

# Effort Dynamics in a Selective Harvesting Prey-Predator Fishery with Beddington- DeAngelis Functional Response

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**ABSTRACT:** In this paper, regulation of selective harvesting of a prey-predator fishery with Beddington-DeAngelis functional response [1,2] is discussed, considering taxation as a control instrument and harvesting effort as a dynamic variable. The Beddington-DeAngelis functional response is similar to the Holling type II [3] functional response but contains one extra term describing mutual interference by the predator. In this paper we assume that fishing is permitted only for the predator fish species after imposing a tax per unit harvested biomass by the regulatory agencies in order to control over exploitation. Considering the effort as the dynamic variable i.e. time dependent variable, the non-trivial interior equilibrium point is determined and the local and global stability of this equilibrium point is discussed. The results are illustrated with the help of a numerical example.

**Keywords:** prey-predator fishery, Beddington-DeAngelis functional response, regulatory agencies, local stability, variational matrix, global stability.

## I. INTRODUCTION

Prey-predator interaction in the fish community is a common problem for extinction of the prey fish population and common harvesting of prey-predator species is a global problem for extinction of both the species. In order to control over exploitation of the species and maintain the biological equilibrium of the species, the Government or private agencies should regulate the harvesting effort. Taxation, license fees, lease of property rights, seasonal harvesting etc, are usually considered as possible governing instruments in fishery regulation. Among these, taxation is superior to other control policies because of its flexibility. As described by Clark ([4], Art.4.6, p-116), "Economists are particularly attracted to taxation, partly because of its flexibility and partly because many of the advantages of a competitive economic system can be better maintained under taxation than under other regulatory methods".

Many researchers developed different types of exploited prey-predator models and discussed the biological equilibrium of the populations and their dynamical behaviour. Pradhan [5] discussed the dynamics of an exploited prey-predator fish species with Beddington-DeAngelis functional response on the basis of CPUE (catch per-unit-effort) hypothesis. Saha Ray and Chaudhuri [6] discussed a Lotka-Volterra prey-predator model with harvesting and environmental perturbations. Bionomic exploitation of a Lotka-Volterra prey-predator system was studied by Chaudhuri and Saha Ray [7]. The optimal policy for combined harvesting of a prey-predator community was discussed by Mesterton-Giobbons [8]. Chaudhuri and Pradhan [9] developed a model of combined harvesting of a prey-predator fishery with low predator density and the prey species obeying the logistic law of growth. Kar [10] developed a prey-predator model with stage-structure for predator and selective harvesting of prey species.

Holling type I ( $f(x) = \alpha x$ ), II ( $f(x) = \frac{\alpha x}{x + \beta}$ ) and III ( $f(x) = \frac{\alpha x^2}{x^2 + \beta}$ ) [3] functional responses are the most popular used functional responses for the prey-predator models. But all these functional responses are prey dependent and so in some situation there may be an unrealistic population dynamics of the prey or predator. This needs a prey-predator

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dependent functional responses  $f(x, y)$ . The Beddington-DeAngelis type  $f(x, y) = \frac{\alpha x}{b + cx + y}$  ( $b > 0, c > 0, \alpha > 0$ ) [1,2] functional response is one of the more realistic functional responses. It is similar to the Holling type II functional response but has an extra term  $y$  in the denominator of  $f(x, y)$  which models mutual interference between the predators.

Since the prey species is killed and eaten by the predator, so if harvesting is allowed for the prey species then the prey species will become extinct in the environment. So, for the existence of the prey species in the environment, in this paper we consider that the prey species is not allowed for harvesting by the fishermen. Harvesting is allowed only for the predator species after imposing a suitable tax per unit harvested biomass by the regulatory agencies. In this model the harvesting effort is considered as a dynamic i.e. time dependent variable. The dynamical behaviour of this selective harvesting prey-predator fishery model with Beddington-DeAngelis functional response is studied. A sufficient condition on the tax limits is found for existence of the non-trivial interior equilibrium point of the system. The local and global stability of this equilibrium point is also studied. A numerical example is taken to illustrate the results.

## II. RELATED WORKS

The problems related to harvesting of multispecies fisheries have drawn the attention of researchers from time to time. Clark [4] studied the problem of combined harvesting of two ecologically independent fish species, each of which obeys the logistic law of growth [11]. Clark [4] considered Gause's model [12] of interspecific competition between two species and studied the effects of selective harvesting of the species. The effects of harvesting both the species in the Gause's model [12] were extensively discussed by Chaudhuri ([13], [14]). Mesterton-Gibbons [15] extended the works of Chaudhuri([13], [14]) giving more emphasis on some economic factors.

