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A Note on Soft Nearly Compact and Soft Nearly Paracompactness in Soft Topological Spaces

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ABSTRACT: In 1999, the concept of a soft set was introduced by D. Molodtsov [1]. Soft topological Space is considered as Mathematical tools for dealing with uncertainties, and a fuzzy topological space is a special case of the soft topological space. Xiao et al. [17] and Pei and Miao [18] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. So it is important to study the structures of soft sets for information systems. Shabir and Naz [2], studied the topological structures of soft sets. The aim of this paper is to study soft topological spaces. I introduce some new notions in soft space such as soft nearly compact spaces, soft nearly paracompact spaces, soft α -nearly paracompact spaces and soft almost continuous mappings. We also investigate some basic properties of these concepts and obtain several interesting results and characterizations of soft nearly compact and nearly paracompact spaces.

KEYWORDS: Soft almost regular spaces, soft almost normal spaces, soft semi-regular spaces, soft paracompact spaces, soft nearly compact spaces, soft nearly paracompact spaces and almost continuous mappings etc.

I. INTRODUCTION

Several theories, such as the theory of fuzzy sets [19] and theory of rough sets [20] can be considered as mathematical tools for dealing with uncertainties. These theories have their inherent difficulties as pointed out in [1]. The reason for these difficulties is, possibly, the inadequency of the parametrization tool of the theories. In the recent years, papers about soft sets theory and their applications in various fields have been writing increasingly. In 1999, Molodtsov [1] initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann integration, theory of measurement, and so on. Soft set theory and its applications have shown great development in recent years. This is because of the general nature of parametrization expressed by a soft set. Shabir and Naz [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna et al.[3], Aygunoglu and Aygun [4] and Hussain et al are continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [5]. M.K. Singal and A. Mathur, [21] introduced nearly compact spaces in topological spaces. In 1969, M.K. Singal and S.P. Arya [22] introduced the notion of nearly paracompact Spaces in topological spaces and studied their some properties. F. Lin [23] introduced the notion of soft connected sets and soft paracompact spaces and explored some of their basic properties. In this work, the notion of soft nearly compact spaces, soft nearly paracompact spaces, soft α -nearly paracompact spaces and soft almost continuous mappings are introduced in Soft Topological Spaces. We also investigate some basic properties of these concepts with example and counter example. Moreover, several interesting results and characterizations of soft nearly compact and nearly paracompact spaces are explored.

II. PRELIMINERIES

Throughout the paper, the space X and Y stand for soft topological spaces with (X,τ, E) and (Y,ν, K) assumed unless otherwise stated. Moreover, throughout this paper, a soft mapping $f : X \rightarrow Y$ stands for a mapping, where $f : (X,\tau, E) \rightarrow (Y,\nu, K)$, $u : X \rightarrow Y$ and $p : E \rightarrow K$ are assumed mappings unless otherwise stated.



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Definition: 2.1[1]: Let X be an initial universe and E be a set of parameters. Let P(X) denotes the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by F: $A \rightarrow P(X)$ defined by $F(e) \in P(X) \quad \forall e \in A$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, F(e) may be considered as the set of e-approximate elements of the soft set (F,A).

Definition 2.2[11]: A soft set (F,A) over X is called a null soft set, denoted by $\tilde{\varphi}$, if $e \in A, F(e) = \varphi$.

Definition 2.3[11]: A soft set (F,A) over X is called an absolute soft set, denoted by \tilde{A} , if $e \in A$, F(e)=X. If A=E, then the A-universal soft set is called a universal soft set, denoted by \tilde{X} .

Theorem 2.4[2]: Let Y be a non-empty subset of X, then \tilde{Y} denotes the soft set (Y,E) over X for which Y(e)=Y, for all $e \in E$.

Definition 2.5 [11]: The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C=A\cup B$ and for all $e\in C$,

$$F(e), if e \in A - B$$

$$H(e) = \begin{cases} G(e), & if e \in B - A \end{cases}$$

 $F(e) \cup G(e), if e \in A \cap B$

We write $(F,A) \widetilde{\cup} (G,B) = (H,C)$

Definition 2.6 [11]: The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X, denoted by $(F,A) \cap (G,B)$, is defined as C=A \cap B and H(e)= F(e) \cap G(e) for all $e \in C$.

Definition 2.7[2]: Let (F,A) be a soft set over a soft topological space (X,τ,E) . We say that $x \in (F,E)$ read as x belongs to the set (F,E) whenever $x \in F(e)$ for all $e \in E$. Note that for any $x \in X$, $x \notin (F,E)$, if $x \notin F(e)$ for some $e \in E$.

Definition 2.8 [11]: Let (F,A) and (G,B) be two soft sets over a common universe X. Then (F,A) $\subseteq (G,B)$ if $A \subseteq B$, and $F(e) \subseteq G(e)$ for all $e \in A$.

Definition 2.9 [2]: Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if it satisfies the following axioms.

(1) $\tilde{\varphi}$, X belong to τ . (2) The union of any number of soft sets in τ belongs to τ .

(3) The intersection of any two soft sets in τ belongs to τ .

The triplet (X,τ,E) is called a soft topological space over X. Let (X,τ,E) be a soft topological space over X, then the members of τ are said to be soft open sets in X. A soft set (F,A) over X is said to be a soft closed set in X, if its relative complement $(F,A)^c$ belongs to τ .

Definition 2.10[12]: For a soft set (F,A) over X, the relative complement of (F,A) is denoted by (F,A)^c and is defined by (F,A)^c = (F^c,A), where $F^c : A \rightarrow P(X)$ is a mapping given by $F^c(e) = X \cdot F(e)$, for all $e \in A$.

Definition 2.1[2]: Let X be an initial universal set and A be the non-empty set of parameters. Then A soft set (P,A) over X is said to be *soft point* if there is exactly one $\lambda \in A$, such that $P(\lambda) = \{x\}$ for some $x \in X$ and $P(\mu) = \varphi$, $\forall \mu \in A \setminus \{\lambda\}$.

