

A Hybrid Soft- Rough Algorithm for Supplier Election Problem

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ABSTRACT: Selection of effective Suppliers in a Supplier Selection Problem is the most challenging task in a supply chain. This is a multi attribute decision making problem where the aim of the companies is to rank the suppliers with respect to different attributes that may have respective weights. It is quite substantial that the better selection of suppliers reduces the purchasing cost and increases the competitive attitude of the companies. The present work deals with this type of problems. A Soft- Rough approach is proposed with a suitable example.

KEYWORDS: Soft Sets, Rough Sets, Lower Approximation, Supplier Selection Problem, MADM.

I. INTRODUCTION

In literature supplier selection problem has been viewed from different angles. We have classified them in mainly five categories: Multi Attribute Decision Making (MADM) Approaches, Multi Objective Decision Making (MODM) Approaches, Statistical Approaches (SA), Intelligent Approaches (IA) and Other Approaches.

Normally in MADM type approaches ([1], [3], [6], [8]), we have to choose the best supplier from a set of n suppliers $\{X_1, X_2, \dots, X_n\}$ where the performance of the alternatives are judged on the basis of a set of m attributes $\{A_1, A_2, \dots, A_m\}$ by a group of p DMs $\{E_1, E_2, \dots, E_p\}$. The comments of each DM are recorded on a rectangular matrix,

$$\begin{matrix} & A_1 & \cdots & A_m \\ X_1 & \left(G_{11}^k & \cdots & G_{1m}^k \right) \\ \vdots & \left(\vdots & \ddots & \vdots \right) \\ X_n & \left(G_{n1}^k & \cdots & G_{nm}^k \right) \end{matrix}$$

where $k=1, 2, \dots, p$. Thus we have p such matrices.

Again sometimes the Experts are asked to weight the attributes. For this, we have another matrix

$$W = \begin{matrix} & E_1 & \cdots & E_p \\ A_1 & \left(W_{11} & \cdots & W_{1p} \right) \\ \vdots & \left(\vdots & \ddots & \vdots \right) \\ A_m & \left(W_{m1} & \cdots & W_{mp} \right) \end{matrix}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 10, October 2016

MODM is a more general view of Mathematical Programming (MP) or Linear Programming (LP) models. It involves the ideal model for the best supplier by interacting within the model constraints that best satisfy the Expert. So there exists a constraint set and the suppliers are expressed in the feasible region of this set. Approaches like ϵ -constraint method, DEA, MP can be classified in this category. MP models actually formulate the given situation in terms of a mathematical objective function that afterwards needs to be either maximized or minimized, by varying the values of the variables in the objective function. In this category, Fuzzy models have also been introduced. The best part in these models is that it impels the evaluator to express the objective function explicitly. Thus they are more objective than the other rating models. On the other hand, Statistical Approaches are mainly based on the hypothesis that stochastic uncertainty is very much related to the supplier selection process. And thus it concentrates on the evaluation over a large number of surveys or deals. Cluster Analysis, Uncertainty Analysis are two sub categories within it. Existing models only accommodate for uncertainty with regard to one criterion at a time. Case Based Reasoning (CBR), Expert System analysis, Genetic Algorithm (GA) techniques, Neural network methods are classified in the category of Intelligent Approaches.

Some other techniques ([2], [3], [5], [8]) are also in the literatures, which belong to none of the above categories. Total Cost of Ownership (TCO), *Interpretive Structural Model (ISM)* and categorical methods are among them.

In this approach, Section 2 deals with preliminaries about Soft Sets and Rough Sets. In Section 3, a novel methodology based on hybrid Soft- Rough technique is introduced. A suitable example is provided in Section 4.

1. Preliminaries: Soft Sets, Fuzzy Soft Sets and Rough Sets

A pair (F, E) is called a Soft set (over the Universe of discourse U) if and only if F is a mapping of E into the set of all subsets of U . In other words, the Soft set is a parameterized family of subsets of the set U . every set $F(\epsilon)$, $\epsilon \in E$, from this family may be considered as the set of ϵ elements of the set (F, E) , or as the set of ϵ approximate elements of the soft set.

Let $U = \{H_1, H_2, H_3, H_4, H_5\}$ be the set of 5 Households in a rural community and $E = \{e_1$ (Poor Household Status), e_2 (Poor Monetary Status), e_3 (Poor Clothing Status), e_4 (Poor Housing Status) $\}$ be the set of parameters and $A = \{e_2, e_3, e_4\}$.

Then $(F, A) = \{F(e_2) = \{H_1, H_2, H_5\}, F(e_3) = \{H_2, H_4\}, F(e_4) = \{H_1, H_3, H_4, H_5\}\}$ is the soft set representing the 'Poverty of a Household' in that community. This Soft Set can be represented in a tabular form as shown in table 2.1.