In a dynamic model of an open-access fishery, the level of fishing effort expands or contracts according as the perceived rent i.e. the net economic revenue to the fishermen is positive or negative. A model reflecting this dynamic interaction between the perceived rent and the effort in fishery is called a dynamic reaction model. In this model it is also assumed that the fishing effort is a dynamic i.e time dependent variable. The fisheries biologist M.B. Schaefer [16] developed a dynamic bioeconomic model for a single species fishery and applied it to the tuna fisheries of the tropical Pacific. Gordan's model [17] is the equilibrium solution of the Schaefer model [16] which is used in commercial marine fisheries inspite of its several drawbacks. Schaefer model [16] is based on the logistic growth function first proposed by P.F. Verhulst [11] and it uses the 'catch-per-unit-effort-hypothesis, for obtaining the functional form of the catch rate. Detailed discussions on the Schaefer model [16] are available in the book of Clark [4].

Various techniques in regulating fisheries been suggested from time to time by the researchers. These include allocation of fishermen's quotas ([18],[19]), imposition of taxes on landed fish [4] license limitation [20], restricting fishing seasons [21],etc. Some of the issues associated with the choice and enforcement of optimal governing instruments in regulating fisheries were discussed Anderson and Lee [22]. The economic implications of enforcing laws for regulating marine fisheries were discussed by Sutinen and Andersen [23]. Among these methods, taxation is superior to the other control policies because of its flexibility described by Clark [4].

A single species fishery model using taxation as a control measure was first discussed by Clark [4]. Chaudhuri and Johnson [24] extended that model using a catch-rate function which was more realistic than that in [4]. Ganguly and Chaudhuri [25] made a capital theoretic study of a single species fishery with taxation as control policy. Pradhan and Chaudhuri [26] developed a mathematical model for growth and exploitation of a schooling fish species, using a realistic catch-rate function and imposing a tax per unit biomass of landed fish to control harvesting. Pradhan and

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Chaudhuri [27] also studied a fully dynamic reaction model of fishery consisting two competing fish species with taxation as a control instrument. Pradhan [28] developed a prey-predator fishery model with low predator density where taxation is the control instrument. In that paper only the predator fish species is allowed for harvesting by the fishermen after imposing suitable tax by the regulatory agencies.

### III. FORMULATION OF THE MODEL

Let us consider a prey-predator fishery model in the form of

$$\left. \begin{aligned} \frac{dx(t)}{dt} &= rx \left(1 - \frac{x}{k}\right) - af(x, y)y \\ \frac{dy(t)}{dt} &= ef(x, y)y - dy \end{aligned} \right\} \quad (1)$$

where  $x(t)$  and  $y(t)$  be the population density of the prey and predator species respectively at any time  $t$ . We assume that the growth rate of prey species obeys the logistic growth function  $rx \left(1 - \frac{x}{k}\right)$  [11] and the natural mortality rate of the predator is directly proportional to the population density. Here  $f(x, y)$  is the functional response which represents per capita predator feeding rate,  $r$  is the intrinsic growth rate of the prey species,  $k$  is the environmental carrying capacity for the prey species,  $a$  is the capturing rate of the prey species by the predator,  $e$  is the growth rate of the predator and  $d$  is the per capita death rate of the predator.

In this model we consider  $f(x, y)$  as the Beddington-DeAngelis [1,2] functional response such that  $f(x, y) = \frac{x}{b+cx+y}$ , ( $b > 0, c > 0$ ). This functional response is more realistic than the Holling type I,II,III functional responses as described by D.T. Dimitrov and H.V. Kajauharov [29].

**Interpretation of the Beddington-DeAngelis functional response**  $f(x, y) = \frac{x}{b+cx+y}$ :

Here  $f(x, y) > 0$ ,

$$\frac{\partial f(x,y)}{\partial x} = \frac{b+y}{(b+cx+y)^2} > 0$$

$$\text{and } \frac{\partial f(x,y)}{\partial y} = -\frac{x}{(b+cx+y)^2} < 0 \quad \forall (x, y) \in R_2^+ = \{(x, y) : x > 0, y > 0\}.$$

These inequalities imply that in the positive quadrant of  $R_2$ , per capita predator feeding rate is always positive and is an increasing function of  $x$  and decreasing function of  $y$ . Biological interpretation of these results is, in the increase of the prey species the predator feeding rate increases and in the increase of the predator species the feeding rate decreases due to interspecies competitions.