It will be denoted by $P_{\frac{x}{2}}^{x}$.

Definition 2.11: A soft set (F,A) in a soft topological space X is called

(i) soft regular open (resp.soft regular closed) set [13] if (F,A) = Int(Cl(F,A)) [resp. (F,A) = Cl(Int(F,A))].

(ii) soft semi-open (resp.soft semi-closed) set [5] if $(F,A) \subseteq Cl(Int(F,A))$ [resp. $Int(Cl(F,A)) \subseteq (F,A)$].

(iii) soft pre-open (resp.soft pre-closed)[13] if $(F,A) \subseteq Int(Cl(F,A))$ [resp. $Cl(Int(F,A)) \subseteq (F,A)$.

(iv) soft α -open (resp.soft α -closed)[13] if (F,A) \subseteq Int(Cl(Int(F,A))) [resp. Cl(Int(Cl(F,A))) \subseteq (F,A)].

(v) soft β -open (resp.soft β -closed) set[13] if (F,A) \subseteq Cl(Int(Cl(F,A))) [resp. Int(Cl(Int(F,A))) \subseteq (F,A)].

(vi) soft γ -open (resp.soft γ -closed) set [6] if (F,A) \subseteq [Int(Cl(F,A)) $\widetilde{\cup}$ Cl(Int(F,A))] [resp. Int(Cl(F,A)) $\widetilde{\cap}$ Cl(Int(F,A)) $\widetilde{\subset}$ (F,A)].



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III. SOFT NEARLY COMPACT AND SOFT NEARLY PARACOMPACT SPACES

In this section soft almost normal, soft almost regular and soft nearly paracompact spaces are defined. Their characterizations and relationships among these notions are also established.

Definition 3.1: A space (X,τ, E) is said to be soft almost normal (briefly, SAN) iff every pair of disjoint sets (F,A) and (G,A), one of which is soft closed and the other is soft regular closed, there exist disjoint soft open sets (U,A) and (V,A) such that (F,A) \subseteq (U,A), (G,A) \subseteq (V,A).

Definition 3.2: A space (X,τ, E) is said to be soft almost regular (briefly, SAR) iff for any soft regular closed set (F,A) and any soft point $P_{\lambda}^{x} \notin (F,A)$, there exist disjoint soft open sets containing (F,A) and P_{λ}^{x} respectably.

Definition 3.3: A soft topological space X is called soft nearly paracompact (briefly, SAP) if every soft regularly open covering admits a locally finite soft open refinement. A subset (U,A) is called soft nearly paracompact if the relative topology defined on it is soft nearly paracompact.

Definition 3.4: For any soft topological space (X, τ, E) , the family of all soft open sets which can be written as the union of an arbitrary number of soft regularly open sets of *X* is a topology on (X, τ, E) , weaker than the initial topology τ on *X* and denoted by τ_s . This τ_s is called soft semi-regular topology and members of τ_s are called soft semi-regular open set.

Lemma 3.1: Let (X,τ, E) be a soft almost normal space and $A \subseteq E$. If $U = \{(U,A)_{\alpha} : \alpha \in I, \text{ finite set}\}$ is any soft regular open locally finite cover of the space X, there is soft open locally finite refinement $V = \{(V,A)_{\alpha} : \alpha \in I\}$ such that $Scl(V,A)_{\alpha} \subseteq (U,A)_{\alpha}$, for all $\alpha \in I$.

Proof: Let (X, τ, E) be a soft almost normal space and $U = \{(U,A)_{\alpha} : \alpha \in I, \text{ finite set}\}$ is any soft regular open locally finite cover of the space X. Then $\tilde{X} = \bigcup_{\alpha} \{(U,A)_{\alpha} : \alpha \in J, a \text{ finite set}\}$. That is $\varphi = \tilde{X} \setminus \bigcup_{\alpha} \{(U,A)_{\alpha} : \alpha \in J\} = \bigcap_{\alpha} \{\tilde{X} \setminus (U,A)_{\alpha} : \alpha \in J\}$, so that $\tilde{X} \setminus (U,A)_1$ and $\tilde{X} \setminus (U,A)_2 \cap \tilde{X} \setminus (U,A)_3 \cap \dots \cap \tilde{X} \setminus (U,A)_n$ form a pair of disjoint sets of which $\tilde{X} \setminus (U,A)_1$ soft closed set(Since, every Soft regular open set is soft closed) and $\tilde{X} \setminus (U,A)_2 \cap \tilde{X} \setminus (U,A)_3 \cap \dots \cap \tilde{X} \setminus (U,A)_n$ is soft regular closed sets in X. As (X, τ, E) be a soft almost normal space, there exists two open sets $(V,A)_1$ and $(H,A)_1$ such that $\tilde{X} \setminus (U,A)_1 \subseteq (V,A)_1$, $\tilde{X} \setminus (U,A)_2 \cap \tilde{X} \setminus (U,A)_3 \cap \dots \cap \tilde{X} \setminus (U,A)_1$ and $(V,A)_1 \cap (H,A)_1 = \varphi$. Then $(H,A)_1 \subseteq \tilde{X} \setminus (V,A)_1$ so that $Scl((H,A)_1) \subseteq Scl(\tilde{X} \setminus (V,A)_1) \subseteq (H,A)_1$ and $(V,A)_1 \cap (H,A)_1 = \varphi$. Then $(H,A)_1 \subseteq \tilde{X} \setminus (V,A)_1$ so that $Scl((H,A)_1) \subseteq Scl(\tilde{X} \setminus (V,A)_1) \subseteq (U,A)_1 \cap \tilde{X} \setminus (U,A)_2 \cup (U,A)_3 \cup \dots \cup (U,A)_n \} = \tilde{X} \setminus (U,A)_2 \cap \tilde{X} \setminus (U,A)_3 \cap \dots \cap \tilde{X} \setminus (U,A)_1 \cap \tilde{X} \setminus (U,A)_2 \cup (U,A)_3 \cup \dots \cup (U,A)_n \} = \tilde{X} \setminus (U,A)_2 \cap \tilde{X} \setminus (U,A)_3 \cap \dots \cap \tilde{X} \setminus (U,A)_1 \cap \tilde{X} \setminus (U,A)_2 \cup (U,A)_3 \cup \dots \cup (U,A)_n \} = \varphi$. Thus $\{(H,A)_1, U(A)_1, U(A)_1, U(A)_2, U(A)_1, U(A)_2, U(A)_1, U(A)_2 \cap (U,A)_2 \cap (U,A)_1 \otimes (U,A)_1 \cap \tilde{X} \setminus (U,A)_2 \cup (U,A)_3 \cup \dots \cup (U,A)_n \} = \tilde{X} \setminus (U,A)_2 \cap \tilde{X} \setminus (U,A)_3 \cap \dots \cap \tilde{X} \setminus (U,A)_n \cap \tilde{X} \setminus (U,A)_2 \cup (U,A)_3 \cup \dots \cup (U,A)_n \} = \varphi$. Thus $\{(H,A)_1, (U,A)_2, (U,A)_3, \dots, (U,A)_n\}$ forms an soft open covering of X, where $Scl((H,A)_1) \cap (U,A)_2 \cap (H,A)_3, \dots, (H,A)_n$, Thus we obtain a soft open locally finite refinement $V = \{(H,A)_\alpha : \alpha = 1, 2, 3, \dots, A\}$ of the given soft open covering U.

