

A Comparative Study - Optimal Path using Fuzzy Distance, Trident Distance and Sub-Trident Distance

A.Praveen Prakash¹, M.GeethaLakshmi²Professor, Department of Mathematics, Hindustan University, Padur, Chennai, India¹Assistant Professor, Department of Mathematics, KCG College of Technology, Chennai, India²

ABSTRACT:In this paper, the comparative study for finding the Optimal Path using Fuzzy Distance, Trident Distance and Sub-Trident Distance are discussed. These three methods are compared and the results are obtained by giving suitable numerical example. Finally the best Method among these three methods is given with proper justification.

KEYWORDS:Trident Form, Sub-Trident Form, Optimal Path, Graded Mean.

I. INTRODUCTION

Most fundamental problems in network theory are to find the shortest path in a network. The distance between source node and a destination node in a transportation network determines the shortest path problem. The numbers associated with the edges of networks may represent characteristics other than lengths, and here the optimum path can be found by using different criteria. This paper deals with the comparison for finding the Fuzzy Shortest Path in the Network using Pascal's Triangle Graded Mean Integration Representation along with the help of Trapezoidal Fuzzy Numbers. Here the trapezoidal fuzzy numbers play the role as edge weights for the connection and the optimal shortest path is obtained by the following three distance methods: 1. Fuzzy Distance method, 2. Trident Distance Method, 3. Fuzzy Sub-Trident Distance Method. The result is obtained by analyzing these three methods and finally the best optimal path is observed through suitable numerical example. This Paper consists of five sections: the first section comes introduction part, the numerical example for the connection network in the second section, third section is the explanation of the first method; here in all adjacent nodes, the fuzzy distance are compared by using fuzzy order relation. The explanation of the second method in the fourth section; here the optimal path is obtained by using Trident Distance method. In the fifth section, the explanation of third method is analysed and the optimal path is observed. Finally the best optimal path is found by comparing all the three methods.

II. RELATED WORK

In 1965, L.A.Zadeh [1] introduced Fuzzy Sets. Later in 1980, D. Dubois and Prade [2] first analysed the fuzzy shortest path problem using Floyd's algorithm. Bortolan G. and Degani.R [3] introduced a review of some methods for ranking fuzzy subsets in the year 1985. In 1994, S. Okada and M. Gen [4] introduced Fuzzy shortest path problem. Then S.Hilpern [5] introduced the Representation and application of fuzzy numbers in 1997. Shan-Huo Chen and Chin Hsun Hseih [6, 7] gives an idea of Graded Mean Integration Representation of Generalized Fuzzy and Representation, Ranking, Distance and Similarity of L-R Type Fuzzy Number and Application in 2000. In the same year, S. Okada and T. Soper [8] introduced a shortest path problem on a network with fuzzy arc lengths. In 2005, Miin-Shen Yang and wen-Liang Hung [9] introduced a Similarity Measure between LR- Type Fuzzy Numbers and its Application to Database Acquisition. Shan-Huo Chen and Chien Chung Wang [10] introduced Fuzzy Distance of Trapezoidal Fuzzy Numbers in 2006. A.Nagoorgani and A.Mumtaj Begam [11] gives idea on a New Approach on Shortest Path in Fuzzy Environment in 2010. Amit Kumar and Manoj Kaur [12] introduced New Algorithm for solving network flow problems with fuzzy arc lengths in 2011.

