

Oscillation of Second Order Nonlinear Neutral Delay Differential Equations

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ABSTRACT: Sufficient conditions for oscillation of second order nonlinear neutral delay differential equations of the form

$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} \{ m(t)y(t) + p(t)y^\alpha(t-\tau) \} \right\} + f(t)G(y(t-\sigma)) = 0$$

are obtained, where $r(t)$, $m(t)$, $p(t)$ are positive real valued continuous functions and $f(t) \geq 0$.

KEY WORDS: Oscillation, Second Order, Neutral Differential Equation.

1. INTRODUCTION

We observe that there has been increasing interest among the researchers in studying the effect of delays or advances in the independent variable have on the nature of solutions of differential equations with deviating arguments. (Recently there has been an increasing interest in the study of the oscillation of differential equations e.g. papers [1]-[16]). In this paper we consider the nonlinear neutral delay differential equation

$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} \{ m(t)y(t) + p(t)y^\alpha(t-\tau) \} \right\} + f(t)G(y(t-\sigma)) = 0 \quad (1)$$

where $r(t) \in C([t_0, \infty), (0, \infty))$, $m(t)$, $p(t)$, $f(t) \in C([t_0, \infty), [0, \infty))$

and $\alpha > 0$.

In the equation (1) if $p(t) = 0$ we get

$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} \{ m(t)y(t) \} \right\} + f(t)G(y(t-\sigma)) = 0 \quad (2)$$

which is a delay equation and further if we take $p(t) \equiv 0$, $m(t) \equiv 1$ and $\sigma = 0$ in equation (1) we get

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$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} (y(t)) \right\} + f(t)G(y(t)) = 0 \quad (3)$$

which is an ordinary differential equation.

The study of behavior of solutions of differential equation (2) has been a subject of interest for several researchers. The nonoscillatory behavior of the differential equation (1) can be found in (18). In this paper we address the oscillatory behavior of solutions.

II. RELATED WORK

The criteria regarding the behavior of solutions of equation (3) can be found in (19) and the references therein. We also mention the works of [1, 2, 10 and 12]. Oscillatory behavior of delay differential equations is extensively studied by several authors [3, 4, 6, 7, 8, and 9].

Now we see some special case of equation (1) ie when

$r(t) \equiv 1$ and $G(y(t)) = y(t)$ equation (1) is reduced to

$$\frac{d^2}{dt^2} \{m(t)y(t) + p(t)y^\alpha(t - \tau)\} + f(t)y(t - \sigma) = 0 \quad (4)$$

if further when $p(t) \equiv 0$, this equation is reduced to

$$\frac{d^2}{dt^2} \{m(t)y(t)\} + f(t)y(t - \sigma) = 0 \quad (5)$$

and to

$$\frac{d^2}{dt^2} \{m(t)y(t)\} + f(t)y(t) = 0 \text{ if } \sigma = 0 \quad (6)$$

and we note that, when $m(t) = 1$, this equation reduces to the equation

$$\frac{d^2}{dt^2} \{y(t)\} + f(t)y(t) = 0 \quad (7)$$

In particular, the works related to neutral equations, equations in which at least one term with a delay (deviating) argument is differentiated, have received much attention. The differential equations of the form (1) and for special cases when $r(t) \equiv 1$, and $G(y(t)) = y(t)$ is a subject of intensive research.

The oscillation for equation (7) has been discussed by many authors. Here we have some interesting results

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(i) Jiqin Deng [6]: If for large $t \in R$,

$$\int_t^{\infty} f(s) ds \geq \frac{\alpha_0}{t} \text{ where } \alpha_0 > \frac{1}{4}$$

then equation (7) is oscillatory.

(ii) CH.G.Philos [10]: Let n be an integer with $n \geq 3$ and ρ be a positive continuously differentiable function on the interval $[t_0, \infty)$ such that

$$\lim_{t \rightarrow \infty} \sup \frac{1}{t^{n-1}} \int_{t_0}^t \frac{(t-s)^{n-3}}{\rho(s)} [(n-1)\rho(s) - (t-s)\rho'(s)]^2 ds < \infty$$

Then (7) is oscillatory if

$$\lim_{t \rightarrow \infty} \sup \frac{1}{t^{n-1}} \int_{t_0}^t (t-s)^{n-1} \rho(s) f(s) ds = \infty$$

Motivated by some of these works, we present oscillation criteria for the solutions of the neutral delay differential equation (1) under particular type of integral conditions.

