

Non-perturbative Quantum Effects in Stringy Degenerate Geometries: Vacuum Created Pair of $(D\bar{D})_3$ -Brane by a Two Form

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ABSTRACT: We obtain axionic charged primordial black holes on a vacuum created gravitational pair of $(3\bar{3})$ -brane by the Kalb-Ramond field on a D4-brane in presence of a background open string metric. The new geometries on an emergent pair of $(3\bar{3})$ -brane universe is shown to be influenced by the non-perturbative quantum effects underlying a geometric torsion in a second order formalism. The presence of small extra dimensions transverse to the pair in the formalism ensures dynamical scalar fields hidden to a 3-brane or a $\bar{3}$ -brane universe. We investigate the non-perturbative quantum vacua for their characteristic properties to explain the accelerated expansion of our universe. Interestingly the emergent brane universe is shown to possess its origin in degenerate stringy de Sitter vacua at an early epoch. A non-perturbative D_p brane world volume correction for $p > 3$ is worked out to explain some of the quantum effects underlying quintessence axion in the string-brane setup. Our analysis reveals that a D-instanton can be a potential candidate to source the dark energy in our brane universe.

KEYWORDS: Emergent Non-perturbative Quantum Gravity, Higher Form Gauge Theory on D-branes, Cosmological Pair Production of brane-Universe and anti-brane Universe, Stringy Degenerate Vacua

I. INTRODUCTION

Generically the low energy string effective actions have been investigated in presence of the Neveu-Schwarz (NS) background [1,2,3] and they have given birth to $(3 + 1)$ -dimensional black holes in string theory [4, 5]. Analysis has revealed the geometrical significance of background NS field, which sources a torsion in space-time. Dirichlet (D) p-brane in type IIA or IIB superstring theory is believed to be a potential candidate to describe a non-perturbative world due to their Ramond-Ramond (RR) charges [6].

In the recent past a string-brane setup underlying such a non-BPS brane configuration was constructed by the authors [7,8,9]. It was shown that the non-BPS brane may well be described by a non-perturbative phenomenon leading to a vacuum created gravitational pair of $(D\bar{D})_3$ -brane by the Kalb-Ramond (KR) quanta on a D4-brane at the cosmological horizon of a background (open string) de Sitter geometry. In fact the cosmological creation of a pair of non-BPS $(D\bar{D})_9$ -brane in a different context was discussed by Majumdar and Davis [10]. Their work was primarily motivated to address the strong coupling regime in type IIA superstring leading to the M-theory. The strength of string-brane setup for an enhanced understanding of the M-theory has been attempted by one of the authors [11].

In the context Schwinger pair production mechanism is known to be instrumental to gain insight on diverse quantum phenomenon in gravitation and cosmology [12]. The novel idea possesses its origin in quantum field theory which

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elegantly describes a (positron and electron) pair production at a vacuum by electromagnetic quanta. The pair production tool was exploited to explain the Hawking radiation phenomenon [13]. Furthermore the mechanism may generalize to produce a pair of charged cosmic string and anti-string at an early era. The emergent pair is governed by the Kibble mechanism [14]. The Schwinger effect has been exploited in string theory to investigate a pair creation of open strings in an electric field [15, 16, 17]. Importantly the pair production mechanism has been extended to obtain a cosmological creation of a pair of $(D\bar{D})$ -brane in superstring theory [10, 18] and the open string pair creation from the world-sheet instanton [19].

The vacuum created pair of $(D\bar{D})$ -brane was inspired by a fact that extra spatial dimensions with small radii of compactifications are believed to play a significant role in the study of quantum cosmology. In particular we have obtained emergent Kerr-brane (anti-brane) geometries, which have been shown to describe the Einstein vacuum in a low energy [20, 21]. A generalization underlying a CFT on a D_5 -brane has been shown to describe Schwarzschild de Sitter and topological de Sitter on a $(D\bar{D})_4$ -brane by one of the authors (SK) in collaborations [22, 23].

The extra dimensions to a vacuum created pair of a lower dimensional $(D\bar{D})$ -brane are identified with the transverse scalar fields and possess their origin in a gauge field on a space filling D_9 -brane. Thus an extra dimension, in the disguise of a scalar field, sources a quintessence in $(3 + 1)$ -dimensional Einstein gravity. Some of the relevant computations pertaining to a quintessence scalar dynamics in higher dimensional gravitation and cosmology have been worked out in refs. [24, 25, 26, 27].

Interestingly the string-brane setup has been exploited to describe the inflationary cosmology underlying a rapid accelerated expansion of our universe [28, 29]. An extra fifth dimension transverse to both the four dimensional world-volumes has been argued to play a significant role to describe a nontrivial vacuum geometries. In fact a large extra dimension incorporates the low energy closed string modes in between a brane and an anti-brane. The torsion condenses in a low energy limit and hence the reduced geometry becomes Riemannian [30, 31, 32].

In the paper we focus on non-perturbative (quantum) new geometries in presence of a propagating torsion on a vacuum created gravitational pair of 3-brane and anti 3-brane. They are indeed a new class of quantum vacua as they are shown to be flat in a lower energy limit, i.e. in absence of a geometric torsion. It would imply an open string propagation in a Minkowski background. It confirms that the quantum new geometries are exact in a perturbative string theory. They do not possess any classical geometric (Einstein) vacuum. Thus, the emergent quantum vacua, obtained in this paper, are indeed different from the brane/anti-brane quantum geometries previously obtained in our other papers [30, 31, 7, 8, 9, 32, 11, 20, 21, 28, 29, 22]. The emergent geometries are shown to be sourced by an axionic scalar underlying a vacuum created gravitational pair of 3-brane and anti 3-brane.

We work out the quantum effects and show that the low energy perturbative string vacuum receives a non-perturbative D-brane world-volume correction in the string-brane model. The axionic sourced non-perturbative quantum geometries are further analysed for two larger length scales to qualitatively describe a laboratory black hole in a medium energy regime and a semi-classical brane universe in a moderately low energy regime. Analysis under Weyl scaling(s) are performed to explore some of the effective (Anti) de Sitter brane vacua in presence of non-perturbative correction(s). A quantum correction is shown to be approximated by a varying energy density underlying an axionic quintessence. It is argued that a D-instanton, sourced by an axion, presumably incorporates the quintessential effects to our universe. These non-perturbative quantum effects indeed provide clue for extra dimension(s) to our $(3 + 1)$ -dimensional universe.

We plan the paper as follows. We begin with a set-up underlying an effective curvature on a D4 brane in presence of a background open string metric in section 2. A gauge choice and various plausible torsion geometries on a vacuum created gravitational pair of $(3\bar{3})$ -brane are obtained in section 3. Non-perturbative quantum effects underlying a lower dimensional D-brane are explicitly worked out in section 4. The brane geometries are Weyl scaled appropriately in section 5 to obtain a de Sitter brane and an AdS brane in the formalism. The brane universe is shown to be described by a quintessence axion. We conclude with a brief summary in section 5.

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II. PRILIMNARIES

A. GEOMETRIC TORSION \mathcal{H}_3 IN 5D:

To illustrate the emergent scenario we begin with a KR field U(1) dynamics, in presence of a background (open) string metric, on a D_4 -brane. Needless to mention that a nontrivial background metric in the world-volume dynamics is sourced by a constant NS field in string theory. It may be noted that the set-up evolves with two independent two forms. One of it is dynamical KR field and the other describes a constant NS field. An appropriate covariant derivative is explored in presence of a world-volume torsion H_3 connection, which in turn incorporates a geometric torsion \mathcal{H}_3 in a second order curvature formalism [30,31,32,11]. The presence of an axion has been explored to address a quintessence scalar dynamics hidden to a vacuum created gravitational 3-brane universe within a pair.