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III. NUMERICAL EXAMPLE

Let us consider the following example for the Connection Network and the edge weights are Trapezoidal Fuzzy Numbers:

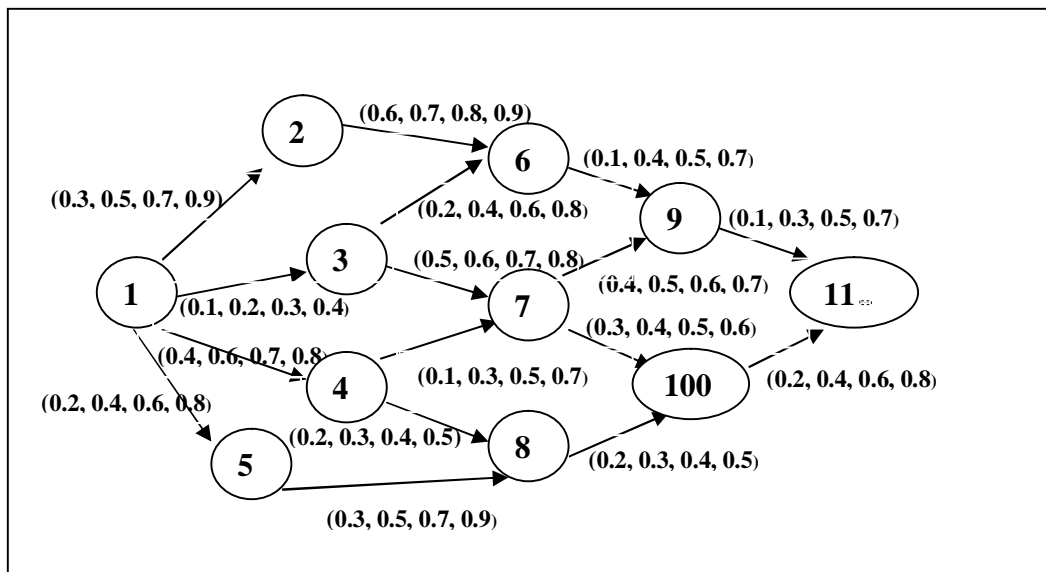


Fig. 1 Example for Connection Network

IV. EXPLANATION OF METHOD : 1

The Optimal Path using Fuzzy Distance is given in Method: 1 as follows:

A. ORDER RELATION

For fuzzy order relation [3], let us consider two trapezoidal fuzzy numbers

$$A = (a_1, a_2, a_3, a_4), B = (b_1, b_2, b_3, b_4)$$

Then $A \leq B$ iff the following inequalities hold: (i) $a_1 \leq b_1$ (ii) $a_2 \leq b_2$ (iii) $a_3 \leq b_3$ (iv) $a_4 \leq b_4$.

B. FUZZY DISTANCE

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers and their Graded Mean Integration Representation are $P(A), P(B)$ respectively. Assume

$$S_i = \frac{(a_i - P(A) + b_i - P(B))}{2}, i = 1, 2, 3, 4;$$

$$C_i = |P(A) - P(B)| + S_i, i = 1, 2, 3, 4;$$

Then the fuzzy distance of A, B is $c = (c_1, c_2, c_3, c_4)$.

C. PASCAL'S TRIANGLE GRADED MEAN APPROACH

The Graded Mean Integration Representation for generalized fuzzy number by Chen and Hsieh [5, 6]. But the present approach is very simple for analyzing fuzzy variables to get the optimum shortest path. This procedure is taken from the following Pascal's triangle. We take the coefficients of fuzzy variables as Pascal's triangle numbers.

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Then we just add and divide by the total of Pascal's number and we call it as Pascal's Triangle Graded Mean Approach [13].