By a solution of equation (1) we mean a function $y(t) \in C([T_y, \infty))$ where $T_y \geq t_0$ which satisfies(1) on $[T_y, \infty)$. We consider only those solutions of $y(t)$ of (1) which satisfy $\sup \{|y(t)| : t \geq T\} > 0$ for all $T \geq T_y$ and assume that (1) possesses such solutions.

A solution of equation (1) is called oscillatory if it has arbitrary large zeros on $[T_y, \infty)$; otherwise it is called nonoscillatory. Equation (1) is said to be oscillatory if all its solutions oscillate. Unless otherwise stated, when we write a functional inequality, it will be assumed to hold for sufficiently large t in our subsequent discussion.

III. MAIN RESULTS

We need the following in our discussion

$$(H_1) : r(t), m(t) \in C([t_0, \infty), R), \quad r(t) > 0, m(t) > 0$$

$$(H_2) : f(t) \in C([t_0, \infty), R), \quad f(t) > 0$$

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$(H_3): p(t) \in C(t_0, \infty), \mathbb{R}$ and $0 \leq p(t) \leq 1$

$(H_4): G(u) \in C(\mathbb{R}, \mathbb{R})$ and $uG(u) > 0, u \neq 0$

$(H_5):$ There exists $q \in C([t_0, \infty), [0, \infty))$ and $f(t)G(t) \geq q(t)x$

$(H_6): m(t - \tau) > p(t)$

(H_7) $0 < \alpha \leq 1$ is the ratio of odd positive integers.

We set

$$z(t) = m(t)y(t) + p(t)y^\alpha(t - \tau) \tag{8}$$

and we have the following Lemma

Lemma 3.1 Assume that there exists $T \in [t_0, \infty)$, sufficiently large such that

$$z(t) > 0, z'(t) > 0, (r(t)z'(t))' < 0, t \in [T, \infty)$$

Then

$$\frac{z(t - \sigma)}{z(t)} \geq \frac{\int_T^{t-\sigma} \frac{1}{r(s)} ds}{\int_T^t \frac{1}{r(s)} ds}$$

Now we present the oscillation criteria:

Theorem 3.2

Assume that conditions $(H_1) - (H_7)$ are satisfied and Let

$$B_k = \{u \in C^1[s_k, t_k] : u(t) \neq 0, u(s_k) = u(t_k) = 0\}, k = 1, 2.$$

If there exists a function $u \in B_k, \rho \in C^1([t_0, \infty), (0, \infty))$, such that for $k=1, 2$,

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$$J_k(u, \rho, \sigma) = \left\{ \int_{s_k}^{t_k} \rho(t)q(t)[u^2(t) \left(\frac{1}{m(t-\sigma)} \left[1 - \alpha 2^{1-\alpha} p(t-\sigma) \frac{\int_{\tau}^{t-\sigma} \frac{1}{r(s)} ds}{\int_{\tau}^t \frac{1}{r(s)} ds} - \frac{(1-2^{1-\alpha})p(t-\sigma)}{M} \right] \right. \right. \\ \left. \left. + \left\{ \frac{1}{q(t)} \sigma^2(t) \right\} - \left\{ \frac{1}{q(t)} [r(t)\sigma(t)] \right\} \right) - r(t) \left(u'(t) + \frac{u'(t)\rho'(t)}{2\rho(t)} + u(t)\sigma(t) \right)^2 \right\} dt > 0$$

Then every solution of equation (1) is oscillatory.

Proof: Suppose to the contrary. And let $y(t)$ be a nonoscillatory solution of equation (1). Without loss of generality we may assume that $y(t)$ is eventually positive. From (H_5) and (1)

we have

$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} \left\{ m(t)y(t) + p(t)y^\alpha(t-\tau) \right\} \right\} = -f(t)G(y(t-\sigma)) \\ \frac{d}{dt} \left\{ r(t) \frac{d}{dt} \left\{ m(t)y(t) + p(t)y^\alpha(t-\tau) \right\} \right\} \leq -q(t)y(t-\sigma) \leq 0 \tag{9}$$

