

On the Generalization of $g^{*+}b$ Sets in Simple Extended Topological Spaces

B. Kanchana¹, F. Nirmala Irudayam²

Research Scholar, Department of Mathematics, Nirmala College for Women, Red Fields, Coimbatore, India¹

Assistant Professor, Department of Mathematics, Nirmala College for Women, Red Fields, Coimbatore, India²

ABSTRACT: In this paper, we introduce a new class of sets called generalized $g^{*+}b$ -closed sets in simple extension topology (briefly $g^{*+}b$ -closed set) and study some of the characterizations of newly introduced sets.

KEYWORDS: $g^{*+}b$ -closed set, gb^+ -closed set, gp^+ -closed set, gs^+ -closed set, sg^+ -closed set, gsp^+ -closed set, g^{*+} -closed set.

I. INTRODUCTION

N.Levine [6] proposed the notion of generalized closed (briefly g -closed) sets in topological spaces. J.Dontchev [4], H.Maki, R.Devi and K.Balachandran [9], A.S.Mashhour, M.E.Abd-El-Monsef and SN.El-Deeb [10], and D.Andrijevic [1] introduced and investigated the concepts of generalized semi-preclosed sets, generalized α -closed sets, preclosed sets and semi-preclosed sets respectively. D.Andrijevic [2], initiated a new class of generalized open sets in a topological space called b -open sets. A.A.Omari and M.S.M.Noorani [13] studied the concept of generalized b -closed sets (briefly gb -closed) in topological spaces. A new class of sets called g^*b -closed set was defined by D.Vidhya and R.Parimelazhagan [15], which is between the class of b -closed sets and the class of gb -closed sets.

In 1963, Levine [8] formulated the idea of simple extension of a topology τ as $\tau(B) = \{O \cup (O' \cap B) / O, O' \in \tau\}$. Sr.I.Arockiarani and F.Nirmala Irudayam [11] introduced the concept of b^+ -open sets in extended topological spaces. In this paper, we have coined the concept of generalized $g^{*+}b$ -closed sets in simple extension topology and study some of their characterizations. Throughout this paper X is a simple extension topological space in which no separation axioms are assumed unless and otherwise stated. For any subset A of X , the interior of A is same as the interior in usual topology and the closure of A is newly defined in simple extension respectively.

II. PRELIMINARIES

Definition 2.1 [10]: Let A subset A of a topological space (X, τ) is called a pre-open set if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.

Definition 2.2 [7]: A subset A of a topological space (X, τ) is called a semi-open set if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$.

Definition 2.3 [12]: A subset A of a topological space (X, τ) is called a α -open set if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and a α -closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

Definition 2.4 [1]: A subset A of a topological space (X, τ) is called a semi-preopen set (β -open set) if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and semi-preclosed set if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Definition 2.5 [7]: A subset A of a topological space (X, τ) is called a b -open set if $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$ and b -closed set if $\text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A)) \subseteq A$.

Definition 2.6 [6]: A subset A of a topological space (X, τ) is called a generalized closed set (briefly g -closed) if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

Definition 2.7 [9]: A subset A of a topological space (X, τ) is called a generalized α -closed set (briefly $g\alpha$ -closed) if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is α -open in (X, τ) .

Definition 2.8 [13]: A subset A of a topological space (X, τ) is called a generalized b -closed set (briefly gb -closed) if $\text{bcl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

Definition 2.9 [13]: A subset A of a topological space (X, τ) is called a generalized preclosed set (briefly gp -closed) if $\text{pcl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

Definition 2.10 [13]: A subset A of a topological space (X, τ) is called a generalized semiclosed set (briefly gs -closed) if $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

Definition 2.11 [13]: A subset A of a topological space (X, τ) is called a generalized semiclosed set (briefly sg -closed) if $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in X .

Definition 2.12 [4]: A subset A of a topological space (X, τ) is called a generalized semi-preclosed set (briefly gsp -closed) if $\text{spcl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

Definition 2.13 [14]: A subset A of a topological space (X, τ) is called a generalized g^* -closed set (briefly g^* -closed) if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is g -open in X .