On the other hand, a D_4 -brane is a dynamical object in ten dimensional type IIA superstring theory. Its five dimensional world-volume dynamics may be approximated by a Dirac-Born-Infeld (DBI) action, which describes a non-linear one form \mathcal{A}_μ in a U(1) gauge theory. For a constant background metric $g_{\mu\nu}$, the dynamics of \mathcal{A}_μ leads to a non-linear field strength $\bar{F}_{\mu\nu} = (2\pi\alpha')F_{\mu\nu} = F_{\mu\nu}^{linear} + B_{\mu\nu}^{NS}$. It is given by

$$S = -\frac{1}{C_1^2} \int d^5x \sqrt{-\det(g + \bar{F})_{\mu\nu}}, \quad (1)$$

where $C_1^2 = (4\pi^2 g_s) \alpha'^{5/2}$. Alternately, the Hodge dual to the field strength on a D_4 -brane underlies a dynamical KR field $B_{\mu\nu}$ and is known to incorporate a gauge theoretic torsion $H_3 = dB_2$. The KR field dynamics, in presence of a background open string metric $\bar{G}_{\mu\nu}^{(NS)} = (g_{\mu\nu} - B_{\mu\nu}^{(NS)} B^{\lambda\nu(NS)})$, is given by

$$S = -\frac{1}{12C_2^2} \int d^5x \sqrt{-\det \bar{G}^{(NS)}} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \quad \text{where } C_2^2 = (8\pi^3 g_s) \alpha'^{3/2}. \quad (2)$$

The significance of the constant NS field in addition to the local modes of the KR field have been addressed in an effective curvature formalism on a D_4 -brane [30,31,7,8,32]. A priori, a gauge theoretic torsion H_3 has been identified with a connection, which in turn modifies the covariant derivative to \mathcal{D}_μ on a D_4 -brane. The mathematical construction implies an absorption of KR field quanta and hence the \mathcal{D}_μ describes a “fat” brane or a string-brane model. It has been argued to describe a vacuum created pair of gravitational ($3\bar{3}$)-brane underlying a geometric torsion curvature in the string-brane model. The gauge connections are appropriately coupled to the NS field and has been shown to define a geometric torsion \mathcal{H}_3 . Generically the geometric torsion for an arbitrary two form takes an usual form:

$$\mathcal{H}_{\mu\nu\lambda} = \mathcal{D}_\mu B_{\nu\lambda} + \text{cyclic in } (\mu, \nu, \lambda), \quad (3)$$

$$\text{where } \mathcal{D}_\lambda B_{\mu\nu} = \nabla_\lambda B_{\mu\nu} + \frac{1}{2} H_{\lambda\mu}^\rho B_{\rho\nu} - \frac{1}{2} H_{\lambda\nu}^\rho B_{\rho\mu} \quad (4)$$

An iterative incorporation of the constant NS field correction in the covariant derivative leads to an exact derivative in a perturbative gauge theory. It may seem to define a non-perturbative covariant derivative in a second order formalism underlying a geometric realization. Then, the geometric torsion may be given by

$$\begin{aligned} \mathcal{H}_{\mu\nu\lambda} &= \nabla_\lambda B_{\mu\nu} + \frac{1}{2} \mathcal{H}_{\lambda\mu}^\rho B_{\rho\nu} - \frac{1}{2} \mathcal{H}_{\lambda\nu}^\rho B_{\rho\mu} + \text{cyclic in } (\mu, \nu, \lambda) \\ &= H_{\mu\nu\lambda} + 3 \mathcal{H}_{[\mu\nu}^\alpha B_{\lambda]}^\beta g_{\alpha\beta} \\ &= H_{\mu\nu\lambda} + (H_{\mu\nu\alpha} B_\lambda^\alpha + \text{cyclic in } \mu, \nu, \lambda) + H_{\mu\nu\beta} B_\alpha^\beta B_\lambda^\alpha + \dots \dots \dots (5) \end{aligned}$$

B. NS FIELD DYNAMICS AND “FAT” BRANE:

Since all the local degrees in KR field have been transformed to the constant NS field, i.e. $\nabla_\mu B_{\mu\nu}^{(NS)} = 0$, the geometric torsion should be viewed via a dynamical NS field. As a result, the brane setup in presence of an open string background metric may be identified as a string-brane model. Then, the geometric torsion in the string-brane model may precisely be defined as:

$$\mathcal{H}_{\mu\nu\lambda} \rightarrow \frac{1}{2} \mathcal{H}_{\lambda\mu}^\rho B_{\rho\nu} - \frac{1}{2} \mathcal{H}_{\lambda\nu}^\rho B_{\rho\mu} + \text{cyclic in } (\mu, \nu, \lambda)$$

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$$= 3 \mathcal{H}[\alpha_{\mu\nu} B_{\lambda}^{(NS)\beta}] g_{\alpha\beta}$$

$$= (H_{\mu\nu\alpha} B_{\lambda}^{(NS)\alpha} + \text{cyclic in } \mu, \nu, \lambda) + H_{\mu\nu\beta} B_{\alpha}^{(NS)\beta} B_{\lambda}^{(NS)\alpha} + \dots \dots (6)$$

Importantly the geometric torsion takes into account the dynamics of NS field at the expense of the KR field dynamics on a D₄-brane. Furthermore an arbitrary NS field in the string bulk is known to source a dynamical NS field in target space leading to a torsion in the string effective action [1,2].

Intuitively the closed string modes are pulled back into the D4-brane and break the supersymmetry. The resultant configuration defines a non-BPS “fat” brane. Interestingly a “fat” brane in the setup has been viewed as an emergent gravitational pair of (3,3̄)-brane by the KR field quanta at the event of a background string black hole [7,8]. The absorption of photon in Schwinger mechanism may be replaced with a higher form (p + 1) quanta to describe a vacuum created pair of, p-brane and anti p-brane (p̄-brane) across an event horizon. In a classical limit, the torsion or D-brane correction becomes insignificant and the non-perturbative gravity reduces to Einstein vacuum. In other words, the quantum effect is indeed a D-brane correction underlying a geometric torsion in a second order formalism [7]. Our analysis has presumably ensured a non-perturbative quantum theory sourced by a D-instanton correction to the Einstein vacuum [7,20,21,28,29]. On the other hand the perturbation gauge theory in the setup re-assures a constant NS field coupling to a gauge theoretic (KR field) torsion H₃ and formally identifies with a geometric torsion \mathcal{H}_3 obtained in eq(6). It shows that the constant NS field serves as a perturbation parameter in the modified gauge theory a priori defined with \mathcal{H} . The iterative corrections, i.e. $H_3 \rightarrow \mathcal{H}_3$, ensure an exact gauge theory. Alternately, an exact gauge theory in presence of a background metric may equivalently be viewed as a non-perturbative quantum theory of gravity underlying a geometric torsion \mathcal{H}_3 dynamics in a second order formalism [7]. The five dimensional geometric torsion dynamics has been shown to generate an irreducible curvature scalar K on S^1 underlying a gravitational pair of (3,3̄)-brane. It was shown the low energy string vacuum receives a non-perturbative correction due to a D-brane which in turn was shown to describe an emergent non-perturbative quantum theory of gravity. Formally a constant NS field perturbative parameter couples with a torsion in the gauge theory appears to source a non-perturbative theory of gravity in a second order formalism underlying a geometric torsion. Thus \mathcal{H}_3 may be identified with a dynamical parameter in our construction for the non-perturbative quantum gravity.

C. SPACE-TIME CURVATURE ON A “FAT” BRANE:

The first order KR field dynamics on a D4-brane in presence of a background open string metric may lead to an emergent geometric torsion dynamics in (3+1)-dimensions in a second order formalism [7]. A priori, the emergent torsion curvature was shown to be governed by a fourth order generalized tensor underlying a vacuum created gravitational pair of (3,3̄)-brane [8]. The non-trivial curvature was shown to emerge from the commutator $[D_{\mu}, D_{\nu}] \mathcal{A}_{\lambda} = \mathcal{K}_{\mu\nu\lambda}{}^{\rho} \mathcal{A}_{\rho}$. The reducible curvature takes a form:

$$4\mathcal{K}_{\mu\nu\lambda}{}^{\rho} = 2\partial_{\mu} \mathcal{H}_{\nu\lambda}^{\rho} - 2\partial_{\nu} \mathcal{H}_{\mu\lambda}^{\rho} + \mathcal{H}_{\mu\lambda}^{\sigma} \mathcal{H}_{\sigma\nu}^{\rho} - \mathcal{H}_{\nu\lambda}^{\sigma} \mathcal{H}_{\sigma\mu}^{\rho} \quad (7)$$

The generalized tensor is antisymmetric under an exchange of indices within a pair and is not symmetric under an exchange of its first pair of indices with the second. It differs from the Riemannian tensor $R_{\mu\nu\lambda\rho}$. However for a non-propagating torsion $\mathcal{K}_{\mu\nu\lambda\rho} \rightarrow R_{\mu\nu\lambda\rho}$. The second order curvature tensor and the irreducible curvature scalar are worked out to yield:

$$4\mathcal{K}_{\mu\nu} = -(2\partial_{\lambda} \mathcal{H}_{\mu\nu}^{\lambda} + \mathcal{H}_{\mu\rho}^{\lambda} \mathcal{H}_{\lambda\nu}^{\rho})$$

and
$$\mathcal{K} = -\frac{1}{4} \mathcal{H}_{\mu\nu\lambda} \mathcal{H}^{\mu\nu\lambda} \quad (8)$$

As a result, the effective torsion curvature constructed in a string-brane set-up describes a non-perturbative curvature tensor. In a second order formalism \mathcal{H}_3 , an effective dynamics underlying a vacuum created pair of (3,3̄)-brane may be approximated by

$$S \rightarrow -\frac{1}{3C_2^2} \int d^5x \sqrt{-\bar{g}} \mathcal{K} \quad (9)$$

