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Vol. 5, Issue 10, October 2016

# Topological Effect on Reachable Work Space for Single Degree of Freedom Planar Kinematic Chains

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**ABSTRACT**: The workspaces and kinematic characterization of serial chain manipulator geometries and the geometric optimization have been considered widely. The designer must consider the number of joints and their geometric arrangement. These considerations affect the manipulator's workspace. In this paper, a method for obtaining reachable workspace for kinematic chains is offered. Reachable workspace is proposed as a valuable measure of kinematic performance. The methodology of delineating the workspace is illustrated with examples.

KEYWORDS: Reachable Workspace, Kinematic chains.

### I. INTRODUCTION

The Kinematic structure of a robot must have six-degrees-of-freedom, three to locate the end-effector and the remaining three to orient it. The accuracy, ease of control, size of the working space, computational effort and accessibility depends upon the structure of the open chain. The determination of the workspace of robotic manipulators is a very vital issue in the context of kinematic design, especially because of fact that parallel manipulators have a small workspace.

Workspace is defined as regions which can be reached by a reference point located on the mobile platform. There are several definitions for the workspace of mechanisms. Constant orientation workspace corresponds to the set of positions reachable by the end effectors as it translates at a fixed orientation. Maximal workspace is defined as the region which can be reached by a point with at least one orientation. The maximal workspace is also referred to as a reachable workspace. Inclusive workspace is defined as the region which can be obtained by a point with at least one orientation in a given range. The total orientation workspace corresponds to the region which can be reached by a point with every orientation of a platform in a given range. Dexterous workspace is given as a region which can be reached by a point with any orientation of the platform.



FIGURE (1) TWO-LINK MANIPULATOR WITH LINK LENGTH  $L_1$  AND  $L_2\;$  WITH IT'S WORKSPACE



(An ISO 3297: 2007 Certified Organization)

### Vol. 5, Issue 10, October 2016

Considering the workspace of the two-link manipulator in Figure (1), If  $L_1 = L_2$ , then the reachable workspace consists of a disc of radius  $2L_1$ . The dexterous workspace consists of only a single point, the origin. If  $l_2$ , then there is no dexterous workspace, and the reachable workspace becomes a ring of outer radius  $L_1+L_2$  and inner radius  $L_2$ . Inside the reachable workspace there are two possible orientations of the end-effector. On the boundaries of the workspace there is only one possible orientation.

These considerations of workspace for the two-link manipulator have assumed that all the joints can rotate 360 degrees. This is rarely true for actual mechanisms. When joint limits are a subset of the full 360 degrees, then the workspace is obviously correspondingly reduced, either in extent, or in the number of possible orientations attainable. Complete rotation of input link is predicted on the basis of grashof condition.

The Grashof condition [15] is a very simple relationship which predicts the rotation behavior or rotatability of a fourbar linkage's inversions based only on the link lengths. Let: S = length of shortest link, L = length of longest link, P = length of one remaining link, Q = length of other remaining link. Considering S + L < P + Q:

The linkage is Grashof and at least one link will be capable of making a full revolution with respect to the ground plane. This is called a Class I kinematic chain. If the inequality is not true, then the linkage is non-Grashof and no link will be capable of a complete revolution relative to any other link. This is a Class II kinematic chain.

The motions possible from a four bar linkage will depend on both the Grashof condition and the inversion chosen. The inversions will be defined with respect to the shortest link. The motions are:

Ground either link adjacent to the shortest and we get a crank-rocker, in which the shortest link will fully rotate and the other link pivoted to ground will oscillate. Ground the shortest link and we will get a double-crank, in which both links pivoted to ground make complete revolutions as does the coupler. Ground the link opposite the shortest and we will get a Grashof double-rocker, in which both links pivoted to ground oscillate and only the coupler makes a full revolution. For the Class II case, S + L > P + Q: All inversions will be triple-rockers in which no link can fully rotate. For the Class III case, S + L = P + Q:as special-case Grashof and also as a Class III kinematic chain, all inversions will be either double-cranks or crank-rockers but will have "change points" twice per revolution of the input crank when the links all become collinear. Ting et al. [10, 11] have extended rotability criteria to all single loop linkages of n bars connected with revolute joint developed general theorems for linkage rotability and the revolvbility of individual links based on link lengths. Taking in to account above said criterion input link completely rotates.