After the above consideration, the dynamical system (1) becomes

$$\left. \begin{aligned} \frac{dx(t)}{dt} &= rx \left(1 - \frac{x}{k}\right) - \frac{axy}{b+cx+y} \\ \frac{dy(t)}{dt} &= \frac{exy}{b+cx+y} - dy \end{aligned} \right\} x(0) > 0, y(0) > 0. \quad (2)$$

In this model we assume that fishing is allowed only for the predator species. In order to control the over exploitation of the predator species, the regulatory agencies impose the tax  $\tau$  per unit harvested biomass and also impose a penalty if the fishermen harvest the prey species. To avoid the penalty, we assume that the fishermen harvest only the predator fish species by fixing the suitable mesh size of the fishing net or using any other control instruments.

Let  $E(t)$  be the effort for harvesting the predator fish species at any time  $t$ .

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The net economic revenue to the fishermen (perceived rent) is  $\{q(p - \tau) - C\}E$ , where  $p$  is the market price per unit biomass of the harvested fish,  $q$  is the catchability coefficient of the predator fish species and  $C$  is the cost per unit effort for harvesting the fish species.

Considering  $E(t)$  as the dynamic variable i.e. time dependent variable, we have

$\frac{dE(t)}{dt} = \lambda\{q(p - \tau)x - C\}E$ , where  $\lambda$  is the stiffness parameter measuring the effort and the perceived rent.

Therefore, the system (2) becomes

$$\left. \begin{aligned} \frac{dx(t)}{dt} &= rx \left(1 - \frac{x}{k}\right) - \frac{axy}{b+cx+y} \\ \frac{dy(t)}{dt} &= \frac{exy}{b+cx+y} - dy - qEy \\ \frac{dE(t)}{dt} &= \lambda\{q(p - \tau)x - C\}E \end{aligned} \right\} \quad (3)$$

## IV. EXISTENCE OF NON-TRIVIAL INTERIOR EQUILIBRIUM POINT

The equilibrium points of the system (3) are given by  $\frac{dx(t)}{dt} = \frac{dy(t)}{dt} = \frac{dE(t)}{dt} = 0$ .

But we are interested only for the non-trivial interior equilibrium point  $P(x^*, y^*, E^*)$  of the dynamical system (3).

Now,  $\frac{dE(t)}{dt} = 0$  gives  $y^* = \frac{c}{q(p-\tau)}$  (4)

For the fixed harvesting cost and the catchability coefficient of the predator, the result (4) implies that the steady state of the predator species is independent of the prey species but depends only on the market price and the tax imposed by the regulatory agencies. Biologically it is true that since the prey species is not harvested so there are sufficient numbers of prey species for the natural growth of the predator species. So the steady state of the predator depends only on the effort given by the fishermen and the effort depends on the market price of the landed fish and the tax imposed by the agencies. Thus the steady state of the predator species depends on the market price  $p$  of this species and the tax  $\tau$ . Result (4) also shows that steady state of the predator decreases when the market price of this fish species increases when the tax  $\tau$  is fixed. This is due to excess harvest of the species by the fishermen for the high market price. Same result holds when the tax decreases.

$$\frac{dx}{dt} = 0 \text{ implies } r \left(1 - \frac{x^*}{k}\right) - \frac{ay^*}{b+cx^*+y^*} = 0 \quad (5)$$

$$\text{and } \frac{dy}{dt} = 0 \text{ implies } E^* = \frac{1}{q} \left\{ \frac{ex^*}{b+cx^*+y^*} - d \right\} \quad (6)$$

Equation (6) can be written as  $E^* = \frac{1}{q(b+cx^*+y^*)} \{(e - cd)x^* - d(b + y^*)\}$ .

So, one of the necessary conditions for  $E^* > 0$  is  $e > cd$  (7)

Therefore, one of the necessary conditions for existence of the steady state level of the effort is that the per capita growth rate of the predator fish species is greater than  $c$  times of the per capita death rate.

But this condition is not sufficient. Because if  $(e - cd)x^* < d(y^* + b)$  then  $E^* < 0$  and the steady state of effort does not exist.

Therefore, the necessary and sufficient condition for existence of the steady effort level is

$$(e - cd)x^* > d(y^* + b) \quad (8)$$

when (7) holds.