A priori, the underlying U(1) gauge invariance of the curvature scalar incorporates a condition:

$$C(2\pi\alpha') H_{\alpha\beta}^{\mu} \mathcal{H}^{\alpha\beta\nu} = f_q^{\mu\nu} \quad (10)$$

The infinitesimal nature of perturbation parameter allows one to re-express the symmetric fluctuations

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$$C(2\pi\alpha')H_{\alpha\beta}^{\mu}\bar{\mathcal{H}}^{\alpha\beta\nu} = f_q^{\mu\nu} \quad (11)$$

The symmetricity under an interchange $\mu \leftrightarrow \nu$ ensures that the naive condition on a two form becomes insignificant. Interestingly, the symmetricity conditions incorporate quantum corrections to the openstring (background) metric (12) and make it dynamical in an emergent scenario. Thus, the emerging notion of space-time is intrinsically defined with the curvature scalar \mathcal{K} on a “fat” brane. The emergent gravity scenario has been shown to describe a gravitational pair of $(3\bar{3})$ -brane across an event horizon of background open string black hole [7, 8]. Formally the phenomenon, leading to an emergent pair of brane and anti-brane, generalizes the Schwinger pair production mechanism by a higher form quantain a gauge theory at the event horizon of a background black hole. Thus it ensures a non-perturbative quantum phenomenon in the string-brane setup. In other words, a propagating geometric torsion is responsible for the emergent metric fluctuation $f_q^{\mu\nu}$, which in turn is sourced by a non-perturbative D_4 -brane world-volume KR field dynamics. In fact, the emergent pair of gravitational $(3\bar{3})$ -brane in a low energy limit may be approximated by a pair of BPS D_3 -brane and anti BPS \bar{D}_3 -brane across an event horizon. In principle, the RR charge of a D_3 -brane cancels that of \bar{D}_3 -brane in absence of the extra fifth dimension transverse to their world-volumes. However the cancellation of RR charge by an anti RR charge is ruled out by the presence of an horizon in between them. Generically the dynamical effects, of a geometric torsion, appear to modify the Riemannian curvature tensor $R_{\mu\nu\lambda\rho}$ to an effective curvature tensor $\mathcal{K}_{\mu\nu\lambda\rho}$.

In a gauge choice for a non-propagating torsion, the effective $\mathcal{K}_{\mu\nu\lambda\rho}$ reduces to $R_{\mu\nu\lambda\rho}$. In the case the emergent scenario on a brane universe is described with a large extra dimension. Then a D-brane or an anti D-brane correction, sourced by a geometric torsion, may be ignored to describe Einstein vacuum in the string-brane model. Furthermore various brane geometries obtained in the model ensure that a geometric torsion contribution to the emergent metric components are generically higher order in $(1/r)$ and hence they play a significant role for small underlying a microscopic brane universe [7,8,9,20,21]. It provokes thought to believe that the metric fluctuations (11) signify a quantum correction to a (perturbative) low energy closed string vacuum.

Furthermore, the generalized curvature dynamics on S^1 may be analyzed to describe a vacuum created gravitational pair of $(3\bar{3})$ -brane in the formalism, Intuitively, the gravitational brane and anti-brane

$$S_{3-3}^{\text{eff}} = \frac{1}{3} \int d^4x \sqrt{-\bar{g}} \left(K - \frac{3}{4} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} \right) \quad (12)$$

where $\mathcal{F}_{\mu\nu} = \mathcal{D}_\mu A_\nu - \mathcal{D}_\nu A_\mu$,

$$= F_{\mu\nu} + \mathcal{H}_{\mu\nu}^\alpha A_\alpha \quad (13)$$

The four dimensional curvature scalar K takes an analogous form to that of the five dimensional curvature (8). The presence of a geometric torsion in the four dimensional field strength $\bar{\mathcal{F}}_{\mu\nu} = (2\pi\alpha')F_{\mu\nu}$, ensures its geometric nature. The curvature scalar K is essentially sourced by the KR field in a first order world-volume theory. Its field strength is appropriately modified $H_3 \rightarrow \mathcal{H}_3$ to describe a propagating torsion in four dimensions underlying a second order effective curvature theory. The fact that a torsion is dual to an axion on a gravitational 3-brane, ensures one degree of freedom. In addition $\bar{\mathcal{F}}_{\mu\nu}$ describes a geometric one form field with two local degrees on a gravitational 3-brane. A precise match among the (three) local degrees of torsion in \mathcal{K} on S^1 with that in K and $\bar{\mathcal{F}}_{\mu\nu}$ reassures the absence of a dynamical dilaton field in the frame-work. The result is consistent with the fact that a two form on S^1 does not generate a dilaton field. Alternately the emergent metric tensor $\bar{G}_{\mu\nu}$ is not formally affected by a compactification on S^1 . The background metric $g_{\mu\nu}$, being non-dynamical on a D_4 -brane, can not generate a dynamical scalar field or dilaton when compactified on S^1 in the string-brane model.

III. DEGENERATE VACUA

A. GAUGE CHOICE LEADING TO TORSION GEOMETRIES IN 4D:

A geometric torsion, underlying a $U(1)$ gauge theory, on a D_4 -brane may alternately be viewed via a vacuum created gravitational pair of $(3\bar{3})$ -brane. Within a pair, the world volumes are distinctly separated by an extra fifth dimension. Thus an irreducible scalar curvature \mathcal{K} on S^1 generically reduces to K on a gravitational 3-brane and a geometric \mathcal{F}_2 on an anti 3-brane or vice-versa. An ansatz for the gauge fields on a 3-brane is worked out to yield:

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$$B_{t\phi} = -P \cos\theta \text{ and } A_t = \frac{1}{\sqrt{2\pi\alpha'}} \left(Q_1 \cos \left[\frac{P\sqrt{2\pi\alpha'}}{r} \right] - Q_2 \sin \left[\frac{P\sqrt{2\pi\alpha'}}{r} \right] \right), \quad (14)$$

where P and (Q1, Q2) signify the non-linear charges respectively under the NS field and the gaugefield on a vacuum created pair of (3 $\bar{3}$)-brane. The gauge choice implies a non-interacting open string background metric, i.e. $G_{\mu\nu}^{NS} \rightarrow g_{\mu\nu}$. However the gauge ansatz ensures a geometric torsion in the string-brane model. It is given by

$$\mathcal{H}_{t\theta\phi} \rightarrow H_{t\theta\phi} = \frac{-P}{\sqrt{2\pi\alpha'}} \sin\theta \quad (15)$$

The dynamical torsion with a constant background metric $g_{\mu\nu}$ yields:

$$H^{t\theta\phi} = \frac{(2\pi\alpha')^{3/2}}{r^4 \sin\theta} \quad (16)$$

In the gauge choice (14), the KR field satisfies the equations:

$$\partial_\lambda H^{\lambda\mu\nu} - \frac{1}{2} (g^{\alpha\beta} \partial_\lambda g_{\alpha\beta} H^{\lambda\mu\nu}) = 0 \quad (17)$$

A torsion incorporates a new (tensor) connection and modifies the covariant derivative: $\nabla_\mu \rightarrow \mathcal{D}_\mu$. A torsion connection has been worked out in a second order formalism by the authors [7]. A torsion redefines a non-linear gauge theoretic curvature $F_{\mu\nu}$ which may be interpreted as geometric one in the formalism (13). The \mathcal{A}_μ on a 3-brane satisfies: $\mathcal{D}_\mu \mathcal{F}_{\mu\nu} = 0$. Explicitly, the field equations are:

$$\partial_\mu \mathcal{F}^{\mu\nu} + \frac{1}{2} (g^{\alpha\beta} \partial_\mu g_{\alpha\beta}) \mathcal{F}^{\mu\nu} - \frac{1}{2} H_{\alpha\beta}^\nu \mathcal{F}^{\mu\alpha} = 0 \quad (18)$$

The non-zero components of the geometric F2 curvature become

$$\mathcal{F}_{tr} = -\frac{P}{r^2} \left(Q_1 \sin \left[\frac{P\sqrt{2\pi\alpha'}}{r} \right] + Q_2 \cos \left[\frac{P\sqrt{2\pi\alpha'}}{r} \right] \right)$$

$$\text{and } \mathcal{F}_{\theta\phi} = -\frac{P}{2\pi\alpha'} \left(Q_1 \cos \left[\frac{P\sqrt{2\pi\alpha'}}{r} \right] - Q_2 \sin \left[\frac{P\sqrt{2\pi\alpha'}}{r} \right] \right) \sin\theta \quad (19)$$

