

# Application of Fuzzy Graph Coloring in Traffic Light Problem

Rakesh Jaiswal<sup>1</sup>, Shweta Rai<sup>2</sup>

Assistant Professor, Dept. of Mathematics & Statistics, Christian Eminent College, DAVV, Indore, MP, India<sup>1,2</sup>

**ABSTRACT:** Let  $G = (V, E, \sigma, \mu)$  be a simple connected undirected fuzzy graph. In this paper, we use a fuzzy graph model to represent a traffic network of a city and discuss a method to find the different type of accidental zones in a traffic flows. This paper is based on fuzzy coloring of fuzzy graphs and fuzziness of vertices. Using this approach, we can minimize the total waiting time of a traffic flow. It will help to reduce the traffic jam of a road.

**KEYWORDS:** Fuzzy graphs, fuzzy graph coloring, traffic lighting

## I. INTRODUCTION

Fuzzy graphs were introduced by Rosenfeld [2], ten years after Zadeh's landmark paper "Fuzzy Sets" [8]. Fuzzy graph theory is now finding numerous applications in modern science and technology especially in the fields of information theory, neural network, expert systems, cluster analysis, medical diagnosis, control theory, etc. Rosenfeld has obtained the fuzzy analogues of several basic graph-theoretic concepts like bridges, paths, cycles, trees and connectedness and established some of their properties [8]. Bhattacharya [11] has established some connectivity concepts regarding fuzzy cut nodes and fuzzy bridges. Arindam Dey and Anita Pal [17] In graph theory, arc analysis is not very important as all arcs are strong in the sense of [4]. But in fuzzy graphs it is very important to identify the nature of arcs and no such analysis on arcs is available in the literature except the division of arcs as strong and non strong in [12]. Depending on the strength of an arc, the authors Sunil Mathew and Sunitha [13] classify strong arcs into two types namely  $\alpha$ -strong and  $\beta$ -strong and introduce two other types of arcs in fuzzy graphs which are not strong and are termed as  $\delta$  and  $\delta^*$  arcs. Graph coloring is one of the most important concepts in graph theory and is used in many real time applications like job scheduling, aircraft scheduling, computer network security, map coloring and GSM mobile phone networks, automatic channel allocation for small wireless local area networks. The proper coloring of a graph is the coloring of the vertices with minimal number of colors such that no two adjacent vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph. The fuzzy coloring of a fuzzy graph was defined by the authors in Eslahchi and Onagh [9]. Then Pourpasha [10] also introduced different approaches to color the fuzzy graph.

## II. PRELIMINARY NOTES

In this Section, we present some definitions related to fuzzy graph and their properties.

### Definition A:[17]

A fuzzy set  $A$  defined on a non empty set  $X$  is the family  $A = \{(x, \mu_A(x)) / x \in X\}$ , where  $\mu_A : X \rightarrow I$  is the membership function. In classical fuzzy set theory the set  $I$  is usually defined as the interval  $[0, 1]$  such that  $\mu_A(x) = \begin{cases} 0 & : x \notin X \\ 1 & : x \in X \end{cases}$

It takes any intermediate value between 0 and 1 represents the degree in which  $x \in A$ . The set  $I$  could be discrete set of the form  $I = \{0, 1, \dots, k\}$  where  $\mu_A(x) < \mu_A(x')$  indicates that the degree of membership of  $x$  to  $A$  is lower than the degree of membership of  $x'$ .

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**Definition B:**

Let  $V$  be a finite nonempty set. The triple  $\hat{G} = (V, \sigma, \mu)$  is called a fuzzy graph on  $V$  where  $\sigma$  and  $\mu$  are fuzzy sets on  $V$  and  $E$ , respectively, such that  $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$  for all  $u, v \in V$  and  $uv \in E$ . For fuzzy graph  $\hat{G} = (V, \sigma, \mu)$ , the elements  $V$  and  $E$  are called set of vertices and set of edges of  $G$  respectively.

**Definition C:**

A fuzzy graph  $\hat{G} = (V, \sigma, \mu)$  is called a complete fuzzy graph if  $\mu(uv) = \sigma(u) \wedge \sigma(v)$  for all  $u, v \in V$  and  $uv \in E$ . We denote this complete fuzzy graph by  $\hat{G}_k$ .