Theorem 3.1: For an soft almost normal space X, the followings are equivalent:

- (a) X is soft nearly Paracompact
- (b) Every soft regular open cover of X has a soft regular open locally finite refinement.
- (c) Every soft regular open cover of X has a locally finite, soft closed refinement.



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(d) Every soft regular open cover of X has a locally finite, soft regularly closed refinement. **Proof:** (a) \rightarrow (b): X is soft nearly Paracompact and let U = { $(U,A)_{\alpha}$: $\alpha \in \Lambda$, infinite set} be any soft regular open covering of X. Then there exists locally finite soft regular open refinement V of U. Let us suppose that C = { $(V,A)_{\beta}$: $(V,A)_{\beta} \in V$ }. Then C is a locally finite soft regular open refinement of U.

(b) \rightarrow **(c):** It follows easily from lemma 3.1.

(c) \rightarrow (d): It follows easily from lemma 3.1

(d) \rightarrow (a): Let U = {(U,A)_{α}: $\alpha \in \Lambda$, infinite set}be any soft regular open cover of X. Then there exists a locally finite soft regular closed refinement V of U. Since the family V is locally finite, for each P_{λ}^{x} of X, there exists an soft open set (O,A) such that P_{λ}^{x} of (O,A) and (O,A) intersects finitely many members of V. Then α (O,A) is a soft regular open set such that P_{λ}^{x} of α (O,A), which intersects finitely many members of V.

Now consider the soft regular open covering $C = \{\alpha(O,A): P_{\lambda}^{x} \in X\}$ of X. Then there exists a locally finite soft regular closed refinement C^{r} of C. Let $(V,A)^{*} = X \setminus \{(G,A): (G,A) \in C^{r}, (G,A) \cap (V,A) = \phi\}$.

Clearly, $(V,A)^*$ is soft open and contains (V,A). Moreover, $(G,A) \cap (V,A)^* \neq \varphi \Leftrightarrow (G,A) \cap (V,A) \neq \varphi$. Since V is a refinement of U, for each $(V,A) \in V$, there exists a $(U,A)_{\nu}$ such that $(V,A) \subseteq (U,A)_{\nu}$. Let $W = \{(U,A)_{\nu} \cap (V,A)^* : (V,A) \in V\}$. W is an soft open, locally finite refinement of U, thus X is soft nearly Paracompact.

Definition 3.5: Let $U = \{(U,A)_{\alpha} : \alpha \in \Lambda, \text{ infinite set}\}$ be a family of soft subsets of a space X. The star of a soft point P_{λ}^{x}

of X, SSt(P_{λ}^{x} ,U), in U defined to be the union of all members of U which contains P_{λ}^{x} . A family U of subsets of a space X is said to be a soft star refinement of another family V of subsets of X iff the family of all stars of soft points of X in U forms a covering of X which refines U.

Theorem 3.2: Let X be a soft almost normal nearly paracompact space. Then every soft regular open covering of X has a soft regular open star refinement.

Proof: Let $U = \{(U,A)_{\alpha} : \alpha \in \Lambda$, infinite set} be any soft regular open covering of X. Since X is soft almost normal and soft nearly paracompact space, there exists, by Theorem 3.1, a locally finite, soft regular closed refinement $V = \{(V,A)_{\beta} : \beta \in \Lambda$, infinite set} of U. For each $(V,A)_{\beta} \in V$, there exists a $(U,A)_{\nu} \in U$ for some $v \in \Lambda$, such that $(V,A)_{\beta} \subseteq (U,A)_{\nu}$. For each soft point $P_{\lambda}^{x} \in \tilde{X}$, let us suppose that $(F,A)_{x} = \tilde{\cap} \{(U,A)_{\nu} : P_{\lambda}^{x} \in (V,A)_{\beta} \in V\} \in \tilde{\cap} \{(V,A)_{\beta}^{c} : P_{\lambda}^{x} \notin (V,A)_{\beta} \in V\}$. Since V is locally finite family of soft regularly closed sets and every subfamily of locally finite family is locally finite, it follows that $\bigcup_{\beta} \{(V,A)_{\beta} : P_{\lambda}^{x} \notin (V,A)_{\beta} \in V\}$. Therefore, $\bigcap_{\beta} \{(V,A)_{\beta}^{c} : P_{\lambda}^{x} \notin (V,A)_{\beta} \in V\} = \tilde{X} \setminus \bigcup_{\beta} \{(V,A)_{\beta} : P_{\lambda}^{x} \notin (V,A)_{\beta} \in V\}$ is soft regularly open. Also, being locally finite, each soft point of X can belong to at most a finite number of V and therefore the intersection of finitely many soft regularly open sets being regularly open, it follows that $\widetilde{\cap} \{(U,A)_{\nu} : P_{\lambda}^{x} \in (V,A)_{\beta} \in V\}$ is soft regularly open. Thus each $(F,A)_{x}$ is soft regularly open set. Now consider the family $C = \{(F,A)_{x} : P_{\lambda}^{x} \notin (V,A)_{\beta} \in V\}$ the a soft point of X. Then either $P_{\lambda}^{x} \in (V,A)_{\beta} \in V$ or $P_{\lambda}^{x} \notin (V,A)_{\beta} \in V$. Thus in any case, $P_{\lambda}^{x} \in (F,A)_{x}$, and therefore C is a soft regularly open covering of X. We shall prove that C is a soft star refinement of U. Let P_{μ}^{x} be a fixed soft point of X. Since V covers X,