A non-linear magnetic charge, underlying the non-trivial field component $\mathcal{F}_{\theta\phi}$ in absence of a magnetic monopole, is remarkable. The new phenomenon may rule out a theoretical notion of a magnetic point charge. It ensures the significant role played by a torsion in the framework. In fact the observation is in agreement with the Kerr-Newman black hole where an angular momentum is seen to mix an electric field component with a magnetic field [21]. An emergent quantum geometry on a vacuum created 3-brane may primarily be influenced by a geometric torsion \mathcal{H}_3 and a gauge theoretic twoform F2. A priori two independent sources for a nontrivial curvature on a pair of (3 $\bar{3}$)-brane may seem to possess their origin in an effective torsion curvature scalar \mathcal{K} on a D4-brane [7]. The open string metric $G_{\mu\nu}^{NS}$ receives further corrections due to a geometric torsion in five dimensional brane world. The generic curvature scalar \mathcal{K} on S^1 describes \mathcal{H}_3 in eq(15) and \mathcal{F}_2 in eq(19) on a 3-brane. Then, the emergent metric on a gravitational 3-brane [7,8,9,20,21] is given by

$$G_{\mu\nu} = (G_{\mu\nu}^{NS} + C_1 F_{\mu\alpha} g^{\alpha\beta} F_{\beta\nu} + C_2 H_{\mu\alpha\beta} g^{\alpha\delta} g^{\beta\sigma} H_{\delta\sigma\nu}) \quad (20)$$

where (C1, C2) are arbitrary constants. We set (C1 = ± 1 , C2 = $\pm 1/2$) and (2 $\pi\alpha'$) = 1 for simplicity.

B. AXIONIC SOURCED BRANE UNIVERSE:

The non-perturbative quantum geometries, a priori on a vacuum created pair of gravitational (3 $\bar{3}$)-brane are worked out using the metric (20). The geometric patches are given by

$$G_{tt} = -\left(1 \mp \frac{P^2}{r^4} \pm \frac{P^2}{r^4} \left(Q_1 \sin \frac{P}{r} + Q_2 \cos \frac{P}{r} \right)^2 \right), \quad G_{rr} = \left(1 \pm \frac{P^2}{r^4} \left(Q_1 \sin \frac{P}{r} + Q_2 \cos \frac{P}{r} \right)^2 \right)$$

$$G_{\theta\theta} = r^2 \left(1 \mp \frac{P^2}{r^4} \mp \frac{P^2}{r^4} \left(Q_1 \cos \frac{P}{r} - Q_2 \sin \frac{P}{r} \right)^2 \right), \quad G_{\phi\phi} = G_{\theta\theta} \sin^2\theta \quad (21)$$

The emergent patches may be re-expressed with an interchange between $dr \leftrightarrow r dt$. Under an interchange, the conserved charges rename appropriately and geometry may be given by

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$$ds^2 = - \left(1 \mp \frac{P^2}{r^4} \pm \frac{P^2}{r^4} \left(Q_1 \sin \frac{P}{r} + Q_2 \cos \frac{P}{r} \right)^2 \right) dt^2 + \left(1 \pm \frac{P^2}{r^4} \left(Q_1 \sin \frac{P}{r} + Q_2 \cos \frac{P}{r} \right)^2 \right) dr^2 + \left(1 \mp \frac{P^2}{r^4} \mp \frac{P^2}{r^4} \left(Q_1 \cos \frac{P}{r} - Q_2 \sin \frac{P}{r} \right)^2 \right) r^2 d\theta^2 + \left(1 \mp \frac{P^2}{r^4} \mp \frac{P^2}{r^4} \left(Q_1 \cos \frac{P}{r} - Q_2 \sin \frac{P}{r} \right)^2 \right) r^2 \sin^2 \theta d\phi^2 \quad (22)$$

For simplicity, we consider $Q_1 = Q_2 = Q$ in the paper. A priori, the quantum geometries on a gravitational 3-brane reduce to yield:

$$ds^2 = - \left(1 \mp \frac{P^2}{r^4} \pm \frac{P^2 Q^2}{r^4} \left(1 + \sin \frac{2P}{r} \right) \right) dt^2 + \left(1 \mp \frac{P^2}{r^4} \mp \frac{P^2 Q^2}{r^4} \left(1 - \sin \frac{2P}{r} \right) \right) dr^2 + \left(1 \pm \frac{P^2 Q^2}{r^4} \left(1 + \sin \frac{2P}{r} \right) \right) r^2 d\theta^2 + \left(1 \mp \frac{P^2}{r^4} \mp \frac{P^2 Q^2}{r^4} \left(1 - \sin \frac{2P}{r} \right) \right) r^2 \sin^2 \theta d\phi^2 \quad (23)$$

A global scenario is defined with a vacuum created gravitational pair of $(3\bar{3})$ -brane from a BPS D_4 -brane underlying a two form $U(1)$ gauge theory. The momentum conservation of a pair at its creation may intuitively imply a formal swing $r \rightarrow -r$ between the brane geometries and that on an anti-brane. It is interesting to note that a repulsive gravity sourced by a Newtonian like potential would like to drive away a brane universe away from an anti-brane. The interpretation provokes thought to believe the role of a quintessence in the string-brane model. The brane geometries are approximated in a brane window: $r > |P|$ and $r > |Q|$ with $r^8 \gg (P^4, Q^4)$. Then the quantum geometries, sourced by a geometrical torsion on a vacuum created pair of $(3\bar{3})$ -brane, become

$$ds^2 = - \left(1 \mp \frac{P^2}{r^4} \pm \frac{P^2 Q^2}{r^4} \left(1 + \sin \frac{2P}{r} \right) \right) dt^2 + \left(1 \pm \frac{P^2}{r^4} \pm \frac{P^2 Q^2}{r^4} \left(1 - \sin \frac{2P}{r} \right) \right) dr^2 + \left(1 \pm \frac{P^2 Q^2}{r^4} \left(1 + \sin \frac{2P}{r} \right) \right) r^2 d\theta^2 + \left(1 \mp \frac{P^2}{r^4} \mp \frac{P^2 Q^2}{r^4} \left(1 - \sin \frac{2P}{r} \right) \right) r^2 \sin^2 \theta d\phi^2 \quad (24)$$

It is remarkable to note that the quantum geometries in a low energy limit, i.e. for a non-propagating torsion, reduces to Minkowski space-time. This fact makes the quantum geometries special as Einstein vacuum cannot be realized from the quantum vacua. It is indeed a property of the free string theory and hence corresponds to an exact theory of quantum gravity underlying a non-interacting graviton. Analysis reveals that the vacuum created pair of $(3\bar{3})$ -brane, underlying the quantum vacua (24), are non-gravitational in nature! Explicitly, the emergent quantum geometries reduce to yield:

$$ds_1^2 = - \left(1 + \frac{M_Q + M_T}{r^4} \right) dt^2 + \left(1 + \frac{M_Q - M_T}{r^4} \right)^{-1} dr^2 + \left(1 + \frac{M_Q}{r^4} \right) r^2 d\theta^2 + \left(1 + \frac{M_Q + M_T}{r^4} \right) r^2 \sin^2 \theta d\phi^2 \quad (25)$$

$$= - \left(1 - \frac{M_Q - M_T}{r^4} \right) dt^2 + \left(1 - \frac{M_Q + M_T}{r^4} \right)^{-1} dr^2 + \left(1 - \frac{M_Q}{r^4} \right) r^2 d\theta^2 + \left(1 + \frac{M_Q + M_T}{r^4} \right) r^2 \sin^2 \theta d\phi^2 \quad (26)$$

$$ds_3^2 = - \left(1 + \frac{M_Q - M_T}{r^4} \right) dt^2 + \left(1 + \frac{M_Q + M_T}{r^4} \right)^{-1} dr^2 + \left(1 + \frac{M_Q}{r^4} \right) r^2 d\theta^2 + \left(1 - \frac{M_Q + M_T}{r^4} \right) r^2 \sin^2 \theta d\phi^2 \quad (27)$$

$$ds_4^2 = - \left(1 - \frac{M_Q + M_T}{r^4} \right) dt^2 + \left(1 - \frac{M_Q - M_T}{r^4} \right)^{-1} dr^2 + \left(1 - \frac{M_Q}{r^4} \right) r^2 d\theta^2 + \left(1 + \frac{M_Q - M_T}{r^4} \right) r^2 \sin^2 \theta d\phi^2 \quad (28)$$

where $M_Q = (2\pi\alpha')^{-1/2} (PQ)^2$ and $M_T = (2\pi\alpha')^{-1/2} P^2$ may define the microscopic black hole mass on a vacuum created pair of $(3\bar{3})$ -brane. The emergent geometries are asymptotically flat. They are characterized by two independent conserved charges (P, Q) . Firstly a torsion charge P re-assures the significance of string charge to the quantum geometries in the formalism. Secondly a non-linear electric charge Q may seem to incorporate a degeneracy in the quantum geometries (25) and (27) defined with $Q = 0$. Needless to mention that two independent non-trivial universes began on a pair of $(3\bar{3})$ -brane at an event horizon by the KR flux in the $U(1)$ gauge theory on a D_4 -brane. Subsequently, a lower form, i.e. an one form flux, in disguise of a non-linear axion on a vacuum created 3-brane within a pair creates a degenerate brane universes [21]. In principle, a higher form, dual to the gauge field, on an appropriate

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Dp-brane for $p > 3$ may possess a potential strength to vacuum create a large degeneracy and hence may be described by various clones of a semi-classical brane universe.