Table: 2.1: Representation of the Soft Sets

U	e_2	e_3	e_4
H_1	1	0	1
H_2	1	1	0
H_3	0	0	1
H_4	0	1	1
H_5	1	0	1

A pair (F, A) is called a Fuzzy Soft Set over U where $F : A \rightarrow \tilde{P}(U)$ is a mapping from A to $\tilde{P}(U)$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 10, October 2016

Let us consider the previous example.

Then $(F, A) = \{F(e_2) = \{H_1/0.25, H_2/0.35, H_3/0.50, H_4/0.90, H_5/0.15\},$

$F(e_3) = \{H_1/0.50, H_2/0.50, H_3/0.25, H_4/0.60, H_5/0.50\},$

$F(e_4) = \{H_1/0.25, H_2/0.10, H_3/0.50, H_4/0.30, H_5/0.90\}$

is the Fuzzy Soft Set representing the Poor Households in the community.

Definition 2.1: $S = (F, A)$ be a soft set over U . Then the pair $V = (U, S)$ is called a soft approximation space. Based on the soft approximation space V , let us define the following two operations:

$$\underline{apr}_V(P) = \{u \in U : \exists a \in A, u \in F(a) \subseteq P\}$$

$$\overline{apr}_V(P) = \{u \in U : \exists a \in A, u \in F(a), F(a) \cap P \neq \phi\},$$

which are respectively called the soft V Lower approximation and the soft V Upper approximation of P with respect to V .

The soft lower approximation can be construed as the set consisting of the objects which are certainly the optimum candidates. On the other hand, the soft upper approximation can be construed as the set consisting of the objects which are possibly the optimum candidates.

Rough set theory ([2]) can be interpreted as a specific application of Frege's(1848-1925) idea of ambiguity, i.e., uncertainty in this approach is assessed by a boundary region of a set, and not by a partial membership. Rough Set theory cannot be viewed in the form of information about their elements. Basically they are expressed by some rule bases- certain and uncertain. Roughly it can be claimed that the certain decision rule bases describe lower approximation of decisions and the uncertain decision rule bases refer to the boundary region of the decisions.

Let X be a subset of U . The aim is to characterize the set X with respect to R .

- i) The *lower approximation* of a set X with respect to R is the set of all objects, which can be for *certain* classified as X with respect to R (are *certainly* X with respect to R).
- ii) The *upper approximation* of a set X with respect to R is the set of all objects which can be *possibly* classified as X with respect to R (are *possibly* X in view of R).
- iii) The *boundary region* of a set X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R .

Definition 2.2: The set X is *crisp* (exact with respect to R), if the boundary region of X is empty, whether the set X is *rough* (inexact with respect to R), if the boundary region of X is nonempty.

Thus, according to the definition, a set is *rough* (imprecise) if it has nonempty boundary region; otherwise the set is *crisp* (precise).

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 10, October 2016

II. METHODOLOGY

Here a novel method based on soft rough approximation is proposed.

The soft rough sets and soft rough approximation were first proposed by Feng et al. (2010).

Let us consider $U = \{X_1, X_2, \dots, X_l\}$ be a set of Suppliers and E , a set of all possible attributes. Also let $A = \{e_1, e_2, \dots, e_m\} \subseteq E$.

Step 1: At first the company forms the original description soft set "Effective Suppliers" $S = (F, A)$ over U . Let $F(e_i)$ denote the component soft set for the i th Attribute. So, a total number of m such component soft sets are formed and arranged in a matrix form. This is a $m \times l$ matrix F where the elements are either 0 or 1. If f_{ij} denotes the general element of F , then

$$f_{ij} = 1, \text{ if } X_j \in F(e_i)$$

$$= 0, \text{ otherwise.}$$

Step 2: A group $G = \{T_1, T_2, \dots, T_n\}$ of n Experts is appointed to evaluate the Suppliers in U . They are requested to only point out the optimal Suppliers as the outcome of the evaluation. So, each Expert's primary evaluation result is always a subset of G .

Let P_i denote the primary evaluation result of the Expert T_i . It can easily be expressed as an evaluation soft set $S_i = (\nu, G)$ over U , where $\nu: G \rightarrow P(U)$ is given by $\nu(T_i) = P_i$ ($i = 1, 2, \dots, n$) and $P(U)$ being the power set of U .

Step 3: Let us now consider the soft rough approximation of the Expert T_i 's primary evaluation outcome P_i with respect to the soft approximation Space $V = (U, S)$. The soft lower approximation $\underline{apr}_V(P_i)$ and the soft upper approximation $\overline{apr}_V(P_i)$ are evaluated.