Definition 2.14 [15]: A subset A of a topological space (X, τ) is called a g^*b -closed set if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .

Definition 2.15 [11]: Let A subset A of a topological space (X, τ^+) is called

- (1) a pre^+ -open set if $A \subseteq \text{int}(\text{cl}^+(A))$ and semi^+ -open set if $A \subseteq \text{cl}^+(\text{int}(A))$.
- (2) an α^+ -open set if $A \subseteq \text{int}(\text{cl}^+(\text{int}(A)))$ and β^+ -open set if $A \subseteq \text{cl}^+(\text{int}(\text{cl}^+(A)))$.
- (3) a b^+ -open set if $A \subseteq \text{cl}^+(\text{int}(A)) \cup \text{int}(\text{cl}^+(A))$

Definition 2.16 [11]: A subset A of a topological space (X, τ^+) is called a generalized α^+ -closed set (briefly $g\alpha^+$ -closed) if $\alpha^+\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is α^+ -open in (X, τ^+) .

Definition 2.17 [11]: A subset A of a topological space (X, τ^+) is called a generalized b^+ -closed set (briefly gb^+ -closed) if $b^+\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

III. CHARACTERIZATION OF g^*b -CLOSED SETS

Definition 3.1: A subset A of a topological space (X, τ^+) is called a generalized pre^+ -closed set (briefly gp^+ -closed) if $p^+\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Definition 3.2: A subset A of a topological space (X, τ^+) is called a generalized semi⁺closed set (briefly gs^+ -closed) if $s^+cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

Definition 3.3: A subset A of a topological space (X, τ^+) is called a generalized semi⁺closed set (briefly sg^+ -closed) if $s^+cl(A) \subseteq U$, whenever $A \subseteq U$ and U is semi⁺-open in X .

Definition 3.4: A subset A of a topological space (X, τ^+) is called a generalized⁺closed set (briefly g^+ -closed) if $cl^+(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

Definition 3.5: A subset A of a topological space (X, τ^+) is called a generalized semi-pre⁺closed set (briefly gsp^+ -closed) if $sp^+cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X .

Definition 3.6: A subset A of a topological space (X, τ^+) is called a generalized⁺closed set (briefly g^{*+} -closed) if $cl^+(A) \subseteq U$, whenever $A \subseteq U$ and U is g^+ -open in X .

Definition 3.7: A subset A of a topological space (X, τ^+) is called a $g^{*+}b$ -closed set if $b^+cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^+ -open in X .

Theorem 3.8: Every closed set is $g^{*+}b$ -closed set.

Proof: Let A be a closed set in X such that $A \subseteq U$, where U is g^+ -open. Since A is closed, $cl^+(A) = A$. Since $b^+cl(A) \subseteq cl^+(A) = A$. Therefore, $b^+cl(A) \subseteq U$. Hence A is a $g^{*+}b$ -closed set in X .

The converse of the above theorem need not be true as seen from the following example.

Example 3.9: Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. Let $A = \{b\}$. Here A is a $g^{*+}b$ -closed set but not a closed set of (X, τ^+) .

Theorem 3.10: Every b^+ -closed set is $g^{*+}b$ -closed set.

Proof: Let A be a b^+ -closed set in X such that $A \subseteq U$, where U is g^+ -open. Since A is b^+ -closed, $b^+cl(A) = A$. Therefore $b^+cl(A) \subseteq U$. Hence A is a $g^{*+}b$ -closed set in X .

The converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{a,c\}\}$ and $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $A = \{a,b\}$. Here A is a $g^{*+}b$ -closed set but not a b^+ -closed set of (X, τ^+) .

Theorem 3.12: Every $g^{*+}b$ -closed set is gb^+ -closed set.

Proof: Let A be a $g^{*+}b$ -closed set in X such that $A \subseteq U$, where U is open. Since every open set is g^+ -open and $g^{*+}b$ -closed, $b^+cl(A) \subseteq U$. Hence A is gb^+ -closed.