Three torsion geometries, among the four (25)-(28), are characterized by an event horizon and hence they describe various brane universes in the quantum regime. We shall see that the emergent geometry (27) does not describe a brane universe as the curvature singularity described in a fundamental theory is not protected by an event horizon. Hence the brane universe is not allowed by a cosmic censorship hypothesis. The Ricci scalar R blows up at various values in r in the vacuum created brane universe(s).

In addition to a curvature singularity at $r \rightarrow 0$ in all four geometries, they are also described by $R \rightarrow \infty$ when: (i) $r^2 \rightarrow P\sqrt{1+Q^2}$ in eq(25), (ii) $r^2 \rightarrow P\sqrt{Q^2-1}$ and $r^2 \rightarrow (PQ)$ in eq (26), (iii) $r^2 \rightarrow P\sqrt{1-Q^2}$ in eq (27) and (iv) $r^2 \rightarrow (PQ)$ and $r^2 \rightarrow P\sqrt{1-Q^2}$ in eq (28). The lower bound on $r > (P, Q)$ rule out the singularity at $r \rightarrow 0$ in a brane universe. In absence of an upper bound on Q , i.e. $Q^2 > 2$, all the created brane universes (26)-(28) appear to possess a single curvature scalar R singularity at $r^2 \rightarrow (PQ)$. Presumably they are described in an effective type IIA or IIB superstring theory on S^1 underlying a $D4$ -brane. Nevertheless the quantum black holes underlying the brane universes are described by an effective torsion curvature scalar:

$$\mathcal{K} = \frac{3M_T}{2r^4} \quad (29)$$

It shows that a brane universe does not access to a curvature singularity at $r \rightarrow 0$ which is otherwise present in a fundamental theory of gravity. The curvature underlying a varying energy density has been argued to be sourced by a quintessence axion [28] or an anti quintessence axion [29] in the string-brane model. A varying positive energy density in the quantum regime presumably ensures an underlying de Sitter at the Big Bang.

IV. NON-PERTURBATIVE CORRECTION

A. QUANTUM BLACK HOLE:

The vacuum created pair of gravitational brane universes may be approximated by a Schwarzschild black hole in presence of a correction underlying a flat metric. Formally, the \mathcal{F}_2 contribution may be separated out from the \mathcal{H}_3 . The emergent geometries (26) and (28) may be given by

$$ds^2 = -\left(1 - \frac{M_Q}{r^4}\right) dt^2 + \left(1 - \frac{M_Q}{r^4}\right)^{-1} dr^2 + \left(1 - \frac{M_Q}{r^4}\right) r^2 d\theta^2 + \left(1 - \frac{M_Q}{r^4}\right) r^2 \sin^2\theta d\phi^2$$

$$\mp \frac{M_T}{r^4} (-dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \quad (30)$$

The second term with a conformal factor $\frac{M_T}{r^4}$ couples to a trivial metric and may formally be identified with a membrane world-volume in the formalism. The RR charge with a D-membrane ensures a non-perturbative nature [6] underlying the quantum correction to a perturbative (low energy) string vacuum. Thus the lower dimensional D-brane correction term incorporates quantum effects non-perturbatively into the otherwise semi-classical vacuum. Interestingly the D-membrane line-element, sourced by a non-zero torsion charge M_T , ensures no curvature and hence confirms a flat space. Thus a formal decomposition of the on-shell metric in eq(30), into nontrivial and trivial matrices, does not violate the on-shell conditions for the splitted non-linear and linear pieces. It may also be understood from the emergent nature of geometries in eqs(26) and (28), which are being sourced by the on-shell gauge fields (17) and (18). On-shell metric is a requirement of low energy limit in the string-brane model and is always satisfied.

In a low energy limit, the non-perturbative correction decouples to yield a semi-classical Schwarzschild geometry on a vacuum pair of $(3\bar{3})$ -brane. The reduced black hole is characterized by an event horizon at $r \rightarrow r_e = (M_Q)^{1/4} = \sqrt{PQ}$. An event horizon, described by a non-linear $U(1)$ charge in a gauge theory, is known to describe an effective black hole on a 3-brane. It implies that the event horizon is essentially described by a non-linear conserved charge (PQ) underlying a geometric torsion in the string-brane model.

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Formally the charge M_Q is identified with a mass of a 4D black hole obtained on a brane universe. The mass appears to break the spherical symmetry in the brane universe. The deformation geometry may also be realized intuitively with the help of an extra fifth hidden dimension transverse to a D_3 -brane or an anti D_3 -brane. A large extra dimension brings in some of the low energy closed string modes into a vacuum created 3-brane or an anti 3-brane. Thus a geometric torsion in the formalism arguably incorporates non-perturbative quantum effects, via D-branes, to the low energy string effective action.

The empirical formula underlying an effective gravitational potential, sourced by a geometric torsion, hints at three extra dimensions in the gauge choice. They may further be analyzed: (i) with a medium energy laboratory scale underlying a large radial coordinate: $r^2 = \rho > 0$ and (ii) in a low energy scale underlying a very large radial coordinate: $r^4 = R > 0$ for a classical black hole presumably in Einstein vacuum.

B. LABORATORY BLACK HOLE:

An effective microscopic black hole on a brane universe at a Planck scale may reduce to a laboratory black hole defined in a medium energy range. With a larger scale, i.e., $\rho = r^2 > 0$ an effective Schwarzschild may approach a Schwarzschild black hole in Einstein vacuum. With a rescaling, the brane universe in a medium energy regime may be approximated by

$$ds^2 = -4\rho \left(1 - \frac{M_Q}{\rho^4}\right) dt^2 + \left(1 - \frac{M_Q}{\rho^4}\right)^{-1} d\rho^2 + \left(1 - \frac{M_Q}{\rho^4}\right) \rho^2 d\theta^2 + \left(1 - \frac{M_Q}{\rho^4}\right) \rho^2 \sin^2\theta d\phi^2$$

$$\mp \frac{M_T}{\rho^4} (-dt^2 + d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2) \quad (31)$$

Some of the geometric patches may seem to decouple from the emergent geometry in the regime. Then the effective brane universe may be approximated by a quantum Schwarzschild black hole a priori without an effective D-string world-volume correction in the framework. It is given by

$$ds^2 = ds_I^2 + ds_{II}^2$$

where $ds_I^2 = -4\rho \left(1 - \frac{M_Q}{\rho^4}\right) dt^2 + \left(1 - \frac{M_Q}{\rho^4}\right)^{-1} d\rho^2 + 4\rho^2 d\Omega^2$

and $ds_{II}^2 = \mp \frac{M_T}{\rho^4} (-4\rho dt^2 + d\rho^2) \quad (32)$

The effective gravitational potential hints at a large extra dimension in a brane universe. It is in conformity with an underlying D4-brane world-volume. However the D-string correction ds_{II}^2 is primarily associated with a torsion or an axionic charge M on a pair of $(3,3)$ -brane. Unlike to a flat metric on a membrane in eq(30), the string world volume appears to possess a nontrivial Ricci curvature scalar $R_{II} = \frac{1}{2\rho^2}$.

Nevertheless, the curvature underlying a correction is evaluated at the event horizon of a semi-classical black hole to yield a positive constant vacuum energy density. The scalar \mathcal{R}_I for the line-element ds_I^2 may be checked to yield a positive constant energy density at an event horizon, i.e. $\rho \rightarrow \rho_e = P\sqrt{2}$. In a low energy limit the non-perturbative geometric correction becomes insignificant and the effective brane universe may describe a spherically symmetric 4D Schwarzschild black hole in presence of a large extra dimension.