#### **II. RELATED WORK**

Kumar and Waldron [2] defined two work spaces, reachable work space and dexterous work space which are important in evaluating the geometric characteristics of manipulator. Reachable work space is the volume within which every point can be reached by a reference point on the manipulator hand. Dexterous space is the volume within which every point can be reached by a reference point on the manipulator hand with the hand in any desired orientation. Tsai and Soni [3] have presented a procedure for determining the accessible region for a two link robot with pin joints and extended it to three link robots. Selfridge [5] developed an algorithm for finding the boundary of the reachable volume of an arbitrary revolute joint manipulator based on extreme reach of a manipulator hand. The extreme reach theorem is that if a manipulator hand is at an extreme distance from some completely arbitrary point then the line of sight from that point to the hand must intersect all turning axes. Gupta and Roth [4] described an attractive idea about the formation of work space and developed basic concepts by introducing total, primary and secondary work spaces. Huang [7] attempted to investigate the work space more analytically. He derived a general expression for the work space of a revolute robot using transformation matrix. He discussed the effect of geometric parameters such as link lengths, offset angles and moving angles on the shape of torus. Mahajan and Mogal [8] described a simple geometric approach to solve for any point on the extreme boundary of the work space for A sixdegree of freedom robot. Rastegar [9] presented work space analysis of 4R manipulators with various degrees of dexterity. A procedure to find out them is described.

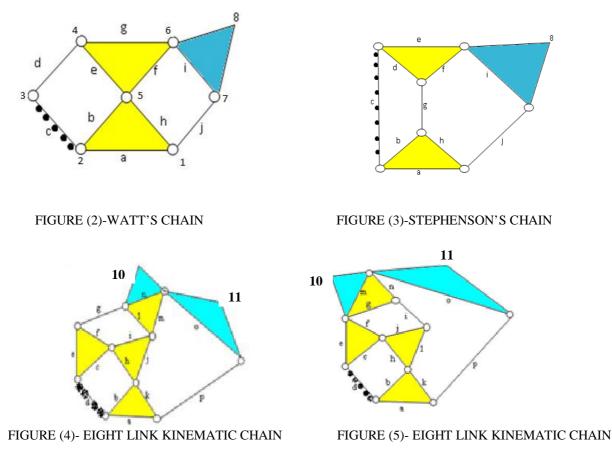
The entire above work can be considered to concern to the area of dimensional analysis and synthesis of mechanisms while no work associated to structural aspects of robots, mainly a comparison of different structures, is



(An ISO 3297: 2007 Certified Organization)

#### Vol. 5, Issue 10, October 2016

reported to the best of the authors Knowledge. If different structures have to be compared, if the dimensions of the robot arms are known i.e., performance can be compared only after the design of robot arms are complete. Rao and Jagdeesh [14] presented a quantitative method based on link assortment, joint assortment and loop type and their adjacency to compare the chains for workspace. The present work aims at helping the designer to compare possible structure before completing the dimensional synthesis. In other words, the work reported so far is a post-design problem while this work deals with a pre-design problem.



#### III. METHODOLOGY

The existence or nonexistence of a kinematic solution defines the workspace of a given manipulator. The lack of a solution means that the manipulator cannot attain the desired position and orientation because it lies outside of the manipulator's workspace. A graphical representation of the workspace can be obtained in 2D.

Link dimensions are known and for single degree of freedom kinematic chain one input position for one input link is known. To obtain all position of other links, for different position of input link following procedure is followed with examples

Step 1-reference coordinate position 1 has been chosen as shown in fig(2) .link perimeters of ternary link are known ,coordinates of joints 2 and 3 are calculated .

Step 2-fixed input link and input angle is used to initiate the procedure .coordinates of joint 4 can be calculated.