**Definition D:**

Two vertices  $u$  and  $v$  in  $\hat{G}$  are called adjacent if  $(\frac{1}{2})[\sigma(u) \wedge \sigma(v)] \leq \mu(uv)$ .

**Definition E:**

The edge  $uv$  of  $\hat{G}$  is called strong if  $u$  and  $v$  are adjacent. Otherwise it is called weak.

**Definition F:**

A family  $\Gamma = \{\gamma_1, \gamma_2, \dots, \gamma_k\}$  of fuzzy sets on  $V$  is called a  $k$ -fuzzy coloring of  $\hat{G} = (V, \sigma, \mu)$  if

a)  $\bigwedge \Gamma = \sigma$

b)  $\gamma_i \wedge \gamma_j = 0$

c) For every strong edge  $uv$  of  $\hat{G}$ ,  $\gamma_i(u) \wedge \gamma_j(v) = 0$  for  $1 \leq i \leq k$ .

The above definition of  $k$ -fuzzy coloring was defined by the authors Eslahchi and Onagh [1] on fuzzy set of vertices.

### III TRAFFIC LIGHTS PROBLEM AND ITS SOLUTION USING FUZZY GRAPH COLOURING[16]

The traffic lights problem consists of controlling a traffic lights system in such a way that certain level of security will be attained. This problem has been studied as an intersection graph. The traffic lights problem can also be modeled as a graph coloring problem. The following example illustrates this approach. Let us consider a traffic flow shown in Fig3.1. Each arrow in Fig 1 indicates the vehicles will go from one direction to another direction and each intersecting point shows the possible accidental zones of two arrows. But numbers of vehicles in all paths are not always equal. Due to this reason, we consider it as fuzzy set whose membership value depends upon on vehicles number. If the number of vehicles in any path is high then its membership value will be high and if the number of vehicles in any path is low then its membership value will be low. Now, consider the traffic flow in each path given below in table 1 with its membership values are represented by linguistic terms H for high traffic flow, M for medium and L for low.

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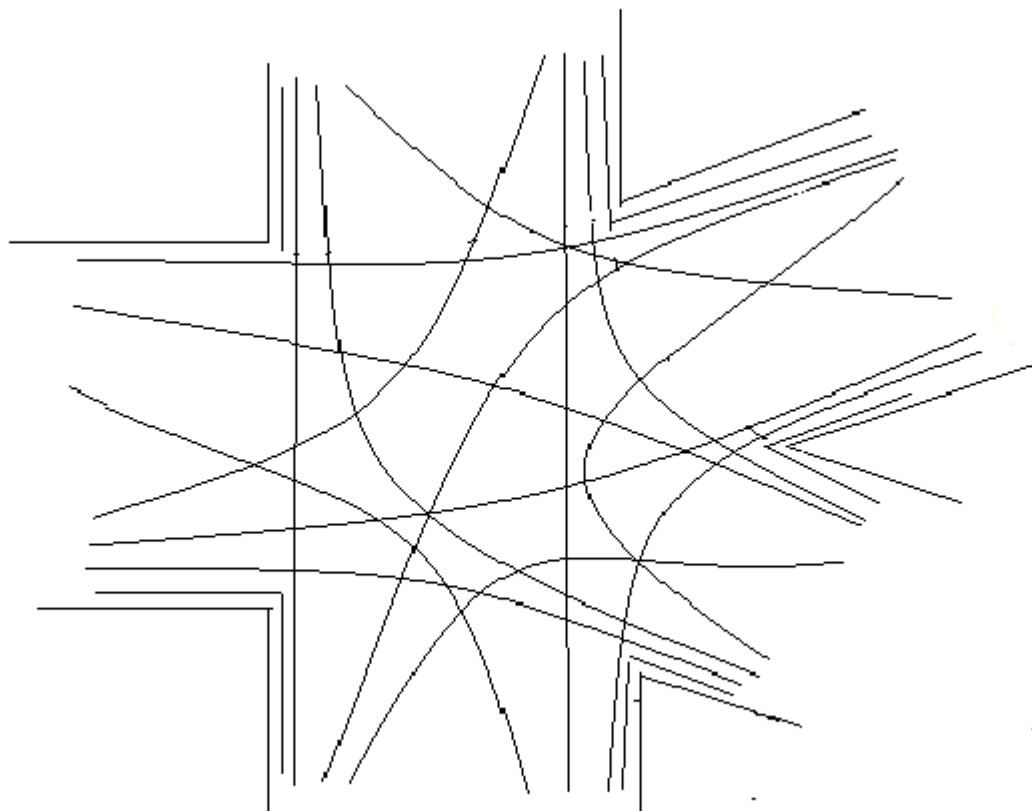


