)) = \text{Int}(Cl^+(U \cap V)) = \text{Int}(Cl^+(U)) \cap \text{Int}(V)$. Therefore, $U \cap V \subset \text{Int}(Cl^+(U)) \cap \text{Int}(V)$. Consider $U \cap V = (U \cap V) \cap U \subset [\text{Int}(Cl^+(U)) \cap \text{Int}(V)] \cap U = U \cap \text{Int}(V)$. $V \subset U \cap \text{Int}(V) \Rightarrow V \subset \text{Int}(V) \Rightarrow V = \text{Int}(V) \Rightarrow A = U \cap V = U \cap \text{Int}(V) \Rightarrow A = \text{Int}(A) \Rightarrow A$ is open.

Theorem 3.7:

For a subset A of a space (X, τ^+) , the following are equivalent:

1. A is regular^+ open
2. A is pre^+ open and a t^+ -set.

Proof:

To prove: (1) \Rightarrow (2): Let A be regular^+ open. Then $A = \text{Int}(Cl^+(A)) \Rightarrow A$ is pre^+ open. Since $A = \text{Int}(Cl^+(A)) \Rightarrow \text{Int}(A) = \text{Int}(Cl^+(A)) \Rightarrow A$ is a t^+ -set.

To prove: (2) \Rightarrow (1): Let A be pre^+ open and a t^+ -set. Then $A \subseteq \text{Int}(Cl^+(A)) \dots (1)$. Since A is a t^+ -set. Then $\text{Int}(Cl^+(A)) \subseteq A \dots (2)$. From (1) and (2), A is regular^+ open.

Theorem 3.8:

For a subset A of a space (X, τ^+) , the following are equivalent:

1. A is regular^+ closed
2. A is pre^+ closed and a t^{+*} -set.

Proof:

To prove: (1) \Rightarrow (2): Let A be regular^+ closed. Then $A = Cl^+(\text{Int}(A))$. $\Rightarrow A$ is pre^+ closed. Since $A = Cl^+(\text{Int}(A)) \Rightarrow Cl^+(A) = Cl^+(\text{Int}(A)) \Rightarrow A$ is a t^{+*} -set.

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To prove: (2) \Rightarrow (1): Let A be pre^+ closed and a t^{+*} -set. Then $\text{Cl}^+(\text{Int}(A)) \subseteq A \dots (1)$. Since A is a t^+ -set. Then $A \subseteq \text{Cl}^+(\text{Int}(A)) \dots (2)$. From (1) and (2), A is regular^+ closed.

Theorem 3.9:

1. The intersection of a b^+ -open set and a $^{*+}\text{b}$ -open set is a b^+ -open set.
2. The intersection of a b^{+**} -open set and a $^{**+}\text{b}$ -open set is a b^{+**} -open set.

Theorem 3.10:

1. The intersection of a locally b^+ -closed set and a locally $^{*+}\text{b}$ -closed set is a locally b^+ -closed set.
2. The intersection of a locally b^{+**} -closed set and a locally $^{**+}\text{b}$ -closed set is a locally b^{+**} -closed set.

Theorem 3.11:

For a subset A of an extremally disconnected space (X, τ^+) , the following are equivalent:

1. A is open
2. A is $^{*+}\text{b}$ -open and locally closed
3. A is b^+ -open and locally closed.

Proof:

Let A be a subset of an extremally disconnected space (X, τ^+) . Then $\text{Cl}^+(\text{Int}(A)) = \text{Int}(\text{Cl}^+(A))$.

To prove: (1) \Rightarrow (2): This is obvious from the definitions.

To prove: (2) \Rightarrow (3): Let A be $^{*+}\text{b}$ -open and locally closed. Since every $^{*+}\text{b}$ -open set is b^+ -open. $\Rightarrow A$ is b^+ -open and locally closed.