Step 3-all joints coordinate are calculated as all link perimeters are known. Links are drawn by joining all joint coordinates.



(An ISO 3297: 2007 Certified Organization)

### Vol. 5, Issue 10, October 2016

Step 4-same procedure is followed for increment in input angle and other coordinates are accordingly determined and next position can be drawn.

Corresponding coupler or floating point can be traced. Input link is completely revolved and reachable workspace is traced. Area of closed curve is calculated and compared for Six link single degree of freedom with binary link fixed and two eight link single degree of freedom kinematic chains with similar link and joint assortment.

The algorithm derived here has been implemented in a Turbo C++. Examples of results obtained with the computer implementation of the algorithm are given. This paper presents an algorithm for the determination of the reachable workspace and compares planar one-degree-of-freedom kinematic chains.

#### CASE 1(A). WATT'S CHAIN, WITH BINARY LINK FIXED

Six link single degree of freedom kinematic chains ie watt's chain is considered for reachable workspace, when binary fixed shown in figure (2) and values for different link is as parameters are a=60, b=50, c=50, d=60, e=60, f=60, g=60, h=60, i=60, j=60, k=60, l=60, m=60, n=60, p=60, q=60. Area of curve traced by point 8 is 34034.263 ..

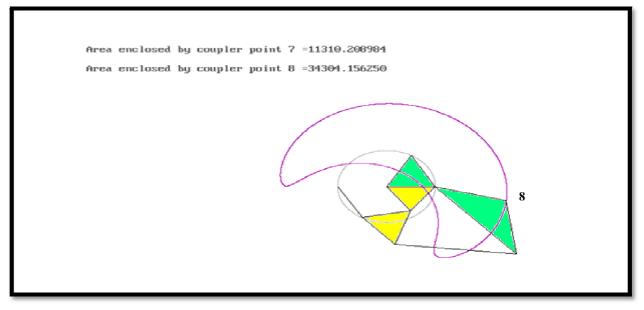


FIG (6) OUTPUT OF WATT'S CHAIN, WITH BINARY LINF 8 ED

#### CASE 1(B). STEPHENSON`S CHAIN WITH BINARY LINK FIXED

Six link single degree of freedom kinematic chains ie stephanson's chain is considered for reachable workspace ,when binary link is fixed as shown in figure (3) .values for different parameters are a=60, b=50, c=50, d=60, e=60, f=60, g=60, h=60, i=60, j=60, k=60, l=60, a=a of curve traced by point 8 is 13251.01.



(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 10, October 2016

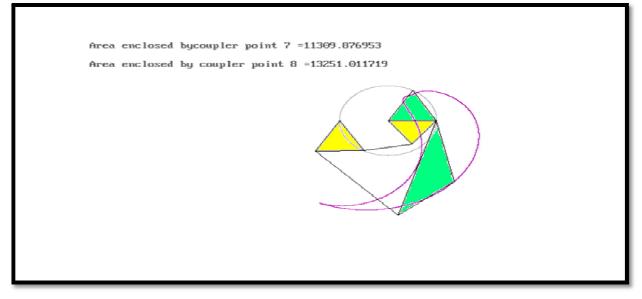


FIG (7) OUTPUT OF STEPHENSON`S CHAIN WITH BINARY LINK FIXED

CASE 2(A). EIGHT LINK KINEMATIC CHAIN FIG (4), WITH BINARY LINK FIXED

Eight link single degree of freedom kinematic chain is considered for reachable workspace ,when binary link is fixed is shown in figure (4) .values for different parameters are a=60, b=60, c=60, d=60, e=60, f=60, g=60, h=60, i=60, i=60, k=60, l=60, m=60, n=60, o=60, p=60, q=60 .area of curve traced by point 8 is 17738.134.