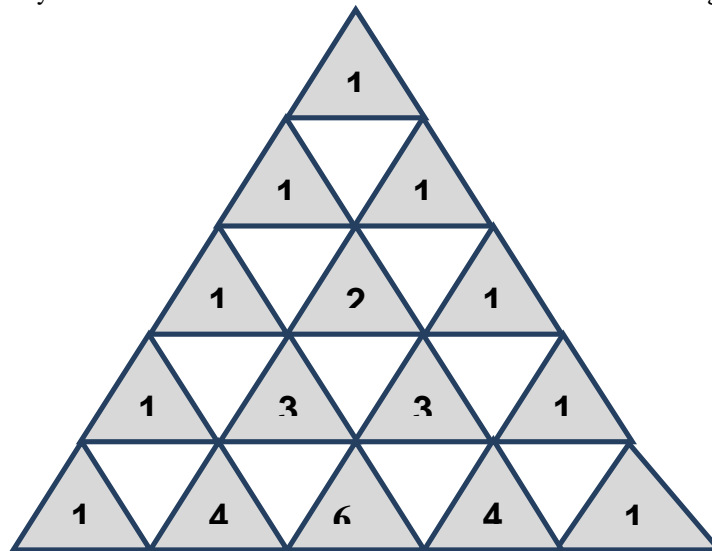


Fig. 2 PASCAL'S TRIANGLE

The following are the Pascal's triangular approach:

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers then we can take the coefficient of fuzzy numbers from Pascal's triangles and apply the approach we get the following formula:

$$P(A) = \frac{a_1 + 3a_2 + 3a_3 + a_4}{8};$$

$$P(B) = \frac{b_1 + 3b_2 + 3b_3 + b_4}{8}$$

The coefficients of a_1, a_2, a_3, a_4 and b_1, b_2, b_3, b_4 are 1, 3, 3, 1. This approach can be extended for n-dimensional Pascal's Triangular fuzzy order also.

D. CALCULATION PART

The Calculations to find the optimal path using Fuzzy Distance with the help of order relation are shown in the tables: 1, 2, and 3 given below: [11]

TABLE 1 CALCULATION PART

NODE(1,2)	NODE(1,3)
P(A)=P(1,1) = (0,0,0,0) = 0/8 =0	P(A)=P(1,1) = (0,0,0,0) = 0/8 =0
P(B)=P(1,2)=(0.3,0.5,0.7,0.9) =(0.3+3(0.5)+3(0.7)+0.9)/8=0.6	P(B)=P(1,3)=(0.1,0.2,0.3,0.4) =(0.1+3(0.2)+3(0.3)+0.4)/8=0.25
S1=(0-0+0.3-0.6)/2= - 0.15	S1=(0-0+0.1-0.25)/2= - 0.075
S2=(0-0+0.5-0.6)/2= 0.05	S2=(0-0+0.2-0.25)/2= -0.025

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$S3=(0-0+0.7-0.6)/2= 0.05$	$S3=(0-0+0.3-0.25)/2= 0.025$
$S4=(0-0+0.9-0.6)/2= 0.15$	$S4=(0-0+0.4-0.25)/2= 0.075$
$C1= 0-0.6 +(- 0.15) = 0.45$	$C1= 0-0.25 +(- 0.075) = 0.175$
$C2= 0-0.6 +(- 0.05) = 0.55$	$C2= 0-0. 25 +(- 0.025) = 0.225$
$C3= 0-0.6 +(0.05) = 0.65$	$C3= 0-0. 25 +(0.025) = 0.275$
$C4= 0-0.6 +(0.15) = 0.75$	$C4= 0-0. 25 +(0.075) = 0.325$
$C=(0.45,0.55,0.65,0.75)$	$C=(0.175,0.225,0.275,0.325)$

TABLE 2 CALCULATION PART

NODE(1,4)	NODE(1,5)
$P(A)=P(1,1) = (0,0,0,0) = 0/8 =0$	$P(A)=P(1,1) = (0,0,0,0) = 0/8 =0$
$P(B)=P(1,4)=(0.4,0.6,0.7,0.8)$ $= (0.4+3(0.6)+3(0.7)+0.8)/8=0.6375$	$P(B)=P(1,5)=(0.2,0.4,0.6,0.8)$ $= (0.2+3(0.4)+3(0.6)+0.8)/8=0.5$
$S1=(0-0+0.4-0.6375)/2= - 0.11875$	$S1=(0-0+0.2-0.5)/2= - 0.15$
$S2=(0-0+0.6-0.6375)/2= - 0.01875$	$S2=(0-0+0.4-0.5)/2= -0.05$
$S3=(0-0+0.7-0.6375)/2= 0.03125$	$S3=(0-0+0.6-0.5)/2= 0.05$
$S4=(0-0+0.8-0.6375)/2= 0.08125$	$S4=(0-0+0.8-0.5)/2= 0.15$
$C1= 0-0.6375 +(- 0.11875) = 0.51875$	$C1= 0-0.5 +(- 0.15) = 0.35$
$C2= 0-0.6375 +(- 0.01875) = 0.61875$	$C2= 0-0.5 +(- 0.05) = 0.45$
$C3= 0-0.6375 +(0.03125) = 0.66875$	$C3= 0-0.5 +(0.05) = 0.55$
$C4= 0-0.6375 +(0.08125) = 0.71875$	$C4= 0-0.5 +(0.15) = 0.65$
$C=(0.51875,0.61875,0.66875,0.71875)$	$C=(0.35,0.45,0.55,0.65)$