C. SEMI-CLASSICAL BRANE UNIVERSE:

A primordial black hole underlying a brane universe may be obtained from the quantum geometries for a relatively large length scale. It may be approximated by a re-defined large radial coordinate with $r^4 = R > 0$. In the case, the quantum brane geometries may formally approach to a Schwarzschild black hole after decoupling an axionic charge. In the regime, the brane geometries may be approximated by

$$ds^2 = -16 R^{3/2} \left(1 - \frac{M_Q}{R}\right) dt^2 + \left(1 - \frac{M_Q}{R}\right)^{-1} dR^2 + 16 \left(1 - \frac{M_Q}{R}\right) R^2 d\theta^2$$

$$+ 16 \left(1 - \frac{M_Q}{R}\right) R^2 \sin^2\theta d\phi^2 \mp \frac{M_T}{R} (-16 R^{3/2} dt^2 + dR^2 + R^2 d\theta^2 + 16 R^2 \sin^2\theta d\phi^2). \quad (33)$$

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A non-perturbative correction underlying an axionic charge M_T , with a very large R scale, may be used to describe a varying negative curvature scalar $\mathcal{R} = -\frac{9}{8R^2}$ on a vacuum created gravitational pair of $(3\bar{3})$ -brane. The correction in energy density is computed at the Schwarzschild event horizon $R \rightarrow R_e = MQ$ to confirm a small negative energy vacuum density. Then the energy density in a Schwarzschild like black hole is worked out at the horizon to confirm a positive vacuum energy density. The Ricci scalar blows up at the event horizon R_e . Then the semi-classical geometry in a brane universe is approximated to yield

$$ds^2 = -16 R^{3/2} \left(1 - \frac{M_Q}{R}\right) dt^2 + \left(1 - \frac{M_Q}{R}\right)^{-1} dR^2 + 16 \left(1 - \frac{M_Q}{R}\right) R^2 d\theta^2 + 16 \left(1 - \frac{M_Q}{R}\right) R^2 \sin^2\theta d\phi^2. \quad (34)$$

It implies that the spherical symmetry is broken in a semi-classical black hole which is described by a torsion-less vacuum in the formalism. Unlike to a Schwarzschild black hole in Einstein vacuum, the Schwarzschild brane universe is not Ricci flat. A computation of Ricci curvature scalar \mathcal{R} in this semi-classical brane universe ensures naive singularities at $R \rightarrow 0$ and $R \rightarrow M$. However they are not accessed by the brane universe as the large radial coordinate is bounded below by $(P, Q) < R$. Importantly, the curvature scalar turns out to be negative on a brane window. The varying curvature presumably ensures an underlying quintessence AdS black hole [29] in the string-brane model. At an event horizon, the vacuum energy density takes a negative constant value to re-assure an AdS brane geometry there. Thus in a low energy limit, a brane universe on a vacuum created gravitational pair of $(3\bar{3})$ -brane is presumably described by an asymptotic AdS in Einstein vacuum. In fact, an asymptotic analysis for the near horizon Schwarzschild brane universe in ref [7] is in agreement with the obtained result.

D. TORSION DECOUPLING REGIME:

A torsion sourced by a string charge decouples from the semi-classical brane universe in a low energy limit. It may as well be described by a large r where a propagating torsion freezes and leads to a condensate. It has been argued that the geometries, right after their vacuum creation at a cosmological horizon, undergo quantum tunneling and may be viewed through radiations [7]. Since an observer in a brane window is not accessible to a curvature singularity otherwise present in Einstein gravity, tunnelling aspects of a vacuum to another become significant in the quantum regime. It has been argued that an effective de Sitter universe cools down via a series of geometric transitions in the early epoch. Finally it leads to a semi-classical universe in $(3 + 1)$ dimensions. Thus, a low energy brane universe may well be approximated by the Ricci scalar and hence the curvature singularity becomes significant there. In fact, we shall see that the brane universes (25), (26) and (28), on a gravitational pair of vacuum created $(3\bar{3})$ -brane, tend to approach a single vacuum in Einstein gravity. Analysis reveals that two of the brane universes (25) and (27) may causally identify with the remaining two brane geometries (26) and (28) respectively across an event horizon where a light cone flips by a right angle.

In a low energy limit, the emergent quantum geometries dissociate a higher order term in $(1/r)$ defined via an effective gravitational potential purely sourced by a two form flux. The reduced semiclassical geometries in the limit may be approximated by the torsion-less effective brane universes. A priori the degenerate quantum vacua (25)-(28) reduce to yield:

$$ds^2 = -\left(1 + \frac{M_Q}{r^4}\right) dt^2 + \left(1 + \frac{M_Q}{r^4}\right)^{-1} dr^2 + \left(1 + \frac{M_Q}{r^4}\right) r^2 d\theta^2 + \left(1 - \frac{M_Q}{r^4}\right) r^2 \sin^2\theta d\phi^2 \quad (35)$$

$$ds^2 = -\left(1 - \frac{M_Q}{r^4}\right) dt^2 + \left(1 - \frac{M_Q}{r^4}\right)^{-1} dr^2 + \left(1 - \frac{M_Q}{r^4}\right) r^2 d\theta^2 + \left(1 + \frac{M_Q}{r^4}\right) r^2 \sin^2\theta d\phi^2 \quad (36)$$

Under a flip of light cone across an event horizon, when considered with an interchange of angular coordinates, the brane universe (36) identifies with the other brane geometry (35). Formally they may be identified under an interchange: $M_Q \leftrightarrow -M_Q$.

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V. QUINTESSENCE

A quintessence axion, presumably on a vacuum created $\bar{3}$ -brane, is argued to source an extra fifth dimension to our brane universe on a 3-brane with a vacuum pair. The curvature scalar K on a gravitational anti 3-brane may describe a quintessence axion and a geometric \mathcal{F}_2 governs the torsion curvature on a gravitational 3-brane. An accelerated and expanding universe in the present day cosmology may seem to be influenced by a growth in extra dimension between a brane and an anti-brane universes. The high energy torsion modes have been argued to decouple with a large fifth dimension and eventually a low energy brane universe may appear to decouple from the anti-brane in the limit. The decoupling of non-perturbative quantum effects may lead to describe Einstein vacuum in the formalism.

In this section we perform Weyl scaling(s) in the effective degenerate vacua (25)-(28) and analyze the de Sitter and AdS brane geometries in the framework to signal the presence of a quintessence. Generically a metric under a Weyl scaling in a low energy effective string action describes Einstein gravity without any change in its causal structure. However a Weyl scaling of the effective metric on a D-brane in the framework may seem to scale the vacuum with a higher or a lower energy density [7, 8, 28, 29]. This is due to a fact that the emergent geometries are defined on a brane window with a bound on its world-volume coordinate r .

A. de Sitter brane:

We obtain effective de Sitter geometries by performing a conformal Weyl transformation among the effective geometric patches approximated by the line-elements (25)-(28) on a gravitational pair ($3\bar{3}$)-brane. In particular, the obtained geometries are viewed within a brane window for the Weyl scaled metric. Hence they correspond to the limiting geometries in an emergent pair of brane and anti-brane scenario. It is worth mentioning that an emergent (metric) gravity does not require on-shell metric. The emergent notion of metric is indeed sourced by a dynamical higher form in an appropriate world-volume gauge theory. However in a low energy limit, the reduced semi-classical geometry should satisfy the on-shell metric condition.

A scale factor $\lambda = r^2$ is worked out to obtain a de Sitter brane vacua in this section. Under a scaling the effective brane universe may be approximated by

$$ds^2 = -\frac{1}{r^2} dt^2 + \left(r^2 - \frac{M_Q - M_T}{r^2} \right)^{-1} dr^2 + d\Omega^2 \quad (37)$$

$$ds^2 = -\frac{1}{r^2} dt^2 + \left(r^2 - \frac{M_Q + M_T}{r^2} \right)^{-1} dr^2 + d\Omega^2 \quad (38)$$

It may be recalled that a NS two form in a string world-sheet is known to incorporate a string charge which may be viewed as a torsion charge in an effective string theory. Thus, a string charge may be identified with an axionic charge on a 3-brane. The brane universe(s) in the quantum phase may imply a decoupling of S^2 -symmetric patch from the causal patches and hence may be viewed through a pair of lower dimensional brane [7]. The curvature scalars \mathcal{R}_\pm computed respectively for the black holes mass ($M_Q \pm M_T$). They are:

$$\mathcal{R}_\pm = \frac{6}{r^4} (M_Q \pm M_T). \quad (39)$$

A positive curvature scalar in the quantum regime ensures a de Sitter in presence of a non-perturbative correction underlying a D-brane in the ten dimensional type IIA and type IIB superstring theories on S^1 . At this point we recall that a superstring theory describes an AdS vacuum. A positive vacuum energy density, underlying a de Sitter vacuum, in the framework is sourced by a non-linear electric field associated with a D-brane correction to the AdS string vacuum. Most importantly a varying positive curvature may seem to be sourced by a quintessence axion underlying a repulsive gravity between a brane and an anti-brane. Nevertheless, the vacuum energy density at the event horizon signifies a small positive cosmological constant in Einstein gravity.