Using these two approximations, we can obtain two other soft sets $\underline{S}_i = (\underline{\nu}, G)$ and $\overline{S}_i = (\overline{\nu}, G)$ over the Suppliers' set U , where

$$\underline{\nu}: G \rightarrow P(U) \text{ and } \overline{\nu}: G \rightarrow P(U) \text{ are given by } \underline{\nu}(P_i) = \underline{apr}_V(P_i) \text{ and } \overline{\nu}(P_i) = \overline{apr}_V(P_i) \text{ respectively,}$$

For $i = 1, 2, \dots, n$.

The matrix representations of the soft sets \underline{S}_i, S_i and \overline{S}_i are formed to simplify the computation of the next steps.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 10, October 2016

Step 4: Let us now consider three Fuzzy sets \tilde{A}_{S_1} , \tilde{A}_{S_1} and $\tilde{A}_{\bar{S}_1}$ for respectively the soft sets \underline{S}_1 , S_1 and \bar{S}_1 . The membership functions of these sets are denoted by $\mu_{\tilde{A}_{S_1}}$, $\mu_{\tilde{A}_{S_1}}$ and $\mu_{\tilde{A}_{\bar{S}_1}}$ and defined by:

$$\mu_{\tilde{A}_{S_1}} : U \rightarrow [0,1] \text{ such that } \mu_{\tilde{A}_{S_1}}(X_k) = \frac{1}{n} \sum_{j=1}^n R_{\underline{v}(T_j)}(X_k)$$

$$\mu_{\tilde{A}_{S_1}} : U \rightarrow [0,1] \text{ such that } \mu_{\tilde{A}_{S_1}}(X_k) = \frac{1}{n} \sum_{j=1}^n R_{v(T_j)}(X_k) \text{ and}$$

$$\mu_{\tilde{A}_{\bar{S}_1}} : U \rightarrow [0,1] \text{ such that } \mu_{\tilde{A}_{\bar{S}_1}}(X_k) = \frac{1}{n} \sum_{j=1}^n R_{\bar{v}(T_j)}(X_k),$$

where $\underline{v}(T_j) = \underline{apr}_v(P_j)$, $\bar{v}(T_j) = \overline{apr}_v(P_j)$, $k = 1, 2, \dots, l$.

Here $R_{v(T_j)}(X_k)$ denotes the number of occurrence of the Supplier X_k in the matrix representation of $v(T_j)$.

Similarly $R_{\underline{v}(T_j)}(X_k)$ and $R_{\bar{v}(T_j)}(X_k)$ are defined.

Since $\underline{S}_1 \subseteq S_1 \subseteq \bar{S}_1$, it can also be observed that $\mu_{\tilde{A}_{S_1}} \leq \mu_{\tilde{A}_{S_1}} \leq \mu_{\tilde{A}_{\bar{S}_1}}$.

Step 5: Define a Fuzzy soft set FS = (α , C) over U where C = {High Confidence Attribute (HCA), Middle Confidence (MCA), Low Confidence (LCA)} is a set of Attributes,

and $\alpha : C \rightarrow P(U)$ is given by $\alpha(HCA) = \mu_{\tilde{A}_{S_1}}$, $\alpha(MCA) = \mu_{\tilde{A}_{S_1}}$, $\alpha(LCA) = \mu_{\tilde{A}_{\bar{S}_1}}$.

Now three weights ω_{HCA} , ω_{MCA} and ω_{LCA} are estimated respectively for HCA, MCA and LCA, which will be associated with $\mu_{\tilde{A}_{S_1}}$, $\mu_{\tilde{A}_{S_1}}$ and $\mu_{\tilde{A}_{\bar{S}_1}}$ respectively. Thus $\sum \omega = 1$.

Step 6: Finally the score of the k th Supplier X_k is defined as $v(X_k) = \omega_{LCA} \times \alpha_{LCA} + \omega_{MCA} \times \alpha_{MCA} + \omega_{HCA} \times \alpha_{HCA}$.

The Suppliers are ranked according to this score. The Supplier with the Maximum value of v is the best supplier.

The ranking is now complete.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 10, October 2016

2. Numerical Example

Let us consider a situation of a company where it needs to rank the five suppliers S_1, S_2, S_3, S_4, S_5 . Also consider $A = \{e_1, e_2, e_3, e_4\}$ is a subset of the whole Attribute Set E . We may consider that e_1, e_2, e_3, e_4 respectively stands ‘product quality’, ‘cost’, ‘delivery time’ and ‘service quality’. Then $U = \{S_1, S_2, S_3, S_4, S_5\}$. A set of 4 experts $G = \{T_1, T_2, T_3, T_4\}$ are appointed to submit their opinion on the Suppliers.

Step 1: The company forms the soft sets (F_i, A) and its representation matrix is formed and displayed in table 4.1.