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there exists $(V,A)_{\beta} \in V$ such that $P_{\mu}^{r} \in (V,A)_{\beta}$. We shall show that $SSt(P_{\lambda}^{x}, \mathbb{C}) \subset (U,A)_{\nu}$. Let $(F,A)_{x}$ be any member of \mathbb{C} which contains P_{μ}^{r} . If $P_{\lambda}^{x} \notin (V,A)_{\beta}$, $P_{\mu}^{r} \in (F,A)_{x} \subset \tilde{X} \setminus \bigcup_{\beta} \{(V,A)_{\beta}\}$ by definition of $(F,A)_{x}$. But this is a

contradiction. So, must belongs to $P_{\lambda}^{x} \in (V,A)_{\beta}$, and then $(F,A)_{x} \subset (U,A)_{y}$. Thus every member of C contains P_{μ}^{r} ,

contained in (U,A)_v. Therefore, SSt(P_{λ}^{x} , C) $\tilde{\subset}$ (U,A)_v. Hence, C is soft regular open star refinement of soft regular open covering U.

Remark 3.1: In Theorem 3.1 and 3.2 if we replace soft almost normal with soft almost regular, the corresponding results hold as well. But in general, soft almost normal soft nearly paracompact space is not necessarily soft almost regular which can be seen from the following example:

Example 3.1: Let X={x₁, x₂, x₃, x₄}, E ={e₁, e₂, e₃} and $\tau = \{\tilde{\varphi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E), (F_{11}, E), (F_{12}, E), (F_{13}, E)\}$ where, (F₁, E), (F₂, E), (F₃, E), (F₄, E), (F₅, E), (F₆, E), (F₇, E), (F₁₀, E), (F_{10}, E), (F_{11}, E), (F_{12}, E), and (F_{13}, E) are soft sets over X, defined as follows:

 $(F_1,E) = \{(e_1,\{x_1\}),(e_2,\{x_2,x_3\}),(e_3,\{x_1,x_3\})\}; (F_2,E) = \{(e_1,\{x_2,x_4\}),(e_2,\{x_1,x_3,x_4\}),(e_3,\{x_1,x_2,x_4\})\}; (F_3,E) = \{(e_1,\phi),(e_2,\{x_3\}),(e_3,\{x_1\})\}; (F_4,E) = \{(e_1,\{x_1,x_2,x_4\}),(e_2,X),(e_3,X)\}; (F_5,E) = \{(e_1,\{x_1,x_3\}),(e_2,\{x_2,x_4\}),(e_3,\{x_2\})\}; (F_6,E) = \{(e_1,\phi),(e_2,\{x_2\}),(e_3,\phi)\}; (F_7,E) = \{(e_1,\{x_1,x_3\}),(e_2,\{x_2,x_3,x_4\}), (e_3,\{x_1,x_2,x_3\})\}; (F_8,E) = \{(e_1,\{x_3\}),(e_2,\{x_4\}),(e_3,\{x_2\})\}; (F_9,E) = \{(e_1,X),(e_2,X),(e_3,\{x_1,x_2,x_3\})\}; (F_1,E) = \{(e_1,\{x_3\}),(e_2,\{x_2,x_4\}),(e_3,\{x_1,x_2,x_3\})\}; (F_1,E) = \{(e_1,\{x_3\}),(e_2,\{x_2,x_4\}),(e_3,\{x_2\})\}; (F_1,E) = \{(e_1,\{x_1\}),(e_2,\{x_2,x_4\}),(e_3,\{x_1\})\}; (F_1,E) = \{(e_1,\{x_1\}),(e_2,\{x_2,x_4\}),(e_3,\{x_1\})\}\}.$

Then (X,τ, E) is a soft almost normal soft nearly paracompact space. But (X,τ, E) is not soft almost regular as $(F_6, E)^c = \{(e_1, X), (e_2, \{x_1, x_3, x_4\}), (e_3, X)\}$ is soft regular closed set not containing the soft point P_{e2}^{x2} there exist soft open sets

 \tilde{X} and $(F_6,E) = \{(e_1,\phi), (e_2,\{x_2\}), (e_3,\phi)\}$ containing respectably $(F_6, E)^c$ and P_{e2}^{x2} such that $\tilde{X} \cap (F_6,E) \neq \tilde{\varphi}$.

Definition 3.6: A soft open cover $U = \{(U,A)_{\alpha} : \alpha \in \Lambda, \text{ indexing set}\}$ is said to be soft δ -even iff there exist a soft semiregular open neighborhood (V,A) of the diagonal in the soft product space(X×X, τ_p ,E) (where, τ_p is the soft product topology) such that for each $P_{\lambda}^x \in \tilde{X}$, $(V,A)[P_{\lambda}^x] \subset (U,A)$ for some $(U,A) \in U$ [where, $(V,A)[P_{\lambda}^x] = \{P_{\lambda}^y : (P_{\lambda}^x, P_{\lambda}^y) \in (V,A)\}$].

Theorem 3.3: Let X be any soft almost normal soft nearly paracompact space. Then every soft regular open covering is soft δ-even.