Among (1, 2), (1, 3), (1, 4), (1, 5) the path (1, 3) is the smallest according to order relation and from node 3 the corresponding adjacent node is 6 & 7.

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TABLE 3 CALCULATION PART

NODE(3,6)	NODE(3,7)
$P(A)=P(1,3)=(0.1,0.2,0.3,0.4)$ $= (0.1+3(0.2)+3(0.3)+0.4)/8=0.25$	$P(A)=P(1,3)=(0.1,0.2,0.3,0.4)$ $= (0.1+3(0.2)+3(0.3)+0.4)/8=0.25$
$P(B)=P(3,6)=(0.2,0.4,0.6,0.8)$ $= (0.2+3(0.4)+3(0.6)+0.8)/8=0.5$	$P(B)=P(3,7)=(0.5,0.6,0.7,0.8)$ $= (0.5+3(0.6)+3(0.7)+0.8)/8=0.65$
$S1=(0.1-0.25+0.2-0.5)/2= - 0.225$	$S1=(0.1-0.25+0.5-0.65)/2= - 0.15$
$S2=(0.2-0.25+0.4-0.5)/2= - 0.075$	$S2=(0.2-0.25+0.6-0.65)/2= - 0.05$
$S3=(0.3-0.25+0.6-0.5)/2= 0.075$	$S3=(0.3-0.25+0.7-0.65)/2= 0.05$
$S4=(0.4-0.25+0.8-0.5)/2= 0.225$	$S4=(0.4-0.25+0.8-0.65)/2= 0.15$
$C1= 0.25-0.5 +(- 0.225) = 0.025$	$C1= 0.25-0.65 +(- 0.15) = 0.25$
$C2= 0.25-0.5 +(- 0.075) = 0.175$	$C2= 0.25-0.65 +(- 0.05) = 0.35$
$C3= 0.25-0.5 +(0.075) = 0.325$	$C3= 0.25-0.65 +(0.05) = 0.45$
$C4= 0.25-0.5 +(0.225) = 0.475$	$C4= 0.25-0.65 +(0.15) = 0.55$
$C=(0.025,0.175,0.325,0.475)$	$C=(0.25,0.35,0.45,0.55)$

Comparing these two nodes (3, 6) and (3, 7), (3, 6) is shortest and the only adjacent nodes are 9&11. Therefore the Shortest Path is 1→3→6→9→11.

V. EXPLANATION OF METHOD : 2

The Optimal Path using Fuzzy Trident Distance is given in Method: 2 as follows:

A. FUZZY TRIDENT DISTANCE

The distance between the two fuzzy numbers are calculated by using the new technique called the Fuzzy Trident Distance as follows:

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ then the Fuzzy Trident Distance is given by

$$F_{Tri} distance(A, B) = \left\{ \frac{1}{3} \left[(a_1 - b_1)^3 + (a_2 - b_2)^3 + (a_3 - b_3)^3 + (a_4 - b_4)^3 \right]^{1/3} \right\}$$

B. CALCULATION PART

The Calculations to find the Fuzzy Trident Distance and Average Fuzzy Trident Distance for the following possible paths by taking the average for the calculated Fuzzy Trident Distance are shown in the table given below: [15]