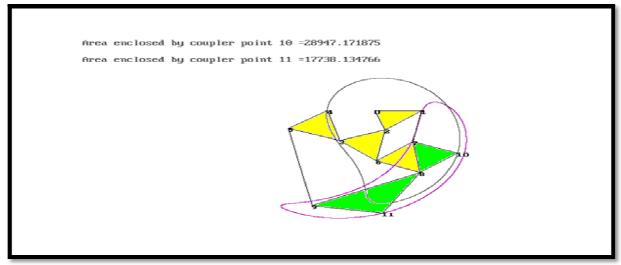


FIG (8) OUTPUT OF EIGHT LINK KINEMATIC CHAIN FIG (4), WITH BINARY LINK FIXED

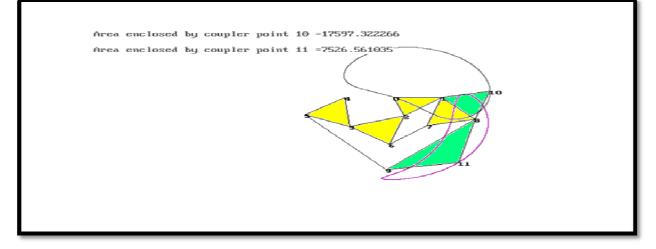


(An ISO 3297: 2007 Certified Organization)

### Vol. 5, Issue 10, October 2016

### CASE 2(B). EIGHT LINK KINEMATIC CHAIN FIG (5), WITH BINARY LINK FIXED

Eight link single degree of freedom kinematic chains is considered for reachable workspace ,when binary link is fixed is shown in figure (5) .values for different parameters are a=60, b=60, c=60, d=60, e=60, f=60, g=60, h=60, i=60, j=60, k=60, l=60, m=60, n=60, n=60, q=60, q=60 .area of curve traced by point 11 is 7526.56.



### FIG (9) OUTPUT OF EIGHT LINK KINEMATIC CHAIN FIG (5), WITH BINARY LINK FIXED

### IV. RESULTS

### COMPARISON OF WORK SPACE

Serial manipulators have the advantage of greater workspace in comparison to parallel manipulators. Closed kinematic chains are considered hybrid chains, have workspaces, which are to be quantified and an index has to be produced for grading on the basis of their workspaces. Reachable workspaces, FIG.(2,3) And Fig (4,5) are obtained following the methodology specified for six and eight link single degree of freedom kinematic chains and are compared in Table 1 and Table 2.

KINEMATIC CHAIN	AREA OF COUPLER POINT 7	AREA OF COUPLER POINT 8
WATT'S CHAIN, WHEN BINARY LINK IS FIXED	11309.634	34034.263
STEPHENSON`S CHAIN WHEN BINARY LINK IS FIXED	11309.634	13251.01

### TABLE 1 – REACHABLE WORKSPACE FOR SIX LINK KINEMATIC CHAINS



(An ISO 3297: 2007 Certified Organization)

### Vol. 5, Issue 10, October 2016

KINEMATIC CHAIN	AREA OF COUPLER POINT 10	AREA OF COUPLER POINT 11
EIGHT LINK KINEMATIC CHAIN FIG (4), WHEN BINARY LINK IS FIXED	28947.171	17738.134
EIGHT LINK KINEMATIC CHAIN FIG (5), WHEN BINARY LINK IS FIXED	17597.322	7526.56

### TABLE 2 – REACHABLE WORKSPACE FOR EIGHT LINK KINEMATIC CHAINS

### V. CONCLUSION

Following conclusions are drawn from the analysis.

1. Two Six link single degree of freedom are analysed and found that reachable workspace for Stephenson's chain is less than watt's chain .it is concluded that Stephenson's chain is more parallel than watt's chain. It has already been established and reported by authors [16] and other researchers.

2. Eight link single degree of freedom kinematic chain figure (5) has lesser reachable work space than Eight link single degree of freedom kinematic chain figure (4), hence figure (5) kinematic chain is more parallel than figure (4) kinematic chain.

Although link and joint assortments are same but because of change in topology the kinematic behaviour of two chains are different .It is observed that one of them is having more reachable work space due to its less parallelism then other hence, it can be concluded with change in topology for similar assortments also more parallel chain will have less reachable workspace

#### ACKNOWLEDGMENT

We are thankful to SSITM Bhilai and NIT Raipur for providing necessary facilities to carry out this research work

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