From (5), we have

$$\frac{rx^*}{k} (b + cx^* + y^*) - r(b + cx^* + y^*) + ay^* = 0$$

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$$\begin{aligned} &\Rightarrow \frac{rc}{k}x^{*2} + \left(\frac{rb}{k} + \frac{r}{k}y^* - rc\right)x^* - (rb + ry^* - ay^*) = 0 \\ &\Rightarrow x^* = \frac{1}{2c} \left\{ -(b + y^* - ck) + \sqrt{(b + y^* - ck)^2 + 4ck \left(b + y^* - \frac{ay^*}{r}\right)} \right\} \end{aligned} \quad (9)$$

We assume that the intrinsic growth rate of the prey species is greater than the capturing rate of this species by the predator i.e.  $r > a$ .

Therefore,  $x^* > 0$  by (9).

Using (9), the inequality (8) becomes

$$\frac{(e-cd)}{2c} \left\{ -(b + y^* - ck) + \sqrt{(b + y^* - ck)^2 + 4ck \left(b + y^* - \frac{ay^*}{r}\right)} \right\} - dy^* - bd > 0.$$

Therefore, the sufficient condition for  $E^* > 0$  is

$$\begin{aligned} &(e - cd)(b + y^* - ck) + 2c(dy^* + bd) < 0 \\ &\Rightarrow y^* < ck \left(\frac{e-cd}{e+cd}\right) - b \text{ provided } \frac{e-cd}{e+cd} > \frac{b}{ck} \end{aligned} \quad (10)$$

$$\Rightarrow \frac{c}{q(p-\tau)} < ck \left(\frac{e-cd}{e+cd}\right) - b \text{ by (4)}$$

$$\Rightarrow q(p - \tau) > \frac{c}{B} \text{ where } B = ck \left(\frac{e-cd}{e+cd}\right) - b > 0 \quad (11)$$

$$\Rightarrow \tau < p - \frac{c}{Bq} \quad (12)$$

Therefore, the condition (11) is the sufficient condition for existence of the non-trivial interior equilibrium point  $P(x^*, y^*, E^*)$  together with the conditions (7) and (10). The non-trivial interior equilibrium point of the system (3) is  $P(x^*, y^*, E^*)$ , where  $x^*$ ,  $y^*$  and  $E^*$  are given by (9), (4) and (6) respectively.

## V. LOCAL STABILITY ANALYSIS OF THE NON-TRIVIAL INTERIOR EQUILIBRIUM POINT

$$P(x^*, y^*, E^*)$$

The variational matrix of the dynamical system (3) is

$$M(x, y, E) = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}, \text{ where}$$

$$\begin{aligned} J_{11} &= \frac{\partial}{\partial x} \left( \frac{dx}{dt} \right) = r - \frac{2rx}{k} - \frac{(b+cx+y)ay-axy}{(b+cx+y)^2} \\ &= r - \frac{rx}{k} - \frac{ay}{b+cx+y} - \frac{rx}{k} + \frac{acxy}{(b+cx+y)^2} \Rightarrow J_{11}(x^*, y^*, E^*) = -\frac{rx^*}{k} + \frac{acx^*y^*}{(b+cx^*+y^*)^2} \text{ by (5)}. \end{aligned}$$

$$J_{12} = \frac{\partial}{\partial y} \left( \frac{dx}{dt} \right) = -\frac{(b+cx+y)ax-axy}{(b+cx+y)^2} \Rightarrow J_{12}(x^*, y^*, E^*) = -\frac{a(b+cx^*)x^*}{(b+cx^*+y^*)^2}.$$

$$J_{13}(x^*, y^*, E^*) = \frac{\partial}{\partial E} \left( \frac{dx}{dt} \right) = 0. J_{21} = \frac{\partial}{\partial x} \left( \frac{dy}{dt} \right) = \frac{(b+cx+y)ey-ecxy}{(b+cx+y)^2} \Rightarrow J_{21}(x^*, y^*, E^*) = \frac{e(b+y^*)y^*}{(b+cx^*+y^*)^2}.$$

$$J_{22} = \frac{\partial}{\partial y} \left( \frac{dy}{dt} \right) = \frac{(b+cx+y)ex-ecxy}{(b+cx+y)^2} - d - qE \Rightarrow J_{22}(x^*, y^*, E^*) = -\frac{ex^*y^*}{(b+cx^*+y^*)^2} \text{ by (6)}$$

$$J_{23} = \frac{\partial}{\partial E} \left( \frac{dy}{dt} \right) = -qE \Rightarrow J_{23}(x^*, y^*, E^*) = -qE^*, J_{31}(x^*, y^*, E^*) = \frac{\partial}{\partial x} \left( \frac{dE}{dt} \right) = 0.$$

$$J_{32} = \frac{\partial}{\partial y} \left( \frac{dE}{dt} \right) = \lambda q(p - \tau)E \Rightarrow J_{32}(x^*, y^*, E^*) = \lambda q(p - \tau)E^*,$$

$$J_{33} = \frac{\partial}{\partial E} \left( \frac{dE}{dt} \right) = \lambda \{ q(p - \tau) - C \} \Rightarrow J_{33}(x^*, y^*, E^*) = 0 \text{ by (4)}.$$