The converse of the above theorem need not be true as seen from the following example.

Example 3.13: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$ and $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$. Let $A = \{d\}$. Here A is a gb^+ -closed set but not a $g^{*+}b$ -closed set of (X, τ^+) .

Theorem 3.14: Every α^+ -closed set is $g^{*+}b$ -closed set.

Proof: Let A be a α^+ -closed set in X such that $A \subseteq U$, where U is g^+ -open. Since A is α^+ -closed, $b^+cl(A) \subseteq \alpha^+cl(A) \subseteq U$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

U. Therefore, $b^+cl(A) \subseteq U$. Hence A is $g^{*+}b$ -closed.

The converse of the above theorem need not be true as seen from the following example.

Example 3.15: Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a, b\}\}$ and $\tau^+ = \{X, \emptyset, \{b\}, \{a, b\}\}$. Let $A = \{b, c\}$. Here A is a $g^{*+}b$ -closed set but not a α^+ -closed set of (X, τ^+) .

Theorem 3.16: Every $semi^+$ -closed set is $g^{*+}b$ -closed set .

Proof: Let A be a $semi^+$ -closed set in X such that $A \subseteq U$, where U is g^+ -open. Since A is $semi^+$ -closed, $b^+cl(A) \subseteq s^+cl(A) \subseteq U$. Therefore, $b^+cl(A) \subseteq U$. Hence A is a $g^{*+}b$ -closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.17: Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a, b\}\}$ and $\tau^+ = \{X, \emptyset, \{b\}, \{a, b\}\}$. Let $A = \{b, c\}$. Here A is a $g^{*+}b$ -closed set but not a $semi^+$ -closed set of (X, τ^+) .

Theorem 3.18: Every pre^+ -closed set is $g^{*+}b$ -closed set .

Proof: Let A be a pre^+ -closed set in X such that $A \subseteq U$, where U is g^+ -open. Since A is pre^+ -closed, $b^+cl(A) \subseteq p^+cl(A) \subseteq U$. Therefore, $b^+cl(A) \subseteq U$. Hence A is $g^{*+}b$ -closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.19: Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{a, c\}\}$ and $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $A = \{a, b\}$. Here A is a $g^{*+}b$ -closed set but not a pre^+ -closed set of (X, τ^+) .

Theorem 3.20: Every g^{+*} -closed set is $g^{*+}b$ -closed set .

Proof: Let A be a g^{+*} -closed set in X such that $A \subseteq U$, where U is g^+ -open. Since A is g^{+*} -closed, $b^+cl(A) \subseteq cl^+(A) \subseteq U$. Therefore, $b^+cl(A) \subseteq U$. Hence A is a $g^{*+}b$ -closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.21: Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a\}\}$ and $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{b\}$. Here A is a $g^{*+}b$ -closed set but not a g^{+*} -closed set of (X, τ^+) .

Theorem 3.22: Every $g\alpha^+$ -closed set is $g^{*+}b$ -closed set.

Proof: Let A be a $g\alpha^+$ -closed set in X such that $A \subseteq U$, where U is g^+ -open. Since A is $g\alpha^+$ -closed, $b^+cl(A) \subseteq \alpha^+cl(A) \subseteq U$. Therefore, $b^+cl(A) \subseteq U$. Hence A is a $g^{*+}b$ -closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.23: Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a\}\}$ and $\tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$. Let $A = \{a, c\}$. Here A is a $g^{*+}b$ -closed set but not a $g\alpha^+$ -closed set of (X, τ^+) .

Theorem 3.24: Every g^+ -closed set is $g^{*+}b$ -closed set.

Proof: Let A be a g^+ -closed set in X such that $A \subseteq U$, where U is g^+ -open. Since A is g^+ -closed, $b^+cl(A) \subseteq cl^+(A) \subseteq U$. Hence A is $g^{*+}b$ -closed.

The converse of the above theorem need not be true as seen from the following example.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Example 3.25: Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{a,c\}\}$ and $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $A = \{a,b\}$. Here A is a $g^{*+}b$ -closed set but not a g^+ -closed set of (X, τ^+) .