Proof: Let $U = \{(U,A)_{\alpha} : \alpha \in \Lambda, \text{ infinite set}\}$ be any soft regular open covering of X. $V = \{(V,A)_{\beta} : \beta \in \Lambda\}$ be any soft regular open star refinement of U. Let $(V,A) = \bigcup_{\beta} \{(V,A)_{\beta} \times (V,A)_{\beta} : \beta \in \Lambda\}$. Then (V,A) is soft semi-regular open

neighborhood of the diagonal in X×X. We shall prove that, for each soft point P_{λ}^{x} of X, $(V,A)[P_{\lambda}^{x}] \subset (U,A)_{\alpha}$ for some $\alpha \in \Lambda$. If $P_{\lambda}^{y} \in (V,A)[P_{\lambda}^{x}]$, then $(P_{\lambda}^{x}, P_{\lambda}^{y}) \in (V,A)$ and hence there exists $\beta \in \Lambda$ such that $(P_{\lambda}^{x}, P_{\lambda}^{y}) \in (V,A)_{\beta} \times (V,A)_{\beta}$. Then $P_{\lambda}^{x} \in (V,A)_{\beta}$ and $P_{\lambda}^{y} \in (V,A)_{\beta}$. Hence $P_{\lambda}^{y} \in SSt(P_{\lambda}^{x}, V)$. Then $(V,A)[P_{\lambda}^{x}] \subset SSt(P_{\lambda}^{x}, V)$. since V is a soft star refinement of U, therefore there exists a $\alpha \in \Lambda$ such that $(V,A)[P_{\lambda}^{x}] \subset SSt(P_{\lambda}^{x}, V) \subset (U,A)_{\alpha}$.

Definition 3.7: A space (X,τ, E) is said to be soft nearly Lindelof iff every soft regular open covering of X has a countable soft subcovering. **Lemma 3.2:** Let X be any soft almost regular soft nearly paracompact space. Then every soft regular open cover U =

 $\{(U,A)_{\alpha} : \alpha \in \Lambda, \text{ infinite set}\}$ admits a soft regular open locally finite refinement $V = \{(V,A)_{\alpha} : \alpha \in \Lambda\}$ such that Scl(V,A) $\alpha \subset (U,A)_{\alpha}$ for each $\alpha \in \Lambda$.

Proof: Let $U = \{(U,A)_{\alpha} : \alpha \in \Lambda\}$ be any soft regular open cover of X. By soft almost regularity, there exists a soft



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regular open refinement P of U such that for any soft set $(P,A) \in P$, $Scl(P,A) \subset (U,A)_{\alpha}$ for some $\alpha \in \Lambda$. Since X is soft nearly paracompact, there exists a soft regular open refinement $Q = \{(Q,A)_{\beta} : \beta \in J, a \text{ finite set}\}$ of P. For every, $\beta \in J$, we choose $\kappa \in \Lambda$ such that $Scl(Q,A)_{\beta} \subset (U,A)_{\kappa}$. Let $(G,A)_{\kappa} = \bigcup_{\kappa=\alpha} \{(P,A)_{\beta} : \beta \in J, a \text{ finite set}\} \subset (U,A)_{\alpha}$. Then $Scl(G,A)_{\kappa} = Scl\{\bigcup_{\kappa=\alpha} (P,A)_{\beta} : \beta \in J, a \text{ finite set}\} \subset (U,A)_{\alpha}$. Then Scl(G,A) is a soft regularly open locally finite soft refinement of U such that $Scl(G,A)_{\kappa} = (V,A)_{\alpha} \subset Scl(U,A)_{\alpha}$ for each $\alpha \in \Lambda$.

Theorem 3.4: Let X be any soft almost regular soft nearly paracompact space such that there exists a soft dense soft nearly Lindelof (that is soft nearly Lindelof set as a soft subspace of X) set (F,A). Then X is soft nearly Lindelof space. **Proof :** Let U = {(U,A)_{α} : $\alpha \in \Lambda$, infinite set} be any soft regular open covering of X. By preceding Lemma, there exists a soft regular open locally finite soft refinement V = {(V,A)_{$\alpha} : \alpha \in \Lambda$ } of U such that Scl((V,A)_{α}) \subset (U,A)_{α}, for each $\alpha \in \Lambda$. Then {(F,A) \cap (V,A)_{α} : $\alpha \in \Lambda$ } is relatively soft regular open covering of (F,A), by hypothesis there exists a countable set J \subset Λ such that (F,A) = \bigcup_{α} {(F,A) \cap (V,A)_{$\alpha} : <math>\alpha \in J$ }. Then X = Scl(F,A) = $\bigcup_{\alpha \in J}$ Scl(((F,A) \cap (V,A)_{α}) \subset \bigcup_{α} Scl(((V,A)_{α}), and thus X is soft nearly Lindelof space.</sub></sub>

 $\operatorname{Befinition}_{\alpha} 3 \mathfrak{e}$: A space X is said to be a soft α party Lindelof iff every soft regular open of

Definition 3.8: A space X is said to be a soft α -nearly Lindelof iff every soft regular open covering of X has admits a countable soft subcovering.

Theorem 3.5: Let X be soft locally nearly compact and soft nearly paracompact space. Then $X = \bigcup_{\alpha} (V,A)_{\alpha}$, where,

 $(V,A)_{\alpha}$ are pairwise soft disjoint and soft α -nearly Lindelof sets.