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$$\text{Therefore, } M(x^*, y^*, E^*) = \begin{pmatrix} Aacx^*y^* - \frac{rx^*}{k} & -Aa(b + cx^*)x^* & 0 \\ Ae(b + y^*)y^* & -Aex^*y^* & -qy^* \\ 0 & \lambda q(p - \tau)E^* & 0 \end{pmatrix},$$

$$\text{where } A = \frac{1}{(b + cx^* + y^*)^2}.$$

The characteristic equation of the matrix  $M(x^*, y^*, E^*)$  is

$$\begin{vmatrix} Aacx^*y^* - \frac{rx^*}{k} - \mu & -Aa(b + cx^*)x^* & 0 \\ Ae(b + y^*)y^* & -Aex^*y^* - \mu & -qy^* \\ 0 & \lambda q(p - \tau)E^* & -\mu \end{vmatrix} = 0$$

$$\begin{aligned} \Rightarrow \mu^3 + \left( Aex^*y^* + \frac{rx^*}{k} - Aacx^*y^* \right) \mu^2 \\ + \left\{ A^2(abcex^{*2}y^* + abex^*y^{*2} + ab^2ex^*y^*) + \frac{1}{k} \cdot Arex^{*2}y^* + \lambda q^2(p - \tau)y^*E^* \right\} \mu \\ - Aac\lambda q^2(p - \tau)x^*y^{*2}E^* + \frac{r}{k} \cdot \lambda q^2(p - \tau)x^*y^*E^* = 0 \end{aligned}$$

$$\Rightarrow \mu^3 + m_1\mu^2 + m_2\mu + m_3 = 0 \text{ where}$$

$$m_1 = Aex^*y^* + \frac{rx^*}{k} - Aacx^*y^*,$$

$$m_2 = A^2(abcex^{*2}y^* + abex^*y^{*2} + ab^2ex^*y^*) + \frac{1}{k} \cdot Arex^{*2}y^* + \lambda q^2(p - \tau)y^*E^* \text{ and}$$

$$m_3 = -Aac\lambda q^2(p - \tau)x^*y^{*2}E^* + \frac{r}{k} \cdot \lambda q^2(p - \tau)x^*y^*E^*.$$

$$\begin{aligned} \text{Here, } m_1 &= Aex^*y^* + x^* \left\{ \frac{r}{k} - \frac{c}{b + cx^* + y^*} \cdot \frac{ay^*}{b + cx^* + y^*} \right\} \\ &= Aex^*y^* + x^* \left\{ \frac{r}{k} - \frac{c}{b + cx^* + y^*} \cdot r \left( 1 - \frac{x^*}{k} \right) \right\} \text{ by (5)} \\ &= Aex^*y^* + \frac{rx^*}{k(b + cx^* + y^*)} (b + 2cx^* + y^* - ck) \end{aligned}$$

$$= Aex^*y^* + \frac{rx^*}{k(b + cx^* + y^*)} \sqrt{(b + y^* - ck)^2 + 4ck \left( b + y^* - \frac{ay^*}{r} \right)} \text{ by (9)}$$

$$> 0, \text{ since } r > a.$$

$$m_2 = A^2(abcex^{*2}y^* + abex^*y^{*2} + ab^2ex^*y^*) + \frac{1}{k} \cdot Arex^{*2}y^* + \lambda q^2(p - \tau)y^*E^* > 0, \text{ since } 0 < \tau < p.$$