Further we obtain from the definition of z , [2 Lemma 2.2] and Bernoulli inequality that

$$m(t)y(t) = z(t) - p(t)y^\alpha(t-\tau) \\ m(t)y(t) = z(t) - p(t)[1 + y^\alpha(t-\tau)] + p(t) \\ \geq z(t) - 2^{1-\alpha} p(t)[1 + y(t-\tau)]^\alpha + p(t) \\ \geq z(t) - 2^{1-\alpha} p(t)[1 + \alpha y(t-\tau)]^\alpha + p(t) \\ \geq z(t) - \alpha 2^{1-\alpha} p(t)z(t-\sigma) + (1 - 2^{1-\alpha})p(t) \\ \geq [1 - \alpha 2^{1-\alpha} p(t)]z(t) + (1 - 2^{1-\alpha})p(t) \\ m(t)y(t) \geq [1 - \alpha 2^{1-\alpha} p(t)]z(t) + (1 - 2^{1-\alpha})p(t)$$

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$$y(t) \geq \frac{1}{m(t)} \{ [1 - \alpha 2^{1-\alpha} p(t)] z(t) + (1 - 2^{1-\alpha}) p(t) \} \quad (10)$$

From (9) and (10) we have

$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} \{ m(t)y(t) + p(t)y^\alpha(t-\tau) \} \right\} \leq -q(t) \frac{1}{m(t-\sigma)} \{ [1 - \alpha 2^{1-\alpha} p(t-\sigma)] z(t-\sigma) + (1 - 2^{1-\alpha}) p(t-\sigma) \} \quad (11)$$

From **Lemma 3.1** we have

$$z(t-\sigma) \geq \frac{\int_T^{t-\sigma} \frac{1}{r(s)} ds}{\int_T^t \frac{1}{r(s)} ds} z(t) - z(t-\sigma) \leq - \frac{\int_T^{t-\sigma} r(s) ds}{\int_T^t r(s) ds} z(t) \quad (12)$$

Substituting (12) in (11) we get

$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} \{ m(t)y(t) + p(t)y^\alpha(t-\tau) \} \right\} \leq -q(t) \frac{1}{m(t-\sigma)} \left\{ [1 - \alpha 2^{1-\alpha} p(t-\sigma)] \frac{\int_T^{t-\sigma} \frac{1}{r(s)} ds}{\int_T^t \frac{1}{r(s)} ds} z(t) + (1 - 2^{1-\alpha}) p(t-\sigma) \right\} \quad (13)$$

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Define

$$\omega(t) = -\rho(t) \left[\frac{r(t)z'(t)}{z(t)} + r(t)\sigma(t) \right] \quad \text{for } t \geq t_1 \quad (14)$$

$$\begin{aligned} \omega'(t) &= -\rho'(t) \left[\frac{r(t)z'(t)}{z(t)} + r(t)\sigma(t) \right] - \rho(t) \left[\frac{r(t)z'(t)}{z(t)} + r(t)\sigma(t) \right]' \\ \omega'(t) &= -\rho'(t) \left[\frac{r(t)z'(t)}{z(t)} + r(t)\sigma(t) \right] - \rho(t) \left[\frac{r(t)z'(t)}{z(t)} \right]' - \rho(t) [r(t)\sigma(t)]' \\ \omega'(t) &= -\rho'(t) \left[\frac{r(t)z'(t)}{z(t)} + r(t)\sigma(t) \right] - \rho(t) \left[\frac{z(t)\{r(t)z'(t)\}' - \{r(t)z'(t)\}z'(t)}{z^2(t)} \right] - \rho(t) [r(t)\sigma(t)]' \\ \omega'(t) &= -\rho'(t) \left[\frac{r(t)z'(t)}{z(t)} + r(t)\sigma(t) \right] \\ &\quad - \rho(t) \left[\frac{\{r(t)z'(t)\}'}{z(t)} \right] + \rho(t) \left[\frac{\{r(t)z'(t)\}z'(t)}{z^2(t)} \right] - \rho(t) [r(t)\sigma(t)]' \end{aligned} \quad (15)$$

$$\omega'(t) = \rho'(t) \left[\frac{\omega(t)}{\rho(t)} \right] - \rho(t) \left[\frac{\{r(t)z'(t)\}'}{z(t)} \right] + \rho(t) \left[\frac{r(t)\{z'(t)\}^2}{z^2(t)} \right] - \rho(t) [r(t)\sigma(t)]' \quad (16)$$