Fig 3.1

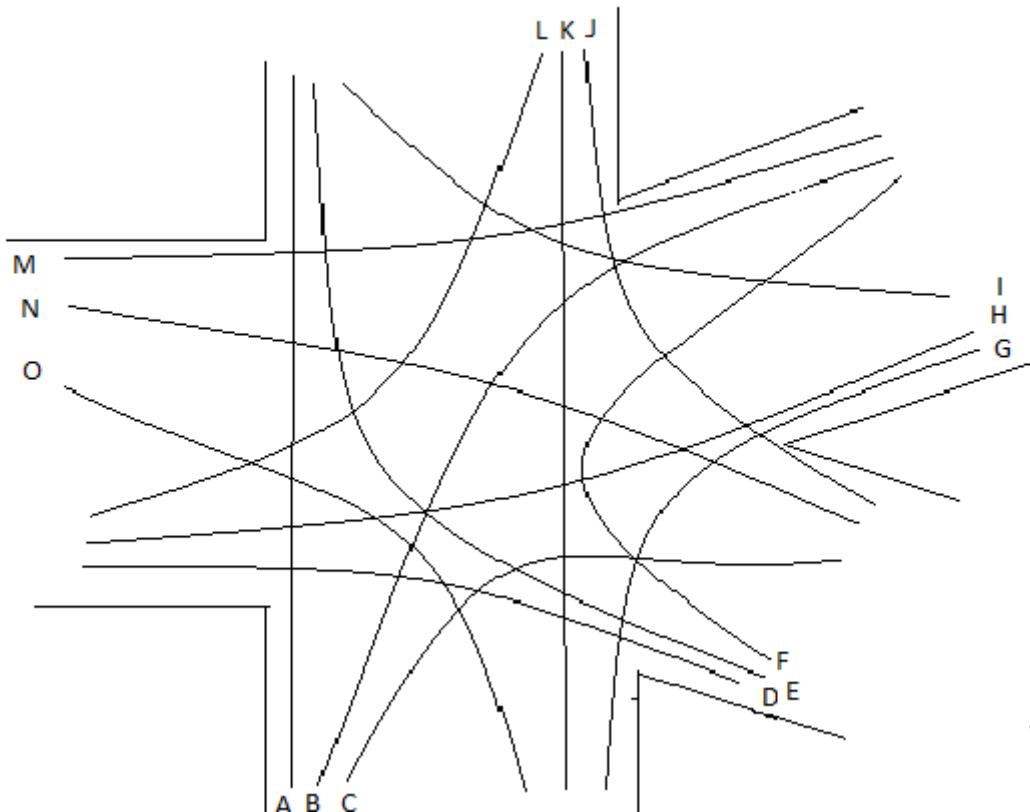
<b>vertex</b>	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
<b>Membership value</b>	L	M	H	L	L	M	M	H	H	L	M	M	L	L	M

In this paper, we represent each traffic flow using the vertices of fuzzy graph and their membership value depend upon the number of vehicles of that road. Since the five left turns do not interfere with the other traffic flows, they can safely be dropped from our discussion. The remaining traffic directions are shown in figure 3.2. We need to develop a traffic pattern so that vehicles can pass through the intersection without interfering with other traffic flows. In this problem, we represent each traffic flow using the vertices of the fuzzy graph and their membership value depends upon the number of the vehicle of that road. Two vertices are adjacent if the corresponding traffic flows cross each other. If two vertices are adjacent then there is a possibility of accident. The possibility of accident depends on the adjacent vertices membership value which represents number of vehicles on the road. We consider the arc membership value is high (H) if both the vertices have high (H) membership or one adjacent vertex has high (H) membership value and another node has medium (M) membership value. We consider the arc membership value is low (L) if both the vertices have low (L) membership or one adjacent vertex has low (L) membership value and another node has medium (M) membership value. In another condition, if the membership value of one adjacent vertex is high (H) and another has low (L) membership value or both node has medium (M) membership value then we consider the membership value of that arc is medium (M).