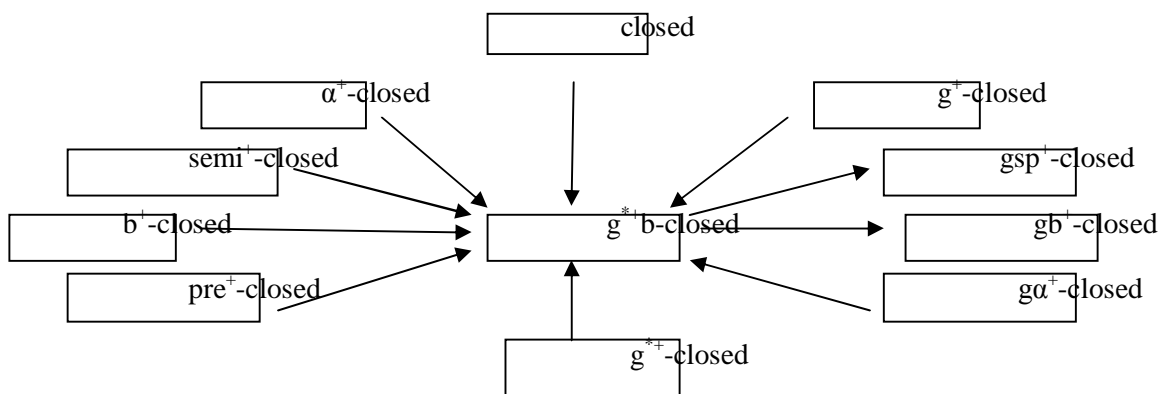
Theorem 3.26: Every $g^{*+}b$ -closed set is gsp^+ -closed set.

Proof: Let A be a $g^{*+}b$ -closed set in X such that $A \subseteq U$, where U is open. Since every open set is g^+ -open and $g^{*+}b$ -closed, $sp^+cl(A) \subseteq b^+cl(A) \subseteq U$. Hence A is gsp^+ -closed.

The converse of the above theorem need not be true as seen from the following example.

Example 3.27: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$ and $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$. Let $A = \{a,b,c\}$. Here A is a gsp^+ -closed set but not a $g^{*+}b$ -closed set of (X, τ^+) .

Remark 3.28: We have the following implications but none of this implications are reversible.



Theorem 3.29: A set A is $g^{*+}b$ -closed iff $b^+cl(A) - A$ contains no non-empty g^+ -closed set.

Proof:

Necessity: Let F be a g^+ -closed set of (X, τ^+) such that $F \subseteq b^+cl(A) - A$. Then $A \subseteq X - F$. Since A is $g^{*+}b$ -closed and $X - F$ is g^+ -open then $b^+cl(A) \subseteq X - F$. This implies $F \subseteq X - b^+cl(A)$. So $F \subseteq (X - b^+cl(A)) \cap (b^+cl(A) - A) \subseteq (X - b^+cl(A)) \cap b^+cl(A) = \emptyset$. Therefore $F = \emptyset$.

Sufficiency: Assume that $b^+cl(A) - A$ contains no non-empty g^+ -closed set. Let $A \subseteq U$, U is g^+ -open. Suppose that $b^+cl(A)$ is not contained in U , $b^+cl(A) \cap U^c$ is a non-empty g^+ -closed set of $b^+cl(A) - A$ which is a contradiction. Therefore $b^+cl(A) \subseteq U$ and hence A is $g^{*+}b$ -closed.

Theorem 3.30: A $g^{*+}b$ -closed set A is b^+ -closed iff $b^+cl(A) - A$ is b^+ -closed.

Proof: Let A is b^+ -closed, then $b^+cl(A) - A = \emptyset$. Conversely, suppose $b^+cl(A) - A$ is b^+ -closed in X . Since A is $g^{*+}b$ -closed, by theorem 3.29, $b^+cl(A) - A$ contains no non-empty g^+ -closed set in X . Then $b^+cl(A) - A = \emptyset$. Hence A is b^+ -closed.