By (9) we have

$$r(t)z'(t) \leq r(t - \sigma)z'(t - \sigma)$$

From $z' > 0$, there exists a constant $M > 0$ such that $z(t) \geq M$ for all t large enough

from (14) we have

$$\begin{aligned} \frac{\omega(t)}{-\rho(t)} &= \left[\frac{r(t)z'(t)}{z(t)} + r(t)\sigma(t) \right] \\ \frac{\omega(t)}{-\rho(t)} - r(t)\sigma(t) &= \frac{r(t)z'(t)}{z(t)} \end{aligned}$$

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$$\frac{1}{r(t)} \left[\frac{\omega(t)}{-\rho(t)} - r(t)\sigma(t) \right] = \frac{z'(t)}{z(t)}$$

i.e.
$$\frac{z'(t)}{z(t)} = \left[\frac{\omega(t)}{-\rho(t)r(t)} - \sigma(t) \right]$$

$$\left\{ \frac{z'(t)}{z(t)} \right\}^2 = \left[\frac{\omega(t)}{-\rho(t)r(t)} - \sigma(t) \right]^2$$

$$\left\{ \frac{z'(t)}{z(t)} \right\}^2 = \frac{\omega^2(t)}{\rho^2(t)r^2(t)} + \sigma^2(t) + 2 \frac{\omega(t)}{\rho(t)r(t)} \cdot \sigma(t) \tag{17}$$

Substituting (13) and (17) in (16) we get

$$\begin{aligned} \omega'(t) \geq & \frac{\rho'(t)}{\rho(t)} \omega(t) - \rho(t) \left[-q(t) \frac{1}{m(t-\sigma)} \left\{ \left[1 - \alpha 2^{1-\alpha} p(t-\sigma) \right] \frac{\int_T^{t-\sigma} \frac{1}{r(s)} ds}{\int_T^t \frac{1}{r(s)} ds} + \frac{(1-2^{1-\alpha})p(t-\sigma)}{M} \right\} \right. \\ & \left. + \rho(t) \left[r(t) \left\{ \frac{\omega^2(t)}{\rho^2(t)r^2(t)} + \sigma^2(t) + 2 \frac{\omega(t)}{\rho(t)r(t)} \sigma(t) \right\} \right] - \rho(t) [r(t)\sigma(t)] \right] \end{aligned} \tag{18}$$

$\omega'(t) \geq$

$$\begin{aligned} & \rho(t)q(t) \left\{ \left[\frac{1}{m(t-\sigma)} \left\{ \left[1 - \alpha 2^{1-\alpha} p(t-\sigma) \right] \frac{\int_T^{t-\sigma} \frac{1}{r(s)} ds}{\int_T^t \frac{1}{r(s)} ds} - \frac{(1-2^{1-\alpha})p(t-\sigma)}{M} \right\} \right] + \left\{ \frac{1}{q(t)} \sigma^2(t) \right\} - \left\{ \frac{1}{q(t)} [r(t)\sigma(t)] \right\} \right\} \\ & + \left\{ \frac{\rho'(t)}{\rho(t)} + 2\sigma(t) \right\} \omega(t) + \left\{ \frac{1}{\rho(t)r(t)} \right\} \omega^2(t) \end{aligned} \tag{19}$$

Let $u \in B_1$ be given as in the hypothesis. Multiplying (19) by u^2 and integrating the resulting inequality from s_1 to t_1 we have

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$$\begin{aligned}
 & \int_{s_1}^{t_1} u^2(t) \omega'(t) dt \geq \\
 & \int_{s_1}^{t_1} u^2 \rho(t) q(t) \left\{ \left[\frac{1}{m(t-\sigma)} \left\{ [1 - \alpha 2^{1-\alpha} p(t-\sigma)] \frac{\int_T^{t-\sigma} \frac{1}{r(s)} ds}{\int_T^t \frac{1}{r(s)} ds} - \frac{(1-2^{1-\alpha}) p(t-\sigma)}{M} \right\} + \left\{ \frac{1}{q(t)} \sigma^2(t) \right\} \right. \right. \\
 & \left. \left. - \left\{ \frac{1}{q(t)} [r(t)\sigma(t)] \right\} \right\} dt \\
 & \quad + \int_{s_1}^{t_1} u^2 \left\{ \frac{\rho'(t)}{\rho(t)} + 2\sigma(t) \right\} \omega(t) dt + \int_{s_1}^{t_1} u^2 \left\{ \frac{1}{\rho(t)r(t)} \right\} \omega^2(t) dt
 \end{aligned} \tag{20}$$