Table 4.1: matrix representation of the original description soft sets

	S_1	S_2	S_3	S_4	S_5
e_1	0	1	0	1	1
e_2	0	1	1	1	0
e_3	1	0	1	1	0
e_4	1	0	1	0	0

Clearly, here $F(e_1) = \{S_2, S_4, S_5\}$, $F(e_2) = \{S_2, S_3, S_4\}$, $F(e_3) = \{S_1, S_3, S_4\}$, and $F(e_4) = \{S_1, S_3\}$

Step 2: The decision matrix is formed by collecting the opinion of the Experts on the Suppliers. It is displayed in table 5.35.

Table 4.2: Decision matrix of the Experts on the Suppliers

	S_1	S_2	S_3	S_4	S_5
T_1	1	0	1	1	0
T_2	1	1	0	1	0
T_3	0	1	1	1	1
T_4	1	1	0	1	1

So clearly, we have the following:

$$v(T_1) = \{S_1, S_3, S_4\}, v(T_2) = \{S_1, S_2, S_4\}, v(T_3) = \{S_2, S_3, S_4\}, \text{ and } v(T_4) = \{S_1, S_2, S_4, S_5\}.$$

Step 3: According to the definitions, let us now construct the soft rough approximations of the Experts’ primary evaluation outcomes.

The lower approximations are given by:

$$\underline{v}(T_1) = F(e_3) = \{S_1, S_3, S_4\}, \underline{v}(T_2) = \phi, \underline{v}(T_3) = F(e_2) = \{S_2, S_3, S_4\}, \text{ and } \underline{v}(T_4) = \phi.$$

The upper approximations are given by:

$$\bar{v}(T_1) = F(e_1) \cup F(e_2) \cup F(e_3) \cup F(e_4) = \{S_1, S_2, S_3, S_4, S_5\},$$

$$\bar{v}(T_2) = F(e_2) \cup F(e_3) \cup F(e_4) = \{S_1, S_2, S_3, S_4\},$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 10, October 2016

$$\bar{v}(T_3) = F(e_1) \cup F(e_2) \cup F(e_3) \cup F(e_4) = \{S_1, S_2, S_3, S_4, S_5\}, \text{ and}$$

$$\bar{v}(T_4) = F(e_1) \cup F(e_2) \cup F(e_3) \cup F(e_4) = \{S_1, S_2, S_3, S_4, S_5\}.$$

From these results the lower and upper Approximation matrices are formed and displayed in table 5.36 and 5.37.

Table 4.3: Lower Approximation Matrix

	S ₁	S ₂	S ₃	S ₄	S ₅
T ₁	1	0	1	1	0
T ₂	0	0	0	0	0
T ₃	0	1	1	1	0
T ₄	0	0	0	0	0

Table 4.4: Upper Approximation Matrix

	S ₁	S ₂	S ₃	S ₄	S ₅
T ₁	1	1	1	1	1
T ₂	1	1	1	1	0
T ₃	1	1	1	1	1
T ₄	1	1	1	1	1

Step 4: From tables 4.2, 4.3 and 4.4, the three Fuzzy membership functions are constructed as defined in the methodology. They are given by:

$$\mu_{\bar{G}_1} = \{(S_1, \frac{1}{4}), (S_2, \frac{1}{4}), (S_3, \frac{1}{2}), (S_4, \frac{1}{2}), (S_5, 0)\}$$

$$\mu_{G_1} = \{(S_1, \frac{3}{4}), (S_2, \frac{3}{4}), (S_3, \frac{1}{2}), (S_4, 1), (S_5, \frac{1}{2})\}$$

$$\mu_{\bar{G}_1} = \{(S_1, 1), (S_2, 1), (S_3, 1), (S_4, 1), (S_5, \frac{3}{4})\}$$

Step 5: In this particular case study, we consider the values of ω_{HCA} , ω_{MCA} and ω_{LCA} as 0.25, 0.50 and 0.25 respectively. Then the scores of the Suppliers are given by:

$$v(X_1) = 0.6875, v(X_2) = 0.6875, v(X_3) = 0.6250, v(X_4) = 0.8750 \text{ and } v(X_5) = 0.4375.$$

So the final rank is $S_4 > S_1 = S_2 > S_3 > S_5$.

It shows that the company should consider S₄ for this dealing.

III. CONCLUSION

Every single object, every single deal, thought, expression carries some sort of information, which, while estimating is not fully understandable or not traceable mathematically. So, when we attempt to interpret these types of information in the framework of Mathematics, it definitely becomes a tougher task for us. The proposed

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(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 10, October 2016

methodology based on Soft Rough Hybrid algorithm is an approach towards uncertainty based modelling of Supplier Selection Problem. The method is novel, easy to implement and effective. Other multi attribute decision making problems may also be addressed using this methodology.

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