To prove: (3) \Rightarrow (1): Let A be b^+ -closed and locally closed, then $A \subseteq \text{Int}(\text{Cl}^+(A)) \cup \text{Cl}^+(\text{Int}(A))$ and $A = U \cap \text{Cl}^+(A)$. Now $A \subseteq U \cap [\text{Int}(\text{Cl}^+(A)) \cup \text{Cl}^+(\text{Int}(A))] \subseteq [\text{Int}(U \cap \text{Cl}^+(A))] \cup [U \cap \text{Cl}^+(\text{Int}(A))]$. Since (X, τ^+) is E.D. space we have, $\subseteq [\text{Int}(U \cap \text{Cl}^+(A))] \cup [U \cap \text{Int}(\text{Cl}^+(A))] \subseteq [\text{Int}(U \cap \text{Cl}^+(A))] \cup [\text{Int}(U \cap \text{Cl}^+(A))] = \text{Int}(A) \cup \text{Int}(A) = \text{Int}(A)$. Therefore, A is open.

Theorem 3.12:

For a subset A of a space (X, τ^+) , the following are equivalent:

1. A is open
2. A is $^{*+}\text{b}$ -open and $\text{D}(c, ^{*+}\text{b})$ -set
3. A is b^+ -open and $\text{D}(c, \text{b}^+)$ -set.

Proof:

To prove: (1) \Rightarrow (2): If A is open then A is $^{*+}\text{b}$ -open. To prove: A is a $\text{D}(c, ^{*+}\text{b})$ -set. Since A is open and every open set is $^{*+}\text{b}$ -open. Then $A = \text{Int}(A) = ^{*+}\text{bInt}(A) \Rightarrow A$ is a $\text{D}(c, ^{*+}\text{b})$ -set.

To prove: (2) \Rightarrow (3): Let A be $^{*+}\text{b}$ -open and $\text{D}(c, ^{*+}\text{b})$ -set. Then $A = ^{*+}\text{bInt}(A)$ and $\text{Int}(A) = ^{*+}\text{bInt}(A) \dots (1)$. Since every $^{*+}\text{b}$ -open set is b^+ -open $\Rightarrow A$ is b^+ -open. $\Rightarrow A = \text{b}^+\text{Int}(A) \dots (2)$. From (1) and (2), $A = \text{b}^+\text{Int}(A) = ^{*+}\text{bInt}(A) \dots (3)$. From (1) and (3), $\text{Int}(A) = \text{b}^+\text{Int}(A) \Rightarrow A$ is a $\text{D}(c, \text{b}^+)$ -set.

To prove: (3) \Rightarrow (1): Let A be b^+ -open and $\text{D}(c, \text{b}^+)$ -set. To prove: A is open. Since $A = \text{b}^+\text{Int}(A)$ and $\text{Int}(A) = \text{b}^+\text{Int}(A) \Rightarrow A$ is open.

Theorem 3.13:

For a subset A of an extremally disconnected space (X, τ^+) , the following are equivalent:

1. A is open
2. A is $^{**+}\text{b}$ -open and locally closed

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3. A is b^{+**} -open and locally closed.

Proof:

Let A be a subset of an extremally disconnected space (X, τ^+) . Then $Cl^+(Int(A)) = Int(Cl^+(A))$.

To prove: (1) \Rightarrow (2): This is obvious from the definitions.

To prove: (2) \Rightarrow (3): Let A be $^{**+}b$ -open and locally closed. Since every $^{**+}b$ -open set is b^{+**} -open. \Rightarrow A is b^{+**} -open and locally closed.

To prove: (3) \Rightarrow (1): Let A be b^{+**} -closed and locally closed, then $A \subseteq Cl^+(Int(Cl^+(A))) \cup Int(Cl^+(Int(A)))$ and $A = U \cap Cl^+(A)$. Now $A \subseteq U \cap [Int(Cl^+(Int(A))) \cup Cl^+(Int(Cl^+(A)))] \subseteq [Int(U \cap Cl^+(Int(A)))] \cup [U \cap Cl^+(Int(Cl^+(A)))]$. Since (X, τ^+) is E.D. space we have, $\subseteq [Int(U \cap Int(Cl^+(A)))] \cup [U \cap Cl^+(Int(Cl^+(A)))] \subseteq [Int(Int(U \cap Cl^+(A)))] \cup [U \cap Cl^+(Int(A))] \subseteq Int(U \cap Cl^+(A)) \cup Int(U \cap Cl^+(A)) = Int(A) \cup Int(A) = Int(A)$. Therefore, A is open.