Integrating (20) by parts and using the fact that $u(s_1) = u(t_1) = 0$ we deduce that

$$\begin{aligned}
 & - \int_{s_1}^{t_1} 2u(t) u'(t) \omega(t) dt \geq \\
 & \int_{s_1}^{t_1} u^2 \rho(t) q(t) \left\{ \left[\frac{1}{m(t-\sigma)} \left\{ [1 - \alpha 2^{1-\alpha} p(t-\sigma)] \frac{\int_T^{t-\sigma} \frac{1}{r(s)} ds}{\int_T^t \frac{1}{r(s)} ds} - \frac{(1-2^{1-\alpha}) p(t-\sigma)}{M} \right\} + \left\{ \frac{1}{q(t)} \sigma^2(t) \right\} \right. \right. \\
 & \left. \left. - \left\{ \frac{1}{q(t)} [r(t)\sigma(t)] \right\} \right\} dt \\
 & \quad + \int_{s_1}^{t_1} u^2 \left\{ \frac{\rho'(t)}{\rho(t)} + 2\sigma(t) \right\} \omega(t) dt + \int_{s_1}^{t_1} u^2 \left\{ \frac{1}{\rho(t)r(t)} \right\} \omega^2(t) dt
 \end{aligned}$$

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$$\int_{s_1}^{t_1} u^2 \left\{ \frac{1}{\rho(t)r(t)} \right\} \omega^2(t) dt + 2u\omega(t) \left[u' + u \left(\frac{\rho'(t)}{2\rho(t)} + \sigma(t) \right) \right] dt$$

$$+ \int_{s_1}^{t_1} u^2 \rho(t)q(t) \left\{ \left[\frac{1}{m(t-\sigma)} \left\{ [1 - \alpha 2^{1-\alpha} p(t-\sigma)] \frac{\int_T^{t-\sigma} \frac{1}{r(t)} ds}{\int_T^t \frac{1}{r(t)} ds} - \frac{(1-2^{1-\alpha})p(t-\sigma)}{M} \right\} \right] \right\}$$

$$+ \left\{ \frac{1}{q(t)} \sigma^2(t) \right\} - \left\{ \frac{1}{q(t)} [r(t)\sigma(t)] \right\} dt \leq 0$$

Hence

$$\int_{s_1}^{t_1} u^2 \rho(t)q(t) \left\{ \left[\frac{1}{m(t-\sigma)} \left\{ [1 - \alpha 2^{1-\alpha} p(t-\sigma)] \frac{\int_T^{t-\sigma} \frac{1}{r(t)} ds}{\int_T^t \frac{1}{r(t)} ds} - \frac{(1-2^{1-\alpha})p(t-\sigma)}{M} \right\} \right] + \left\{ \frac{1}{q(t)} \sigma^2(t) \right\} \right. \right\} dt$$

$$\left. - \left\{ \frac{1}{q(t)} [r(t)\sigma(t)] \right\} \right\} + \left\{ \int_{s_1}^{t_1} \frac{u(t)\omega(t)}{\sqrt{\rho(t)r(t)}} + \sqrt{\rho(t)r(t)} \left[u' + u \frac{\rho'(t)}{2\rho(t)} + u\sigma(t) \right] \right\}^2 dt - \rho(t)r(t) \left[u' + u \frac{\rho'(t)}{2\rho(t)} + u\sigma(t) \right]^2 dt \leq 0$$

which is equivalent to

$$\left\{ \int_{s_1}^{t_1} \frac{u(t)\omega(t)}{\sqrt{\rho(t)r(t)}} + \sqrt{\rho(t)r(t)} \left[u' + u \frac{\rho'(t)}{2\rho(t)} + u\sigma(t) \right] \right\}^2 dt + J_1(u, \rho, \sigma) \leq 0 \quad (21)$$

where $J_1(u, \rho, \sigma)$ is as in **Theorem 3.2**. Since $J_1(u, \rho, \sigma) > 0$, inequality (21) yields

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$$\left\{ \int_{s_1}^{t_1} \frac{u(t)\omega(t)}{\sqrt{\rho(t)r(t)}} + \sqrt{\rho(t)r(t)} \left[u' + u \frac{\rho'(t)}{2\rho(t)} + u\sigma(t) \right] \right\}^2 dt \leq -J_1(u, \rho, \sigma) < 0$$

which is a contradiction. Hence $y(t)$ is oscillatory.