Proof: Let X be any soft locally nearly compact and soft nearly paracompact space. Since X is soft locally nearly compact, for each soft point P_{λ}^{x} of X, there exists a soft regular open set $(U,A)_{x}$, containing the soft point P_{λ}^{x} , such that $Scl((U,A)_{x})$ is soft α -nearly compact subset of X. Let us consider the soft regular open cover $U = \{(U,A)_{x} : P_{\lambda}^{x} \in X\}$ of the space X. Since X is soft nearly paracompact, then there exists a soft regular open locally finite soft refinement $V = \{(V,A)_{\alpha} : \alpha \in A\}$ of U. For $(V,A)_{0} \in V$, let, $P_{k}((V,A)_{0})$ be a set of all $(V,A)_{0} \in V$, such that there exists a sequence $(V,A)_{1}$, $(V,A)_{2}$,..., $(V,A)_{k}$ of members of V and $(V,A)_{k} = (V,A)$, $(V,A)_{i} \cap (V,A)_{i-1} = \varphi$. Let $P((V,A)_{0})$ $= \bigcup_{k=1}^{\infty} P_{k}((V,A)_{0})$ and $S((V,A)_{0}) = \bigcup P((V,A)_{0})$. Clearly, the family $P_{k}((V,A)_{0})$ is finite , and hence $K(P((V,A)_{0}) \in (P_{\lambda}^{x})_{0}$. For $(V,A)_{0}$, $(V,A)_{1} \in V$. $S((V,A)_{0})$, $S((V,A)_{1})$ are soft clopen sets which. $S((V,A)_{0}) \equiv S((V,A)_{1})$ or $S((V,A)_{0}) \cap S((V,A)_{1}) = \varphi$. Then $S((V,A)_{0}) = \bigcup_{\alpha} Scl(V,A)$, hence $S((V,A)_{0})$ is a soft α -nearly Lindelof subset, because $S((V,A)_{0})$ is a countable union of soft α -nearly compact subsets.

IV. SOFT SUBSETS, SOFT NEARLY COMPACT AND SOFT NEARLY PARACOMPACT SPACES.

In this section soft nearly compact and soft α -nearly paracompact spaces are defined and some of their properties and relationships among are also shown.

Definition 4.1: Let (X,τ,E) be a soft topological space and X will be called soft nearly compact (in short, soft N-compact) iff every soft open covering of X has finite subcollection, the soft interiors of soft closures of which covers X.



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Definition 4.2: Let (X,τ,E) be a soft topological space and (U,A) be a soft subset X. The soft set (U,A) is soft α -nearly paracompact (in short, soft α -N-paracompact) iff every soft regular open covering of (U,A), has an soft open refinement which covers (U, A) and is soft locally finite for every point in (X,τ,E) . The subset (U,A) is soft nearly paracompact (in short, soft N-paracompact) iff (U,A) is soft nearly paracompact (in short, soft N-paracompact) as a subspace.

Lemma 4.1: Let (U,A) be a soft α -nearly paracompact subset of a space (X, τ ,E). Then (U,A) is soft α -nearly paracompact subset of (X, τ ,E) if and only if (U,A) is soft α -paracompact subset of (X, τ ,E).

Proof: Let (U,A) be any soft α -nearly paracompact subset of (X,τ,E) . Let $C = \{(C,A)_{\gamma} : \gamma \in \Lambda\}$ be a soft basic τ_s covering of (U,A). Then, C is also a soft τ -regular open covering of (U,A). Hence there exists a soft regular open Xlocally finite refinement $V = \{(V,A)_{\beta} : \beta \in J\}$ of C, which covers (U,A). Hence, (U,A) is a soft α -paracompact subset of

 (X, τ_s, E) . Conversely, let $C = \{(C, A)_{\gamma} : \gamma \in \Lambda\}$ be soft τ - regular open covering of A. Then, $C = \alpha(C)$ for each $\gamma \in \Lambda$, and

 $\{\alpha(C): \gamma \in \Lambda\}$ is soft τ_s -open covering of (U,A), therefore there exists a soft X-locally finite soft τ_s -open refinement V of C, which covers the soft set (U,A). Hence (U,A) is a soft α -nearly paracompact subset of the space (X, τ ,E).

Theorem 4.1: Every soft semi regular closed subset (U,A) of a soft nearly paracompact space (X, τ ,E) is soft α -nearly paracompact space, where, A \subset E.

Proof: Let (U,A) be any soft semi regular closed subset of a nearly paracompact space (X,τ,E) . Then (X, τ_s, E) is soft

paracompact. Thus (U,A) is soft α -paracompact subset of (X, τ_s ,E). Hence by preceding lemma 4.1, (U,A) is soft α -nearly paracompact subset of the space (X, τ ,E).

Theorem 4.2: Let (X,τ,E) be a soft topological space and $A \subset E$. Every soft regular closed subset (B,A) of a soft α -nearly paracompact subset (C,A) is soft α -nearly paracompact space.

Proof: Let (C,A) be any soft α -nearly paracompact subset, (B,A) a soft regular closed subset and (B,A) \subset (C,A). Let U = {(U,A)_{α}: $\alpha \in \Lambda$, indexing set}be any soft regular open covering of (B,A). Then U \cup { $\tilde{X} \setminus (B,A)$ } is a soft regularly open cover of (C,A). Since (C,A) is soft α -nearly paracompact, there exist an open X-locally finite soft refinement V ={(V,A)_{β}: $\beta \in J$ } of U \cup { $\tilde{X} \setminus (B,A)$ } such that (C,A) \subset { $\tilde{X} \setminus (B,A)$ } \cup {(V,A)_{β}: $\beta \in J$ }. from this it follows that (B,A) \subset \cup {(V,A)_{β}: $\beta \in J$ } and hence (B,A) is soft α -nearly paracompact subset.

Theorem 4.3: Let (X,τ,E) be any soft almost regular space and A \subset E, let (U,A) be any soft α -nearly paracompact subset of X. Then Scl(U,A) is soft α -nearly paracompact subset of X.