Theorem 3.14:

For a subset A of a space (X, τ^+) , the following are equivalent:

1. A is open
2. A is $^{**+}b$ -open and $D(c, ^{**+}b)$ -set
3. A is b^{+**} -open and $D(c, b^{+**})$ -set.

Proof:

To prove: (1) \Rightarrow (2): Let A be open then A is $^{**+}b$ -open. To prove: A is a $D(c, ^{**+}b)$ -set. Since A is open and every open set is $^{**+}b$ -open. Then $A = Int(A) = ^{**+}bInt(A) \Rightarrow A$ is a $D(c, ^{**+}b)$ -set.

To prove: (2) \Rightarrow (3): Let A be $^{**+}b$ -open and $D(c, ^{**+}b)$ -set. Then $A = ^{**+}bInt(A)$ and $Int(A) = ^{**+}bInt(A) \dots (1)$. Since every $^{**+}b$ -open set is b^{+**} -open. $\Rightarrow A$ is b^{+**} -open $\Rightarrow A = b^{+**}Int(A) \dots (2)$. From (1) and (2), $A = b^{+**}Int(A) = ^{**+}bInt(A) \dots (3)$. From (1) and (3), $Int(A) = b^{+**}Int(A) \Rightarrow A$ is a $D(c, b^{+**})$ -set.

To prove: (3) \Rightarrow (1): Let A be b^{+**} -open and $D(c, b^{+**})$ -set. To prove: A is open. Since $A = b^{+**}Int(A)$ and $Int(A) = b^{+**}Int(A) \Rightarrow A$ is open.

IV. APPLICATIONS OF $^{**+}b$ -OPEN AND b^{+**} -OPEN SETS

Definition 4.1: A function $f : X \rightarrow Y$ is called:

1. $^{**+}b$ -continuous if $f^{-1}(V)$ is $^{**+}b$ -open in X for each open set V of Y.
2. t^{+*} -continuous if $f^{-1}(V)$ is t^{+*} -open in X for each open set V of Y.
3. B^{+*} -continuous if $f^{-1}(V)$ is B^{+*} -open in X for each open set V of Y.
4. locally $^{**+}b$ -closed continuous if $f^{-1}(V)$ is locally $^{**+}b$ -closed in X for each open set V of Y.
5. $D(c, ^{**+}b)$ -continuous if $f^{-1}(V)$ is $D(c, ^{**+}b)$ -open in X for each open set V of Y.

Theorem 4.2:

Let $f : X \rightarrow Y$ be a function and let X be an extremally disconnected space. Then the following are equivalent:

1. f is continuous
2. f is $^{**+}b$ -continuous and locally closed continuous
3. f is b^{+**} -continuous and locally closed continuous

Proof: It follows from theorem 3.11.

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Theorem 4.3:

Let $f : X \rightarrow Y$ be a function. Then the following are equivalent:

1. f is continuous
2. f is $^{*+}b$ -continuous and $D(c, ^{*+}b)$ -continuous
3. f is b^+ -continuous and $D(c, b^+)$ -continuous

Proof: It follows from theorem 3.12.

Theorem 4.4:

Let $f : X \rightarrow Y$ be a function and let X be an extremally disconnected space. Then the following are equivalent:

1. f is continuous
2. f is $^{**+}b$ -continuous and locally closed continuous
3. f is b^{+**} -continuous and locally closed continuous

Proof: It follows from theorem 3.13.

Theorem 4.5:

Let $f : X \rightarrow Y$ be a function. Then the following are equivalent:

1. f is continuous
2. f is $^{**+}b$ -continuous and $D(c, ^{**+}b)$ -continuous
3. f is b^{+**} -continuous and $D(c, b^{+**})$ -continuous

Proof: It follows from theorem 3.14.

Theorem 4.6:

Let $f : X \rightarrow Y$ be a function. Then

1. f is continuous iff f is pre^+ continuous and B^+ -continuous
2. f is completely continuous iff f is pre^+ continuous and t^+ -continuous
3. f is t^+ -continuous iff f is $semi^+$ continuous

Proof: It follows from theorem 3.6, 3.7, 3.3(2).

V. CONCLUSION

Thus new forms of sets are proposed under the names $^{*+}b$ -open set, b^{+**} -open set, $^{**+}b$ -open set, t^{+*} -set, $D(c, b^+)$ -set, $D(c, b^{+**})$ -set, $D(c, ^{*+}b)$ -set, $D(c, ^{**+}b)$ -set and its respective continuities are discussed. Furthermore, b -sets can be generalized and various mappings may be introduced for further study.

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