$$\begin{aligned} m_3 &= -Aac\lambda q^2(p - \tau)x^*y^{*2}E^* + \frac{r}{k} \cdot \lambda q^2(p - \tau)x^*y^*E^* \\ &= \lambda q^2(p - \tau)x^*y^*E^* \left\{ \frac{r}{k} - Aacy^* \right\} \\ &= \lambda q^2(p - \tau)x^*y^*E^* \left\{ \frac{r}{k} - \frac{cr}{b + cx^* + y^*} \left( 1 - \frac{x^*}{k} \right) \right\} \text{ by (5)} \\ &= \lambda q^2(p - \tau)x^*y^*E^* \cdot \frac{r}{k(b + cx^* + y^*)} (b + 2cx^* + y^* - ck) \\ &= \lambda q^2(p - \tau)x^*y^*E^* \cdot \frac{r}{k(b + cx^* + y^*)} \sqrt{(b + y^* - ck)^2 + 4ck \left( b + y^* - \frac{ay^*}{r} \right)} \text{ by (9)} \\ &> 0, \text{ since } r > a. \end{aligned}$$

$$\begin{aligned} \text{Now, } m_1m_2 - m_3 &= \left( Aex^*y^* + \frac{rx^*}{k} - Aacx^*y^* \right) \times \\ &\left\{ A^2(abcex^{*2}y^* + abex^*y^{*2} + ab^2ex^*y^*) + \frac{1}{k} \cdot Arex^{*2}y^* + \lambda q^2(p - \tau)y^*E^* \right\} \end{aligned}$$

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$$\begin{aligned}
 &+ Aac\lambda q^2(p-\tau)x^*y^{*2}E^* - \frac{r}{k} \cdot \lambda q^2(p-\tau)x^*y^*E^* \\
 &= Aex^*y^*\{D + \lambda q^2(p-\tau)y^*E^*\} + D\left(\frac{rx^*}{k} - Aacx^*y^*\right) \\
 \text{where } D &= A^2(abcex^{*2}y^* + abex^*y^{*2} + ab^2ex^*y^*) + \frac{1}{k} \cdot Arex^{*2}y^* > 0 \\
 &= Aex^*y^*\{D + \lambda q^2(p-\tau)y^*E^*\} + Dx^*\left\{\frac{r}{k} - Aacy^*\right\} \\
 &= Aex^*y^*\{D + \lambda q^2(p-\tau)y^*E^*\} + Dx^*\left\{\frac{r}{k} - \frac{cr}{b+cx^*+y^*}\left(1 - \frac{x^*}{k}\right)\right\} \text{ by (5)} \\
 &= Aex^*y^*\{D + \lambda q^2(p-\tau)y^*E^*\} + Dx^* \cdot \frac{r}{k(b+cx^*+y^*)}(b + 2cx^* + y^* - ck) \\
 &= Aex^*y^*\{D + \lambda q^2(p-\tau)y^*E^*\} \\
 &\quad + \frac{Drx^*}{k(b+cx^*+y^*)} \sqrt{(b + y^* - ck)^2 + 4ck\left(b + y^* - \frac{ay^*}{r}\right)} \text{ by (9)} \\
 &> 0.
 \end{aligned}$$

Therefore, by Routh-Hurwitz stability criterion [30], the non-trivial interior equilibrium point  $P(x^*, y^*, E^*)$  of the dynamical system (3) is always locally asymptotically stable, if it exists.

## VI. GLOBAL STABILITY ANALYSIS

The dynamical system (3) has a unique positive non-trivial equilibrium point  $P(x^*, y^*, E^*)$  provided the condition (11) together with the sufficient conditions (7) and (10) hold. For the limited environmental carrying capacity the prey species must be bounded in the range  $0 < x < k$ . Due to natural mortality, effects of harvesting and crowding effects the predator species has an upper bound. Since the regulatory agencies control the over exploitation by imposing a suitable tax, so the effort level is also bounded. So the solutions of the dynamical system (3) are uniformly bounded in the region  $R_3^+$ . We now examine the global stability of the non-trivial interior equilibrium point  $P(x^*, y^*, E^*)$  of the system (3). For this let us consider the following Lyapunov function [31]

$$L(x, y, E) = x - x^* - x^* \ln\left(\frac{x}{x^*}\right) + L_1\left\{y - y^* - y^* \ln\left(\frac{y}{y^*}\right)\right\} + L_2\left\{E - E^* - E^* \ln\left(\frac{E}{E^*}\right)\right\}$$

where  $L_1$  and  $L_2$  are suitable positive constants to be determined in the subsequent steps.

Here  $L(x^*, y^*, E^*) = 0$  and  $\lim_{(x,y,E) \rightarrow (0,0,0)} L(x, y, E) = \lim_{(x,y,E) \rightarrow (\infty, \infty, \infty)} L(x, y, E) = \infty$ .