B. AdS brane:

We perform a conformal Weyl transformation within a brane window in the effective degenerate geometries obtained (25)-(28) on vacuum created gravitational pair of ($3\bar{3}$)-brane. A priori, the limiting brane geometries do not retain the Weyl invariance and hence causal properties can vary under an emergent metric scaling. Nevertheless the Weyl

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invariance is restored in the semi-classical geometries which are obtained by throwing away the torsion degrees completely in a low energy limit of the quantum patches on a gravitational pair of $(3\bar{3})$ -brane.

A scale factor $\lambda = 1/r^2$, predefined with $r^2 > (M_Q, M_T)$ is worked out to obtain an AdS branevacuum in this section. Under a scaling the effective brane universe may a priori be described by

$$ds^2 = -\left(r^2 + \frac{M_Q \pm M_T}{r^2}\right) dt^2 + \left(r^2 - \frac{M_Q \mp M_T}{r^2}\right)^{-1} r^4 dr^2 + r^4 d\Omega^2$$

$$\text{and } ds^2 = -\left(r^2 + \frac{M_Q \mp M_T}{r^2}\right) dt^2 + \left(r^2 - \frac{M_Q \pm M_T}{r^2}\right)^{-1} r^4 dr^2 + r^4 d\Omega^2, \quad (40)$$

where the brane geometries are approximated in a window for $r^4 > (M_T^2, M_Q^2)$. However the causal patches sourced by an axion appears to be associated with a relative wrong sign. The relative sign in G_{tt} and G_{rr} may be corrected via a matrix projection underlying a discrete transformation among the causal patches [7, 8]. The causal patches define a (2×2) matrix and it is given by

$$M = \frac{1}{2\sqrt{M_Q}} \begin{pmatrix} G_{tt}(+) & G_{rr}(+) \\ G_{rr}(-) & G_{tt}(-) \end{pmatrix}. \quad (41)$$

$$\text{Where } G_{tt}(+) = \left(r^2 + \frac{M_T}{r^2} \pm \frac{M_Q}{r^2}\right), G_{rr}(+) = \left(r^2 + \frac{M_T}{r^2} \pm \frac{M_Q}{r^2}\right)^{-1}, \\ G_{tt}(-) = \left(r^2 - \frac{M_T}{r^2} \pm \frac{M_Q}{r^2}\right), G_{rr}(-) = \left(r^2 - \frac{M_T}{r^2} \pm \frac{M_Q}{r^2}\right)^{-1}. \quad (42)$$

The causal patches in the quantum geometries (40) may be viewed via a projection of the matrix on the column vectors: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

The matrix determinant turns out to be independent of a lower form non-linear charge Q or M_Q and is given by: $\det M = -1$. It may imply a discrete transformation of the geometric causal patches in an effective quantum gravity on a brane universe. The inverse of the matrix becomes

$$M^{-1} = -\frac{1}{2\sqrt{M_Q}} \begin{pmatrix} G_{tt}(-) & -G_{rr}(+) \\ -G_{rr}(-) & G_{tt}(+) \end{pmatrix} \quad (43)$$

An inverse matrix projection on the column vectors may be checked to describe the quantum geometries with appropriate causal patches. In other words the inverse matrix projected geometries are obtained by a discrete transformation from the matrix projected ones in eqs(40). A discrete transformation may seen to reduce the degeneracy in the quantum vacua by fixing a $(-)$ - sign in the effective metric potential sourced by a charge MQ. Two, among the four emergent, geometries may seen to describe a brane universe. Under an analytic continuation to real time they may be given by

$$ds^2 = -\left(r^2 - \frac{M_Q - M_T}{r^2}\right) dt^2 + \left(r^2 - \frac{M_Q - M_T}{r^2}\right)^{-1} r^4 dr^2 + r^4 d\Omega^2 \\ ds^2 = -\left(r^2 - \frac{M_Q + M_T}{r^2}\right) dt^2 + \left(r^2 - \frac{M_Q + M_T}{r^2}\right)^{-1} r^4 dr^2 + r^4 d\Omega^2 \quad (44)$$

The curvature scalars \mathcal{R}_\pm respectively for the effective black holes with mass: $(M_Q \pm M_T)$ are computed to confirm an AdS brane with varying curvature. Explicitly they may be given by

$$\mathcal{R}_\pm = \frac{6}{r^4} \left(\frac{M_Q \pm M_T}{r^4} + 2 \right) \quad (45)$$

The varying curvature presumably imply an accelerating brane universe and hence hint for a quintessence axion. At an event horizon the curvature tends to a constant and precisely describes an AdS brane underlying a constant vacuum energy density. In the low energy limit a string charge on a \bar{D}_3 -brane may be drained off by a gauge theoretic charge. Thus for $M_Q = M_T$ one arrives at a single brane universe and the reduced geometry becomes

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$$ds^2 = -\left(r^2 - \frac{2M_Q}{r^2}\right) dt^2 + \left(r^2 - \frac{2M_Q}{r^2}\right)^{-1} r^4 dr^2 + r^4 d\Omega^2 \quad (46)$$

In the case the cancellation of a torsional effects by a string charge removes the degeneracy in the vacuum solutions. Then the brane universe is described with a torsion free geometry. It may be identified with a semi-classical vacuum in Einstein gravity.

C. DARK ENERGY:

The effective geometries in eq(40) for a large scale may further be analyzed for $r^2 = \rho$ in the string-brane model. The non-perturbative quantum effects apparently described by the four vacua are worked out with euclidean time. Explicitly, they are given by

$$ds^2 = \left(\rho + \frac{M_T + M_Q}{\rho}\right) dt_e^2 + \frac{1}{4} \left(1 - \frac{M_T - M_Q}{\rho}\right)^{-1} d\rho^2 + \rho^2 d\Omega^2 \quad (47)$$

$$ds^2 = \left(\rho + \frac{M_T - M_Q}{\rho}\right) dt_e^2 + \frac{1}{4} \left(1 - \frac{M_T + M_Q}{\rho}\right)^{-1} d\rho^2 + 4\rho^2 d\Omega^2 \quad (48)$$

$$ds^2 = \frac{1}{4} \left(1 - \frac{M_T + M_Q}{\rho^2}\right) dt_e^2 + \left(\rho + \frac{M_T - M_Q}{\rho}\right)^{-1} \rho^2 d\rho^2 + \rho^2 d\Omega^2 \quad (49)$$

$$ds^2 = \frac{1}{4} \left(1 - \frac{M_T - M_Q}{\rho^2}\right) dt_e^2 + \left(\rho + \frac{M_T + M_Q}{\rho}\right)^{-1} \rho^2 d\rho^2 + \rho^2 d\Omega^2 \quad (50)$$

The vacua (47)-(49) among the four geometric patches correspond to an AdS brane universe. Nevertheless, the emergent geometries (49) and (50) under an interchange $dt_e \leftrightarrow d\rho$ may seem to define a de Sitter brane universe. The mixed causal patches in the brane geometries (47)-(48) and in the remaining geometries (49)-(50) with $dt_e \leftrightarrow d\rho$ may be separated out using matrix M projection(s) (41) defined with appropriate normalization(s). The $\det M = -1$ at an event horizon of a primordial black hole ensures a discrete transformation underlying the quantum patches. Three independent matrix projections on the (2×1) column vectors are worked out between the geometries: (47)-(48), (48)-(49) and (47)-(49). The inverse matrix M^{-1} projections on the column vectors are checked to yield six redefined vacua. However two among the six vacua may seem to be defined with a correct relative sign in their effective potentials. With Lorentzian signature they are:

$$ds^2 = -\left(1 - \frac{M_Q - M_T}{\rho^2}\right) dt^2 + \left(1 - \frac{M_Q - M_T}{\rho^2}\right)^{-1} d\rho^2 + \rho d\Omega^2$$

$$ds^2 = -\left(1 - \frac{M_T + M_Q}{\rho^2}\right) dt^2 + \left(1 - \frac{M_T + M_Q}{\rho^2}\right)^{-1} d\rho^2 + 4\rho d\Omega^2 \quad (51)$$