Example 3.3

We consider the nonlinear neutral delay differential equation

$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} \{ m(t)y(t) + p(t)y^\alpha(t-\tau) \} \right\} + f(t)G(y(t-\sigma)) = 0 \quad (22)$$

where

$$r(t) = 1 \quad m(t) = 1 \quad \alpha = 1 \quad p(t) = \frac{1}{2} \quad \sigma = 0$$

and $G(y) = y$

We set

$$\rho(t) = 1 \quad q(t) = 4 \quad u(t) = \sin t \quad M = 1 \quad \alpha = 1 \quad \sigma(t) = 0$$

$$u'(t) = \cos t$$

Clearly $\rho'(t) = 0$

Then for $k=1, 2$.

$$J_k(u, \rho, \sigma) = \left\{ \int_{s_k}^{t_k} \rho(t)q(t) \left[u^2(t) \left(\frac{1}{m(t-\sigma)} \left[1 - \alpha 2^{1-\alpha} p(t-\sigma) \frac{\int_T^{t-\sigma} \frac{1}{r(s)} ds}{\int_T^t \frac{1}{r(s)} ds} - \frac{(1-2^{1-\alpha})p(t-\sigma)}{M} \right] \right) \right. \right. \\ \left. \left. + \left\{ \frac{1}{q(t)} \sigma^2(t) \right\} - \left\{ \frac{1}{q(t)} [r(t)\sigma(t)] \right\} \right] - r(t) \left(u'(t) + \frac{u'(t)\rho'(t)}{2\rho(t)} + u(t)\sigma(t) \right)^2 \right\} dt > 0$$

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$$J_k(u, \rho, \sigma) = \left\{ \int_{s_k}^{t_k} 1.4[\sin^2(t) \left(\frac{1}{1} \left[1 - 1.2^{1-1} \frac{1}{2} \frac{\int_1^t \frac{1}{s} ds}{\int_1^t \frac{1}{s} ds} - (1 - 2^{1-1}) \frac{1}{2} \right] \right) + \left\{ \frac{1}{4}(0) \right\} - \left\{ \frac{1}{4}[1(0)] \right\} \right\} - 1 \left(\cos t + \frac{\cos t(0)}{2(1)} + \sin t(0) \right)^2 \right\} dt > 0$$

$$J_k(u, \rho, \sigma) = \left\{ \int_{(2n+1)\pi}^{(2n+2)\pi} 1.4[\sin^2(t) \left(\frac{1}{1} \left[1 - 1.2^{1-1} \frac{1}{2} \frac{t-T}{t-T} - (1 - 2^{1-1}) \frac{1}{2} \right] \right) + \left\{ \frac{1}{4}(0) \right\} - \left\{ \frac{1}{4}[1(0)] \right\} \right\} - 1 \left(\cos t + \frac{\cos t(0)}{2(1)} + \sin t(0) \right)^2 \right\} dt > 0$$

$$J_k(u, \rho, \sigma) = \left\{ \int_{(2n+1)\pi}^{(2n+2)\pi} 1.4[\sin^2(t) \left(\frac{1}{2} \right) - \cos^2 t] \right\} dt > 0$$

$$J_k(u, \rho, \sigma) = \left\{ \int_{(2n+1)\pi}^{(2n+2)\pi} 2 \sin^2 t - \cos^2 t \right\} dt = \frac{1}{2} \pi .$$

We observe that the conditions of Theorem 3.2 are satisfied and the solutions of the equation (1) are oscillatory.

IV. CONCLUSION

We found criterion under which the solution of equation (1) are oscillatory and provided an example also. However we may think of studying the influence of forcing term on the differential equation (1) ie

$$\frac{d}{dt} \left\{ r(t) \frac{d}{dt} \left\{ m(t)y(t) + p(t)y^\alpha(t-\tau) \right\} \right\} + f(t)G(y(t-\sigma)) = g(t) \quad (23)$$

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