The time derivative of  $L(x, y, E)$  along the solution of (3) is

$$\begin{aligned}
 \frac{dL}{dt} &= \frac{x-x^*}{x} \frac{dx}{dt} + L_1 \frac{y-y^*}{y} \frac{dy}{dt} + L_2 \frac{E-E^*}{E} \frac{dE}{dt} \\
 &= (x-x^*) \left\{ r \left(1 - \frac{x}{k}\right) - \frac{ay}{b+cx+y} \right\} + L_1 (y-y^*) \left\{ \frac{ex}{b+cx+y} - d - qE \right\} + \lambda L_2 (E-E^*) \{q(p-\tau)y - C\} \text{ by (3)} \\
 &= (x-x^*) \left\{ r \left(1 - \frac{x}{k}\right) - \frac{ay}{b+cx+y} - r \left(1 - \frac{x^*}{k}\right) + \frac{ay^*}{b+cx^*+y^*} \right\} + L_1 (y-y^*) \left\{ \frac{ex}{b+cx+y} - d - qE - \frac{ex^*}{b+cx^*+y^*} + d + qE^* \right\} \\
 &\quad + \lambda L_2 (E-E^*) \{q(p-\tau)y - C - q(p-\tau)y^* + C\} \\
 &= (x-x^*) \left\{ -\frac{r}{k}(x-x^*) - \frac{a(by-by^*+cx^*y-cxy^*)}{(b+cx+y)(b+cx^*+y^*)} \right\} \\
 &\quad + L_1 (y-y^*) \left\{ \frac{ebx+exy^*-bex^*-ex^*y}{(b+cx+y)(b+cx^*+y^*)} - q(E-E^*) \right\} + \lambda q L_2 (p-\tau)(y-y^*)(E-E^*) \\
 &= -\frac{r}{k}(x-x^*)^2 - \frac{ab(x-x^*)(y-y^*)}{(b+cx+y)(b+cx^*+y^*)} - \frac{ac(x-x^*)(x^*y-xy^*)}{(b+cx+y)(b+cx^*+y^*)} + L_1 \frac{eb(x-x^*)(y-y^*)}{(b+cx+y)(b+cx^*+y^*)} \\
 &\quad + L_1 \frac{e(y-y^*)(xy^*-x^*y)}{(b+cx+y)(b+cx^*+y^*)} + \lambda q L_2 (p-\tau)(y-y^*)(E-E^*) - L_1 q (y-y^*)(E-E^*) \\
 &= -\frac{r}{k}(x-x^*)^2 - \frac{ac(x-x^*)(x^*y-xy^*)}{(b+cx+y)(b+cx^*+y^*)} - \frac{a(y-y^*)(x^*y-xy^*)}{(b+cx+y)(b+cx^*+y^*)} \text{ for } L_1 = \frac{a}{e} > 0 \text{ and } L_2 = \frac{a}{e\lambda(p-\tau)} > 0.
 \end{aligned}$$

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$$= -\frac{r}{k}(x-x^*)^2 - \frac{a(x^*y-xy^*)}{(b+cx+y)(b+cx^*+y^*)} \{c(x-x^*) + (y-y^*)\}$$

$< 0 \forall (x, y, E) \in \Omega_1 \cup \Omega_2$  which is a subset of  $R^3$ .

$$\Omega_1 = W_1 \cap W_2$$

where  $W_1 = \{(x, y, 0) : y \geq \frac{y^*}{x^*}x, x > 0, y > 0\}$  and

$$W_2 = \{(x, y, 0) : cx + y \geq cx^* + y^*, x > 0, y > 0\}.$$

$$\Omega_2 = W_3 \cap W_4$$

where  $W_3 = \{(x, y, 0) : y \leq \frac{y^*}{x^*}x, x > 0, y > 0\}$  and

$$W_4 = \{(x, y, 0) : cx + y \leq cx^* + y^*, x > 0, y > 0\}$$

Also  $\frac{dL}{dt} = 0$  at  $P(x^*, y^*, E^*)$ .

Hence by Lassaile's invariance principle [32],  $P(x^*, y^*, E^*)$  is globally asymptotically stable for all  $(x, y, E) \in W_1 \cap W_2$ .

## VII. NUMERICAL EXAMPLE

Let  $r = 5, k = 100, a = 0.3, b = 1, c = 2, d = 0.25, e = 4, q = 0.005, p = 25, C = 10$  and  $\lambda = 1$  in appropriate units.

The necessary condition,  $e - cd > 0$  for  $E^* > 0$ , is satisfied.

From (11) we have  $B = 154.556$ .