Proof: Let (U,A) be any soft α -nearly paracompact subset of X. Let $U = \{(U,A)_{\alpha} : \alpha \in \Lambda, \text{ indexing set}\}$ be any soft regular open covering of Scl(U,A). For each soft point P_{λ}^{x} of (U,A), there exists a soft regular open set $(U,A)_{\alpha}$, containing the soft point P_{λ}^{x} . Since X is soft almost regular, there exists a soft regular open set $(V,A)_{x}$ such that $P_{\lambda}^{x} \in (V,A)_{x} \subset Scl((V,A)_{x}) \subset (U,A)_{\alpha}$. Consider the soft regular open covering $V = \{(V,A)_{x} : P_{\lambda}^{x} \in (U,A)\}$ of (U,A). By hypothesis there exists an X-locally finite family of soft open sets $P = \{(P,A)_{\beta} : \beta \in \Lambda\}$ which refines V and covers (U,A). Then $\{\alpha((P,A)_{\beta}):\beta \in J\}$ is an X-locally finite soft regular open family which refines $\{(U,A)_{\alpha} : \alpha \in \Lambda, \text{ indexing set}\}$, and covers (U,A). For each $P_{\lambda}^{x} \in (U,A)$, there exists $(P,A)_{\beta}$ such that $P_{\lambda}^{x} \in (P,A)_{\beta}$. Since X is soft almost regular, there exists a soft regularly open set $(V,A)_{x}^{*}$ such that $P_{\lambda}^{x} \in (V,A)_{x}^{*} \subset Scl(V,A)_{x}^{*}$. Since X is soft almost regular, there exists a soft regularly open set $(V,A)_{x}^{*}$ such that $P_{\lambda}^{x} \in (V,A)_{x}^{*} \subset Scl(V,A)_{x}^{*}$. Since, $\{(V,A)_{x}^{*} : P_{\lambda}^{x} \in (U,A)\}$ is soft regularly open covering of soft α -nearly paracompact subset of (U,A), therefore there exists an X-locally



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finite soft open refinement {(U,A)_v : v ∈ A, indexing set} of {(V,A)^{*}_x : P^x_{\lambda} ∈(U,A)} which covers (U,A) and (U,A)_v $\tilde{\subset}$ (V,A)^{*}_y for some P^y_{\lambda} ∈(U,A).Then, Scl(U,A) $\tilde{\subset}$ Scl(\bigcup_{v} {(U,A)_v : v ∈ A}= \bigcup_{v} {Scl(U,A)_v : v ∈ A}.Then Scl(U,A) $\tilde{\subset}$ Scl((V,A)^{*}_y) $\tilde{\subset}$ α((P,A)_{β*}), for some β* ∈J. Thus, {α((P,A)_{β*}): β* ∈J} is an X-locally finite soft open refinement of the soft regular open family U = {(U,A)_α : α∈A} which covers Scl(U,A).Hence, Scl(U,A) is soft α-nearly paracompact subset of the space X. **Theorem 4.4:** In any soft topological space X, the soft union of a locally finite family of soft regularly open soft α-nearly paracompact sets is soft α-nearly paracompact. **Proof:** Let U = {(U,A)_α : α∈A, indexing set} be any locally finite family of soft regularly open soft α-nearly paracompact sets and let (U,A) = \bigcup_{α} {(U,A)_α : α∈A}.Let {(V,A)_β : β∈J} be any soft regular open covering of (U,A). Then, for each α, {(U,A)_α $\tilde{\cap}$ (V,A)_β : β∈J} is a regular open covering of (U,A)_α : λ∈K^α which refines {(U,A)_α $\tilde{\cap}$ (V,A)_β : β∈J} and covers (U,A)_α : Ωconsider the family of soft α-nearly paracompact, then there exists an X-locally finite family of soft α-nearly paracompact, then there exists an X-locally finite family of soft open sets {(D,A)_λ : λ∈K^α , α∈A}. Then this is an X-locally finite soft open sets {(U,A)_β : β∈J} and covers (U,A)_α : Ωconsider the family of soft α-nearly paracompact sets and soft α-nearly paracompact.

Corollary 4.1: In a soft almost regular space X, the union of soft closures of a locally finite family of soft regular open soft α -nearly paracompact subsets is soft α -nearly paracompact.

Definition 4.3 [14]: Let $f: (X,E) \to (Y,K)$ be a mapping from a soft class (X,E) to another soft class (Y,K), and (G,C) a soft set in soft class (Y,K), where, $C \subseteq K$. Let $u: X \to Y$ and $p: E \to K$ be mappings. Then $(f^{-1}(G,C),D)$, $D=p^{-1}(C)$, is a soft set in the soft classes(X,E), defined as $(f^{-1}(G,C)(\alpha) = u^{-1}(G(p(\alpha))))$ for $\alpha \in D \subseteq E$. $(f^{-1}(G,C),D)$ is called a soft inverse image of (G,C) denoted by $f_{up}^{-1}(G,C)$. Hereafter, we shall write, $(f^{-1}(G,C),E)$ as $(f^{-1}(G,C))$.

Definition 4.4: A soft mapping $f_{up} : (X,\tau_1,E) \to (Y,\tau_2,K)$ is said to be soft almost continuous if $f_{up}^{-1}((F,A)$ is soft open in X for every soft regular open set (F,A) in Y.

Definition 4.5: A mapping $f_{up} : (X,\tau_1,E) \to (Y,\tau_2,K)$ is said to be soft almost open (soft almost closed) if the image of every soft regularly open (soft regularly closed) subset of X is an soft open (soft closed) subset of Y.

Theorem 4.5: Let $f_{up}: (X,\tau_1,E) \to (Y,\tau_2,K) f: (X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ be any soft almost closed, soft almost continuous, soft almost open mapping of a space X onto a Space Y such that for each soft point P_{λ}^{y} of Y, $f_{up}^{-1}(P_{\lambda}^{y})$ is soft nearly compact. If X is *soft nearly* paracompact, then so is Y.

Proof: Let $U = \{(U,A)_{\alpha} : \alpha \in \Lambda, an \text{ index set}\}$ be soft regular open cover of (Y,τ_2,K) . Since f_{up} is soft almost continuous, the collection $V = f_{up}^{-1}(U) = \{f_{up}^{-1}(U,A)_{\alpha} : \alpha \in \Lambda,\}$ is an soft regular open cover of soft nearly paracompact space (X,τ_1,E) and so it has a locally finite soft regular open refinement, say $\Omega = \{f_{up}^{-1}(U,A)_{\alpha} : \alpha \in I, \text{finite set}\}$. Again since, f_{up} is soft almost continuous and soft almost open, the collection $f(\Omega)$ is locally finite soft regular open refinement of U. Thus Y is soft nearly paracompact space.