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**Fig 3.2**

In this paper, we represent each possibility of accident with an edge and their membership value. Membership values of edges are given below in Table 2.

<b>Edge</b>	AD	AH	AL	AM	AN	AO	BD	BE	BH	BI
<b>Membership value</b>	M	L	L	L	L	L	L	L	H	H
<b>Edge</b>	BJ	BK	BN	BO	CD	CE	CF	CG	CK	CO
<b>Membership value</b>	L	M	L	M	M	M	H	H	H	H
<b>Edge</b>	DJ	DK	DO	EG	EH	EK	EL	EM	EN	FG
<b>Membership value</b>	L	L	L	L	M	L	L	L	L	M
<b>Edge</b>	FH	FI	FJ	FN	GJ	GN	HJ	HK	HN	HO
<b>Membership value</b>	H	H	L	L	L	L	M	H	M	H
<b>Edge</b>	IJ	IK	IL	IM	JM	KM	KN	LM	LN	LO
<b>Membership value</b>	M	H	H	M	L	L	L	L	L	M

**Table 2:** Membership values of the edges

The fuzzy graph  $\hat{G} = (V, \sigma, \mu)$  corresponding to our example is defined as follow:  
Let  $V = \{A, B, C, \dots, M, N, O\}$

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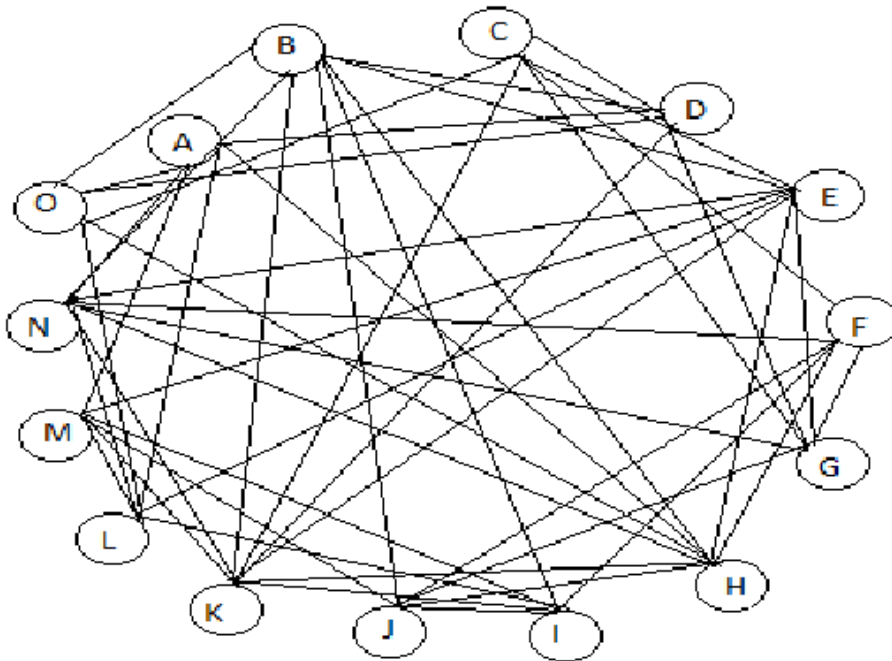
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$$\sigma(i) = \begin{cases} L & \text{for } i = A, D, E, J, M, N \\ M & \text{for } i = B, F, G, K, L, O \\ H & \text{for } i = C, H, I \\ 0 & \text{Otherwise} \end{cases}$$

$$\mu(i, j) = \begin{cases} L & \text{for } i, j = \{AH, AL, AM, AN, AO, BD, BE, BJ, BN, DJ, DK, DO, EG, EK, EL, EM, EN, \\ & \quad \quad \quad FJ, FN, GJ, GN, JM, KM, KN, LM, LN\} \\ M & \text{for } i, j = \{AD, BK, BO, CD, CE, EH, FG, HJ, HN, IJ, IM, LO\} \\ H & \text{for } i, j = \{BH, BI, CF, CG, CK, CO, FH, FI, HK, HO, IK, IL\} \end{cases}$$