By (12) we have,  $\tau < p - \frac{c}{Bq} = 12.06$ .

Therefore, it is sufficient that, if the regulatory agencies impose the tax  $\tau$  such that  $0 < \tau < 12.06$ , then the non-trivial interior equilibrium point  $P(x^*, y^*, E^*)$  of the dynamical system (3) exists and always locally asymptotically stable. This steady state is also globally asymptotically stable in the region  $\Omega_1 \cup \Omega_2$  which is a subset of  $R^3$ .

For different values of tax  $\tau \in [0, 12]$ , the different steady states of the system (3) are given in the following table:

**Table-1:** Steady states for different taxes

Tax $\tau$	Steady states		
	$x^*$	$y^*$	$E^*$
0	98.271	80.000	233.261
1	98.219	83.333	229.855
2	98.165	86.957	226.242
3	98.107	90.910	222.402
4	98.045	95.238	218.315
5	97.980	100.000	213.954
6	97.909	105.263	209.292
7	97.834	111.111	204.297
8	97.753	117.647	198.931
9	97.666	125.000	193.153
10	97.572	133.333	186.914
11	97.470	142.857	180.156
12	97.359	153.846	172.812



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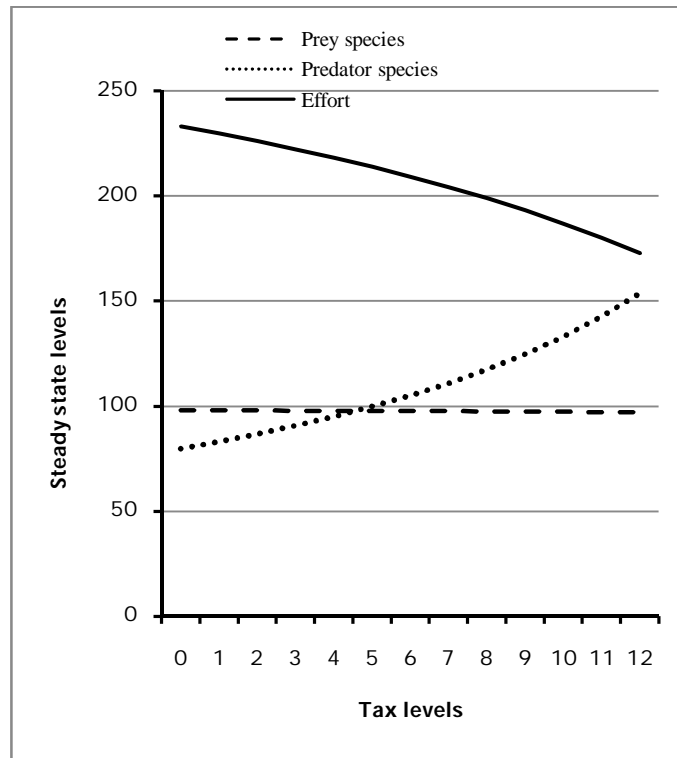


Fig.-1: Steady states for different tax levels

From table-1, we see that when the tax level increases, the steady state level of effort gradually decreases and as a result the steady state level of predator species gradually increases. Due to increase of the steady state level of predator species, the steady state level of prey species slightly decreases and this steady state is very close to the environmental carrying capacity of the prey species. Since the prey species is not harvested so the steady state level of prey species is independent of the effort but slightly depends on the steady state of predator species. So in order to keep the environmental balance between the prey and predator, the regulatory agencies should impose the tax  $\tau$  such that  $0 < \tau \leq 12$ .

From figure-1, it is also clear that the steady state level of the prey species always remains very close to the environmental carrying capacity. Whereas the steady state levels of predator species gradually increases and the steady state level of effort level gradually decreases with the increase of tax level.

## VIII. CONCLUSION

In the selective harvesting of an exploited fishery with Beddington-DeAngelis functional response, if the prey species is not allowed for harvesting and the over exploitation of the predator species is controlled by imposing a suitable tax per unit harvested bio mass of the predator fish species, the non trivial interior equilibrium point of the system always exists under a sufficient condition on the tax limits. This non trivial interior equilibrium point is always locally asymptotically stable and globally asymptotically stable in an unbounded region. Pradhan T. [5] discussed the dynamical behaviour of an exploited prey-predator fishery with Beddington-DeAngelis [1,2] functional response by considering the constant harvesting effort on the basis of CPUE hypothesis and both species are subjected to harvesting. But in this model the harvesting effort is considered as dynamic variable and only the predator species is allowed for harvesting.

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