Theorem 4.6: Let $f_{up}: (X,\tau_1,E) \to (Y,\tau_2,K) f: (X,\tau_1,\tau_2) \to (Y,\rho_1,\rho_2)$ be any soft almost closed, soft almost continuous, soft almost open mapping of a space X onto a Space Y such that for each soft point P_{λ}^{y} of Y, $f_{up}^{-1}(P_{\lambda}^{y})$ is soft α -



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nearly compact. If X is *soft* α -nearly paracompact, then so is Y.

Proof: It is similar to the proof of corresponding theorem for soft nearly paracompact spaces.

Theorem 4.7: Let X be soft nearly compact and Y be soft nearly paracompact then $X \times Y$ is soft nearly paracompact.

Proof: If *X* is soft nearly paracompact and *Y* is soft nearly compact, then $X_s \times Y_s$ is soft paracompact. The equality $W_x \times U_Y = W_x \times V_X$ says that $X_s \times Y_s = (X \times Y)_s$ i.e. the soft semi-regularization space of $X \times Y$ is soft nearly paracompact, where W_x and U_y are any basic soft neighborhoods of *x* and *y* in the soft spaces *X* and *Y* respectively.

Theorem 4.8: The product of a soft α -nearly paracompact subset and a soft α -nearly compact sets is a soft α -nearly paracompact.

Proof: It is similar to the proof of corresponding theorem for soft nearly paracompact and soft nearly compact spaces.

V. CONCLUSION

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. The soft set theory proposed by Molodtsov [1] offers a general mathematical tool for dealing with uncertain or vague objects. It is shown that soft sets are special type of information system known as single valued information system. In the present work properties of soft topological spaces are studied. Some new notions in soft space such as soft nearly compact spaces, soft nearly paracompact spaces and soft almost continuous mappings are introduced. Some basic properties of these concepts with examples and counter examples are examined and several interesting results and characterizations of soft nearly compact and nearly paracompact spaces are investigated. Hope that the findings in this paper will help researcher enhance and promote the further study on soft topological spaces to carry out a general framework for their applications in practical life.

REFERENCES

1. Molodtsov, D.: Soft set theory-First results. Comput. Math. Appl. 37(4–5), 19–31, 1999.

2. Shabir, M., Naz, M.: On soft topological spaces. Comput. Math. Appl. 61, 1786–1799, 2011.

3. Zorlutuna, I., Akdag, M., Min, W.K., Atmaca, S.: Remarks on soft topological spaces. Ann. Fuzzy Math. Inf. 3(2), 171–185, 2012.

4. Aygunoglu, A., Aygun, H: Some notes on soft topological spaces. Neural Comput. Appl. doi:10.1007/s00521-011-0722-3.

5. Chen, B.: Soft semi-open sets and related properties in soft topological spaces. Appl. Math. Inf. Sci. 7(1), 287–294, 2013.

6. Akdag, M., Ozkan, A: Soft b-open sets and soft b-continuous functions. Math. Sci. 8:124, DOI 10.1007/s40096-014-0124-71-7, 2014.

7. Akdag, M., Ozkan, A: Soft α -open sets and soft α -continuous functions. Abstr. Appl. Anal. Art ID 891341, 1-7, 2014.

8. A.S. Mashour and et al: On Supra Topological space, Indian J. Pure and Appl. Math., 4(4), 502-510, 1983.

. A.S. Mashour and et al. On Supra Topological space, indian J. Fure and Appl. Math., 4(4), 502-510, 1965.

9. Kandil and et al: Supra Generalised Closed Sets with respect to an soft ideal in supra soft topological spaces, Appl. Math.Infor.Science,8(4), 1731-174, 2014.

10. Yumak, Y., Kaymakcı, A.K: Soft β-open sets and their aplications. http://arxiv.org/abs/1312.6964V1[Math.GN] 25, Dec 2013.

11. Maji, P.K., Biswas, R., Roy, A.R.: Soft set theory. Comput. Math. Appl. 45, 555-562, 2003.

12. Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M.: On some new operations in soft set theory. Comput. Math. Appl. 57, 1547–1553 , 2009. 13. Arockiarani, I., Arokialancy, A.: Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces. Int. J. Math. Arch. 4(2), 1–7 2013

14. Kharal A., Ahmad B.: Mappings on soft classes. New Math. Nat. Comput. 7(3), 471-481, 2011.

15. Zorlutana, I., Akdag M., Min W.K., Atmaca S.: Remarks on Soft Topological Spaces. Annals of Fuzzy Mathematics and Informatics. 3(2), 171-185,2012.

16. Debnath, B.: On Maximal soft δ -open (Minimal soft δ -closed) Sets in Soft topological Spaces, International Journal of Computer Applications , Vol. 168, No. 5, pp.8-13, June 2017.

17. Z. Xiao, L. Chen, B. Zhong, S. Ye: Recognition for soft information based on the theory of soft sets in: J. Chen (Ed.), Proceedings of ICSSSM-05, vol. 2, IEEE, pp. 1104-1106, 2005.

18. D. Pei, D. Miao: From soft sets to information systems, in: X. Hu, Q. Liu, A. Skowron, T.Y. Lin, R.R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, vol. 2, IEEE, pp. 617-621, 2005.

19. L. A. Zadeh, "Fuzzy Sets", Inform. Control., Vol. 8, pp. 338-353, 1965

20. Z. Pawlak, "Rough Sets", Int. Journal of Computer and Information Sciences, Vol. 11, pp. 341-356,1982.

21. M.K. Singal and A. Mathur, On Nearly compact Spaces-II, Bull. U. M. Ital. 4(9), pp 669-678,1974.

22. M.K. Singal and S.P. Arya, On Nearly paracompact Spaces, Matematicki Vesnik 6(21), pp 3-16,1969.

23. F. Lin, Soft Connected Space and Soft Paracompact Spaces, Int. J. Math., Computational, Physical, Electrical and Computer Engineering, Vol.7, No.2, 2013.