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TABLE 4 CALCULATION PART

Possible Paths	Paths	Fuzzy Trident Distance ($F_{Tri} distance$)	Average F_{Tri} distance ($Avg F_{Tri} distance$)
1→2→6→9→11	1→2	0.11006	0.071695
	2→6		
	6→9	0.03333	
	9→11		
1→3→6→9→11	1→3	0.15472	0.094025
	3→6		
	6→9	0.03333	
	9→11		
1→3→7→9→11	1→3	0.211653	0.16086
	3→7		
	7→9	0.11006	
	9→11		
1→3→7→10→11	1→3	0.211653	0.1392
	3→7		
	7→10	0.06667	
	10→11		
1→4→7→9→11	1→4	0.13264	0.12135
	4→7		
	7→9	0.11006	
	9→11		
1→4→7→10→11	1→4	0.13264	
	4→7		

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	7→10		0.099655
	10→11	0.06667	
1→4→8→10→11	1→4	0.14882	0.12944
	4→8		
	8→10	0.11006	
	10→11		
1→5→8→10→11	1→5	0.05291	0.081485
	5→8		
	8→10	0.11006	
	10→11		

The minimum Average Trident Distance for all possible paths is 0.071695. Therefore the corresponding path 1→2→6→9→11 is the shortest path.

VI. EXPLANATION OF METHOD: 3

The Optimal Path using Fuzzy Sub-Trident Distance is given in Method: 3 are as follows:

A. FUZZY SUB-TRIDENT DISTANCE

The distance between the two fuzzy numbers are calculated by using the new technique called the Fuzzy Sub-Trident Distance as follows:

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ then the Fuzzy Sub-Trident Distance is given by

$$FS_{Tri} \text{ distance}(A, B) = \left\{ \frac{1}{3} [(a_1 - b_1)^3 + (a_2 - b_2)^3 + (a_3 - b_3)^3] \right\}^{1/3}$$

B. CALCULATION PART

The Calculations to find the Fuzzy Sub-Trident Distance and Average Fuzzy Sub-Trident Distance for the following possible paths by taking the average for the calculated Fuzzy Sub-Trident Distance are shown in the table given below:

TABLE 5 CALCULATION PART

Possible Paths	Paths	Fuzzy Sub-Trident Distance ($FS_{Tri} \text{ distance}$)	Average $FS_{Tri} \text{ distance}$ ($AvgFS_{Tri} \text{ distance}$)
1→2→6→9→11	1→2	0.012	1.6725
	2→6		

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	6→9 9→11	3.333	
1→3→6→9→11	1→3 3→6	0.01267	1.6728
	6→9 9→11	0.03333	
1→3→7→9→11	1→3 3→7	0.064	0.326
	7→9 9→11	0.012	
1→3→7→10→11	1→3 3→7	0.064	0.032
	7→10 10→11	0	
1→4→7→9→11	1→4 4→7	0.02067	0.016335
	7→9 9→11	0.012	
1→4→7→10→11	1→4 4→7	0.0313	0.01565
	7→10 10→11	0	
	1→4		

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1→4→8→10→11	4→8	0.02067	0.011835
	8→10	0.003	
	10→11		
1→5→8→10→11	1→5	0.001	0.002
	5→8		
	8→10	0.003	
	10→11		

The minimum Average Sub-Trident Distance for all possible paths is 0.002. Therefore the corresponding path 1→5→8→10→11 is the shortest path.

VII. CONCLUSION

From the Comparison of three methods Fuzzy Distance, Trident Distance and Sub-Trident Distance it is observed that, the optimal path for Method: 1 is 1→3→6→9→11, the optimal path for Method: 2 is 1→2→6→9→11 and the optimal path for Method: 3 is 1→5→8→10→11. Among all these three methods, the average value using Sub-Trident Distance is very small i.e., 0.002. So the path 1→5→8→10→11 obtained from Sub-Trident Distance is the best optimal path.

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