Theorem 3.31: If A and B are $g^{*+}b$ -closed, then $A \cap B$ is $g^{*+}b$ -closed.

Proof: Let A and B are two $g^{*+}b$ -closed sets in X . Let $A \cap B \subseteq U$, U is g^+ -open set in X . Since A is $g^{*+}b$ -closed,

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

$b^+cl(A) \subseteq U$, whenever $A \subseteq U$, U is g^+ -open in X . Since B is $g^{*+}b$ -closed, $b^+cl(A) \subseteq U$ whenever $B \subseteq U$, U is g^+ -open in X .

Corollary 3.32: The intersection of a $g^{*+}b$ -closed set and a closed set is a $g^{*+}b$ -closed set.

Definition 3.33: let A be a subset of a space X . A point $x \in X$ is said to be a b^+ -limit point of A if for each b^+ -open set U containing x , we have $U \cap (A - \{x\}) \neq \emptyset$. The set of all b^+ -limit points of A is called the b^+ -derived set of A and is denoted by $D_b^+(A)$.

Since every open set is b^+ -open, we have $D_b^+(A) \subseteq D^+(A)$ for any subset $A \subseteq X$, where $D^+(A)$ is the derived set of A . Moreover, since every closed set is b^+ -closed, we have $A \subseteq b^+cl(A) \subseteq cl^+(A)$.

The proof of the following result is straightforward and thus omitted.

Lemma 3.34: If $D^+(A) = D_b^+(A)$, then we have $cl^+(A) = b^+cl(A)$.

Corollary 3.35: If $D^+(A) \subseteq D_b^+(A)$ for every subset A of X . Then for any subsets F and B of X , we have $b^+cl(F \cup B) = b^+cl(F) \cup b^+cl(B)$.

Corollary 3.36: If A and B are $g^{*+}b$ -closed sets such that $D^+(A) \subseteq D_b^+(A)$ and $D^+(B) \subseteq D_b^+(B)$. Then $A \cup B$ is $g^{*+}b$ -closed.

Proof: Let U be an open set such that $A \cup B \subseteq U$. Then since A and B be $g^{*+}b$ -closed sets we have $b^+cl(A) \subseteq U$ and $b^+cl(B) \subseteq U$. Since $D^+(A) \subseteq D_b^+(A)$, thus $D^+(A) = D_b^+(A)$ and by Lemma 3.34, $cl^+(A) = b^+cl(A)$. Similarly, $cl^+(B) = b^+cl(B)$. Thus $b^+cl(A \cup B) \subseteq cl^+(A \cup B) = cl^+(A) \cup cl^+(B) = b^+cl(A) \cup b^+cl(B) \subseteq U$, which implies that $A \cup B$ is $g^{*+}b$ -closed.

Let $B \subseteq A \subseteq X$. Then we say that B is $g^{*+}b$ -closed relative to A if $b^+cl_A(B) \subseteq U$ where $B \subseteq U$ and U is g^+ -open in A .

Theorem 3.37: Let $B \subseteq A \subseteq X$ where A is a g^+ -open and $g^{*+}b$ -closed set. Then B is $g^{*+}b$ -closed relative to A if and only if B is $g^{*+}b$ -closed in X .

Proof: We first note that since $B \subseteq A$ and A is both a $g^{*+}b$ -closed and g^+ -open set, then $b^+cl(A) \subseteq A$ and thus $b^+cl(B) \subseteq b^+cl(A) \subseteq A$. Now from the fact that $A \cap b^+cl(B) = b^+cl_A(B)$, we have $b^+cl(B) = b^+cl_A(B) \subseteq A$. If B is $g^{*+}b$ -closed relative to A and U is g^+ -open subset of X such that $B \subseteq U$, then $B = B \cap A \subseteq U \cap A$ where $U \cap A$ is g^+ -open in A . Hence as B is $g^{*+}b$ -closed relative to A , $b^+cl(B) = b^+cl_A(B) \subseteq U \cap A \subseteq U$. Therefore B is $g^{*+}b$ -closed in X . Conversely if B is $g^{*+}b$ -closed in X and U is an g^+ -open subset of A such that $B \subseteq U$, then $U = V \cap A$ for some open subset V of X . As $B \subseteq V$ and B is $g^{*+}b$ -closed in X , $b^+cl(B) \subseteq V$. Thus $b^+cl_A(B) = b^+cl(B) \cap A \subseteq V \cap A = U$. Therefore B is $g^{*+}b$ -closed relative to A .