The quantum vacua may further be re-expressed in term of the decoupled geometries. They are:

$$ds^2 = -\left(1 - \frac{\rho}{b} - \frac{M_Q - M_T}{\rho^2}\right) dt^2 + \left(1 - \frac{\rho}{b} - \frac{M_Q - M_T}{\rho^2}\right)^{-1} d\rho^2 - \frac{\rho}{b} (dt^2 + d\rho^2 - b d\Omega^2) \quad (52)$$

$$\text{and } ds^2 = -\left(1 - \frac{\rho}{b} - \frac{M_Q + M_T}{\rho^2}\right) dt^2 + \left(1 - \frac{\rho}{b} - \frac{M_Q + M_T}{\rho^2}\right)^{-1} d\rho^2 + 4\rho^2 (d\Omega^2) - \frac{\rho}{b} (dt^2 + d\rho^2) \quad (53)$$

where $b > 0$ is a constant. A decoupled causal patch from the S2-symmetric geometry in eq(52) may signify an early epoch in de Sitter [7]. The curvature scalars \mathcal{R}_- and \mathcal{R}_+ respectively for the black holes in eqs(52) and (53) are computed to yield:

$$\mathcal{R}_- = \frac{6}{b} \left(\frac{1}{b} - \frac{1}{4\rho}\right) \text{ and } \mathcal{R}_+ = \frac{6}{\rho^4} (M_Q - M_T) \quad (54)$$

The varying positive scalar curvatures underlying the semi-classical black holes presumably describe a quintessence de Sitter brane in the framework. At an event horizon they may seem to underlie a de Sitter in Einstein gravity. The non-perturbative brane corrections, underlying the quantum effects, to the black holes are checked for their curvature scalar. They ensure a positive curvature and the conformal factor further hints at a quintessence axion. Interestingly the effective black holes on a vacuum created gravitational pair of $(3\bar{3})$ -brane may formally be seen to be in agreement with a quintessence de Sitter black hole without a cosmological constant in five dimensions [26]. The state parameter is worked out to yield $w_q = -3/4$ for a quintessence de Sitter black hole on a brane (53) which is within the observed range $(-1 < w_q < -1/3)$ in cosmology.

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For a special case, i.e. $M_Q = M_T$, the semi-classical geometry (53) reduces to a Ricci flat and describes a Schwarzschild black. It implies that the energy density is not a constant ($\Lambda = 0$) in the formalism. Nevertheless a quintessence axion incorporates a varying vacuum energy density into the brane universe(s). The absence of a cosmological constant in the low energy vacua may well be understood from the absence of a constant two form in the ansatz (14). Thus a constant NS field incorporates a cosmological constant while a dynamical NS field appears to govern a varying vacuum energy density underlying a quintessence axion in the string-brane model. Since a D-instanton is sourced by a quintessence axionic scalar field in our model, it may provoke thought to believe that D-instanton is a potential candidate to the dark energy in our universe.

VI. CONCLUDING REMARKS

Primarily the research work has been shown to explore new quantum geometries in (3+1)-dimensions on a pair of $(3\bar{3})$ -brane underlying a type IIA or IIB superstring theory on S^1 . In particular, an effective torsion curvature in a second order formalism underlying a non-linear $U(1)$ gauge theory on a D_4 -brane was explored to investigate certain aspects of a quintessence axion in a vacuum created brane universe. It was argued that a quintessence dynamics underlie an effective torsion curvature on an anti 3-brane, while a 3-brane dynamics may be approximated by a DBI action. A five dimensional propagating torsion underlying an effective curvature scalar on a D_4 -brane was further supported by a pair production mechanism in a non-linear gauge theory. It was argued that a vacuum created gravitational pair of $(3\bar{3})$ -brane with nontrivial geometries may be sourced by the KR quanta on a D_4 -brane. Interestingly, a propagating geometric torsion was shown to generate a non-linear magnetic charge from an electric point charge. The phenomenon may rule out the possibility for a magnetic pole in the string-brane model.

An extra (or hidden) fifth dimension transverse to both the world-volumes has been shown to play a significant role in the string-brane model. A large extra dimension was argued to incorporate the low energy closed string vacuum between a gravitational pair of $(3\bar{3})$ -brane. In other words, the string-brane set-up underlying a geometric torsion may seem to describe a low energy perturbative closed string vacuum in presence of non-perturbative D-brane and anti D-brane corrections. In the context, a flat metric has been shown to be associated with the quantum corrections. They indeed reassure the absence of closed string modes on a D-brane.

The quantum geometries were shown to degenerate in presence of a string charge in the string-brane model. The emergent vacua are analyzed for various non-perturbative quantum corrections underlying a geometric torsion. It was shown that a lower dimensional D-brane incorporates quantum effects to the semi-classical vacuum. The quantum effects in an effective gravity were shown to be sourced by a quintessence axion, which may provide a clue to unfold the origin of dark energy in our universe. Arguably the quintessence possesses its origin in a D-instanton, which is also sourced by an axionic scalar in the string-brane model.

Furthermore the emergent brane/anti-brane geometries were Weyl scaled to obtain a de Sitter brane and an AdS brane in the regime. It was shown that the high energy geometric patches in the brane universe are sourced by a dynamical geometric torsion. In a moderately low energy limit, the torsion charge decouples from a brane universe and the vacuum may formally be identified with Einstein gravity. A Ricci curvature scalar underlying various brane universes further re-assured the vital role of a quintessence axion in the model. One of our results was shown to be in formal agreement with that in higher dimensional Einstein gravity described in presence of a quintessence scalar. It may suggest that our analysis underlying a torsion curvature may play a significant role to explain the accelerated expansion of our universe.

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REFERENCES

- [1] S. Candelas, G.T. Horowitz and A. Strominger, Nucl. Phys. B258 (1985) 46
- [2] D.S. Freed, Comm.Math.Phys. 107 (1986) 483
- [3] C.G. Callan, R.C. Meyers, M.J. Perry, Nucl. Phys. B311 (1988/89) 673
- [4] D. Garfinkle, G.T. Horowitz and A. Strominger, Phys.Rev. D43 (1991) 31403143
- [5] S.B. Giddings and A. Strominger, Phys.Rev.Lett.67 (1991) 2930
- [6] J. Polchinski, Phys.Rev.Lett.75 (1995) 4724
- [7] A.K. Singh, K.P. Pandey, S. Singh and S. Kar, JHEP 05 (2013) 033
- [8] A.K. Singh, K.P. Pandey, S. Singh and S. Kar, Phys.Rev. D88 (2013) 066001
- [9] A.K. Singh, K.P. Pandey, S. Singh and S. Kar, Nucl. Phys. B252-252 (2014) 241
- [10] M. Majumdar and A.-C. Davis, JHEP 03 (2002) 056
- [11] S. Kar, J.Astrophys.Aerosp.Technol. 03 (2013) e106
- [12] J. Schwinger, Phys.Rev. 82 (1951) 664
- [13] S.W. Hawking, Comm.Math.Phys. 43 (1975) 199
- [14] T.W.B. Kibble, Physics Report 67 (1980) 183
- [15] C.P. Burgess, Nucl. Phys. B294 (1987) 427
- [16] C. Bachas, M. Porrati, Phys. Lett. B296 (1992) 77
- [17] C. Bachas, hep-th/9303063, World Scientific (1993), Conference Proceeding C9209-21.3
- [18] M. Majumdar and A.-C. Davis, JHEP 0312 (2003) 012
- [19] C. Schubert and A. Torrielli, J. Phys A 43 (2010) 402003
- [20] S. Singh, K.P. Pandey, A.K. Singh and S. Kar, Nucl. Phys. B879 (2014) 216
- [21] S. Singh, K.P. Pandey, A.K. Singh and S. Kar, Int.J.Mod.Phys. A29 (2014) 1450164
- [22] R. Kapoor, S. Kar and D. Singh, Int.J.Mod.Phys. D24 (2015) 02, 1550015
- [23] D. Singh, R. Kapoor, S. Kar, Springer Proceedings Physics 174 (2016) 507
- [24] S. Chen and J. Jing, Class. & Quant. Grav. 22 (2005) 4651
- [25] S.B. Chen, B. Wang and R.K. Su, Phys.Rev. D77 (2008) 2401
- [26] S. Chen, Q. Pan and J. Jing, Class. & Quant. Grav.30 (2013) 145001
- [27] W.Y. Huan and R. Jun, Chin.Phys. D 22 (2013) 030402
- [28] K.P. Pandey, A.K. Singh, S. Singh, R. Kapoor and S. Kar, Eur.Phys.J. C74 (2014) 11, 3173 19
- [29] K.P. Pandey, A.K. Singh, S. Singh and S. Kar, Int.J.Mod.Phys. A30 (2015) 1550065
- [30] S. Kar, K.P. Pandey, S. Singh and A.K. Singh; World Scientific C10-02-24 (2011) 559, Proceeding of the conference in honour of Gell-Mann's 80th birthday; DOI: 10.1142/9789814335614 0058
- [31] A.K. Singh, K.P. Pandey, S. Singh and S. Kar; World Scientific C10-02-24 (2011) 567, Proceeding of the conference in honour of Gell-Mann's 80th birthday; DOI: 10.1142/9789814335614 0059
- [32] K.P. Pandey, A.K. Singh, S. Singh and S. Kar, J.Astrophys.Aerosp.Technol. 03 (2013) 01