The fuzzy graph for above problem is shown below



Let Let  $\Gamma_1 = \{\gamma_1, \gamma_2, \dots, \gamma_5\}$  be a family of fuzzy set defined on  $V$  where

$$\gamma_1(i) = \begin{cases} L, & \text{for } i = L \\ M, & \text{for } i = M \\ H, & \text{for } i = H \\ 0, & \text{otherwise} \end{cases}, \gamma_2(i) = \begin{cases} L, & \text{for } i = D, F \\ M, & \text{for } i = f \\ 0, & \text{otherwise} \end{cases}, \gamma_3(i) = \begin{cases} M, & \text{for } i = G \\ H, & \text{for } i = H, I \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma_4(i) = \begin{cases} L, & \text{for } i = J \\ M, & \text{for } i = K, L \\ 0, & \text{otherwise} \end{cases}, \gamma_5(i) = \begin{cases} L, & \text{for } i = M, N \\ M, & \text{for } i = O \\ 0, & \text{otherwise} \end{cases}$$

Here we can see that the family  $\Gamma_1 = \{\gamma_1, \gamma_2, \dots, \gamma_5\}$  satisfies the conditions of definition of fuzzy graph coloring. Thus the fuzzy graph in our example have 5-fuzzy vertex coloring i.e.  $\chi(G) = 5$ .

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Thus we get a efficient way of designing the traffic signal pattern which consist of five phases as follow

### 1. Traffic light pattern

Phase I	Only A, B, C proceeds
Phase II	Only D, E, F proceeds
Phase III	Only G, H, I proceeds
Phase IV	Only J, K, L proceeds
Phase V	Only M, N, O proceeds

Since the number of vehicles in all the path are not equal, therefore we need minimum time duration for low traffic flow, maximum time duration for high traffic flow and medium time duration for medium traffic flow. But in these type of traffic light pattern waiting time for each phase is more, due to these total waiting time of vehicles will be more. To minimize the waiting time for vehicles we can follow the following traffic light pattern

Let  $\Gamma_2 = \{\gamma_1, \gamma_2, \dots, \gamma_5\}$  be a family of fuzzy set defined on  $V$  where

$$\gamma_1(i) = \begin{cases} L, & \text{for } i = A, E, J \\ o, & \text{otherwise} \end{cases}, \gamma_2(i) = \begin{cases} M, & \text{for } i = B, F, L \\ o, & \text{otherwise} \end{cases}, \gamma_3(i) = \begin{cases} L, & \text{for } i = D, M, N \\ o, & \text{otherwise} \end{cases}$$

$$\gamma_4(i) = \begin{cases} M, & \text{for } i = G, K, L \\ o, & \text{otherwise} \end{cases}, \gamma_5(i) = \begin{cases} H, & \text{for } i = C, H, I \\ o, & \text{otherwise} \end{cases}$$

Here we can see that the family  $\Gamma_2 = \{\gamma_1, \gamma_2, \dots, \gamma_5\}$  also satisfies the conditions of definition of fuzzy graph coloring. Now new traffic light pattern is as follow

### 2. Traffic light pattern

Phase I	Only A, E, J proceeds
Phase II	Only B, F, L proceeds
Phase III	Only D, M, N proceeds
Phase IV	Only G, K, O proceeds
Phase V	Only C, H, I proceeds

## IV. CONCLUSION

In the concept of crisp incompatibly among the nodes of graph we cannot describe the vagueness or partial information about a problem. In this paper, we represent the traffic flows using a fuzzy graph whose vertices and edges both are fuzzy vertices and fuzzy arcs. So we can describe vagueness in vertices and also in edges. With the help of these type of fuzzy graph colouring we can minimize the waiting time of vehicles.

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