Corollary 3.38: Let A be g^+ -open and $g^{*+}b$ -closed set. Then $A \cap F$ is $g^{*+}b$ -closed whenever $F \in b^+cl(X)$.

Proof: Let A is $g^{*+}b$ -closed and g^+ -open, then $b^+cl(A) \subseteq A$ and thus A is b^+ -closed. Hence $A \cap F$ is b^+ -closed in X which implies that $A \cap F$ is $g^{*+}b$ -closed in X .

Theorem 3.39: If A is $g^{*+}b$ -closed and $A \subseteq B \subseteq b^+cl(A)$, then B is $g^{*+}b$ -closed.

Proof: Let U be a g^+ -open set of X such that $B \subseteq U$. Then $A \subseteq U$. Since A is $g^{*+}b$ -closed, then $b^+cl(A) \subseteq U$. Now

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

$b^+cl(B) \subseteq b^+cl(b^+cl(A)) = b^+cl(A) \subseteq U$. Therefore B is $g^{*+}b$ -closed in X.

Theorem 3.40: A subset $A \subseteq X$ is $g^{*+}b$ -open if and only if $F \subseteq b^+int(A)$ whenever F is closed set and $F \subseteq A$.

Proof: Let A be a $g^{*+}b$ -open set and suppose $F \subseteq A$ where F is closed. Then $X - A$ is a $g^{*+}b$ -closed set contained in the g^+ -open set $X - F$. Hence $b^+cl(X - A) \subseteq X - F$ and $X - b^+int(A) \subseteq X - F$. Thus $F \subseteq b^+int(A)$. Conversely if F is a closed set with $F \subseteq b^+int(A)$ and $F \subseteq A$, then $X - b^+int(A) \subseteq X - F$. Thus $b^+cl(X - A) \subseteq X - F$. Hence $X - A$ is a $g^{*+}b$ -closed set and A is a $g^{*+}b$ -open set.

Proposition 3.41: Let A be a subset of a topological space (X, τ^+) . If $A \in so^+(X)$, then $p^+cl(A) = cl^+(A)$.

A space (X, τ^+) is extremally disconnected if the closure of every open subset of X is open. The following results about the class of extremally disconnected space:

Theorem 3.42: For a space X, the following statements are equivalent:

- (1) (X, τ^+) is extremally disconnected;
- (2) $s^+cl(A \cup B) = s^+cl(A) \cup s^+cl(B)$ for all $A, B \subseteq X$;
- (3) the union of two semi $^+$ -closed subsets of X is semi $^+$ -closed;
- (4) the union of two sg^+ -closed subsets of X is sg^+ -closed;
- (5) every semi-pre $^+$ closed subset of X is pre $^+$ -closed;
- (6) every sg^+ -closed subset of X is pre $^+$ -closed;
- (7) every semi $^+$ -closed subset of X is pre $^+$ -closed;
- (8) every semi $^+$ -closed subset of X is α^+ -closed;
- (9) every semi $^+$ -closed subset of X is $g\alpha^+$ -closed.

Next, we present one more characterization of extremally disconnectedness using generalized closed subsets.

Theorem 3.43: A space X is extremally disconnected if and only if every gb^+ -closed subset of X is gp^+ -closed.

Proof: suppose that X is extremally disconnected. Let A be gb^+ -closed and let U be an open set containing A. Then $b^+cl(A) = A \cup [int(cl^+(A)) \cap cl^+(int(A))] \subseteq U$, i.e, $[int(cl^+(A)) \cap cl^+(int(A))] \subseteq U$. Since $int(cl^+(A))$ is closed, we have $cl^+(int(A)) \subseteq cl^+[int(cl^+(A)) \cap (int(A))] \subseteq [cl^+(int(cl^+(A))) \cap cl^+(int(A))] \subseteq U$. It follows that $p^+cl(A) = A \cup cl^+(int(A)) \subseteq U$. Hence A is gp^+ -closed. To prove the converse, let every gb^+ -closed subset of X be gp^+ -closed. Let $A \subseteq X$ be regular $^+$ open. Then, $b^+cl(A) = A \cup [int(cl^+(A)) \cap cl^+(int(A))] = A \cup [A \cap cl^+(int(A))] \subseteq A$. Then A is gb^+ -closed and so gp^+ -closed. Since every regular $^+$ open is semi $^+$ -open set by proposition 3.41 and A is gp^+ -closed. we have $cl^+(A) = p^+cl(A) \subseteq A$. Therefore A is closed and X is extremally disconnected.

IV. CONCLUSION

Thus a new class of $g^{*+}b$ -closed sets lies between the class of b^+ -closed sets and the class of gb^+ -closed sets and their characterizations are discussed. In further study the various mappings may be introduced.

REFERENCES

- [1] Andrijevic, D., "Semi-preopen sets", Mat. Vesnik, Vol.38, pp.24 – 32, 1986.
- [2] Andrijevic, D., "On b-open sets", Mat. Vesnik, Vol.48, pp.59 – 64, 1996.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

- [3] Cao, J., Ganster, M., and Reilly, I., "Submaximality, extremal disconnectedness and generalized closed sets", Houston J. Math. Vol.24, No.4, pp.681-688, 1998.
- [4] Dontchev, J., "On generalizing semi-preopen sets", Mem. Fac. Sci. Kochi. Ser. A, Math., Vol.16, pp.35-48, 1995.
- [5] Dontchev, J., Ganster, M., and Noiri, T., "On p-closed spaces", Int. J. Math. Math. Sci. Vol.24, No.3, pp.203-212, 2000.
- [6] Levine, N., "Generalized closed sets in topology", Rend. Circ. Math. Palermo, Vol.19, pp.89 – 96, 1970.
- [7] Levine, N., "Semi-open sets and semi-continuity in topological spaces", Amer. Math. Monthly, Vol.70, pp.36 – 41, 1963.
- [8] Levine, N., "Simple extension of topology", Amer. Math. Monthly, Vol.71, pp.22-105, 1964.
- [9] Maki, H., Devi, R., and Balachandran, K., "Generalized α -closed sets in topology", Bull. Fukuoka. Univ. E. Part III, Vol.42, pp.13 – 21, 1993.
- [10] Mashhour, A. S., Abd.El-Monsef, M. E., and El-Deeb, S. N., "On pre-continuous and weak pre-continuous mapping", Proc. Math., Phys. Soc. Egypt, Vol.53, pp.47 – 53, 1982.
- [11] Nirmala Irudayam, F., and Arockiarani, Sr. I., "A note on the weaker form of bI set and its generalization in SEITS", International Journal of Computer Application, Vol.4, Issue 2, pp.42-54, 2012.
- [12] Njastad, O., "On some classes of nearly open sets", Pacific. J. Math, Vol.15, pp.961-970, 1965.
- [13] Omari, A. A., and Noorani, M. S. M., " On generalized b-closed sets", Bull. Malays. Math. Sci. Soc, vol.32, pp.19 – 30, 2009.
- [14] Veerakumar, M. K. R. S., "Between closed sets and g-closed sets", Mem. Fac. Sci. Kochi. Univ. Ser. A, Math, Vol.21, pp.1-19, 2000.
- [15] Vidhya, D., and Parimelazhagan, R., "g^b-Closed Sets in Topological Spaces", Int. J. Contem. Math. Sciences, Vol.7, No.27, pp.1305-1312, 2012.