

# MHD Nanofluid Radiating Stretching Sheet with Chemical Reaction and Heat Source/Sink

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**ABSTRACT:** Numerical analysis is presented to investigate the chemical reaction and heat source/sink parameter on saturated porous media over a radiating MHD nanofluid stretching sheet. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The governing boundary-layer equations of the problem are formulated and transformed into a non-similar form. The resultant equations are then solved numerically using finite difference method along with Thomas algorithm. The results are analyzed for the effect of different physical parameters on the dimensionless velocity, temperature and nanoparticle concentration fields and are presented through graphs.

**KEYWORDS:** MHD, chemical reaction parameter, heat source/sink, thermophoresis parameter, Brownian motion parameter, convection-radiation parameter, hydrodynamic (momentum) slip parameter.

## I. INTRODUCTION

Interaction of nanoparticles in a fluid is called nanofluid. The nanoparticle used in nanofluid are normally composed of materials, carbides or carbon nanotubes, water, ethylene, glycol and oil which are common examples of based fluids. Nanofluid have their major applications in heat transfer. They demonstrate enhanced thermal conductivity and convective heat transfer coefficient, counter balance to the base fluid. A comprehensive study of convective transport in nanofluids was made by Buongiorno and Hu [1] and Buongiorno [2]. Theories and experiments of thermal convection in porous media and state-of-the-art reviews, with special emphasis on practical applications have been presented in the recent books by Vafai[6], Pop and Ingham[3] and Bejan et al.[4]. Srinivas Maripala, Kishan.N[5], studied the unsteady MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with thermal radiation and chemical reaction” However, such studies for nanofluid are rarely available in the literature especially when conjugate effects of heat and mass transfer are considered [6]. Srinivas Maripala and Kishan Naikoti [7,8] studied the MHD effects on micropolar nanofluid flow over a radiative stretching surface with thermal conductivity. Dessie and Kishan [9] studied the heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink. The above literature survey reveals that all of these studies are restricted to isothermal or isoflux boundary conditions. The use of the thermal convective boundary condition in order to study Blasius flow over a flat plate was first introduced by Aziz [10]. After his pioneering work, several authors used this boundary condition to study transport phenomena. Recently, Srinivas Maripala and Kishan Naikoti [14] studied the, MHD convection slip flow of a thermosolutal nanofluid in a saturated porous media over a radiating stretching sheet with heat source/sink when chemical reaction parameter is absent. In the present investigation is to study the chemical reaction effects of magnetohydrodynamics convection slip flow of a thermosolutal nanofluid in a saturated porous media over a radiating stretching sheet with heat source/sink. The governing equations are solved by using the implicit finite difference scheme using along with the Gauss-Siedel iteration method. The C-programming code is used to compute the values for the same.

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## II. GOVERNANING NANOFLUID TRANSPORT MODEL

The two-dimensional regime with a coordinate system with the  $\bar{x}$ -axis aligned horizontally and the  $\bar{y}$ -axis is normal to it. A transverse magnetic field  $B_0$  acts normal to the bounding surface. The magnetic Reynolds number is small so that the induced magnetic field is effectively negligible when compared to the applied magnetic field. We neglect the electric field associated with the polarization of charges and Hall effects. It is further assumed that the left of the plate is heated by the convection from the hot fluid of temperature  $T_f (> T_w > T_\infty)$  which provides a variable heat transfer coefficient  $h_f(\bar{x})$ . Consequently, a thermal convective boundary condition arises. It is further assumed that the concentration in the left of the plate  $C_f$  is higher than that of the plate concentration and free stream concentration  $C_\infty$  which provides a variable mass transfer coefficient  $h_m(\bar{x})$ . As a result, a mass convective boundary condition arises. The Oberbeck- Boussinesq approximation is utilized and the four field equations are the conservation of mass, momentum, thermal energy and the nanoparticles volume fraction. These equations can be written in terms of dimensional forms, extending the formulations of Buongiorno [11] and Makinde and Aziz [12]:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\rho_f \left( \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = \mu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\mu}{Kp} \bar{u} - \sigma B_0^2 \bar{u}, \tag{2}$$

$$\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} + \tau \left\{ D_B \frac{\partial C}{\partial \bar{y}} \frac{\partial T}{\partial \bar{y}} + \left( \frac{D_T}{T_\infty} \right) \left( \frac{\partial T}{\partial \bar{y}} \right)^2 \right\} - \frac{1}{\rho_f c_f} \frac{\partial q_r}{\partial \bar{y}} + \frac{Q_0}{(\rho c)_f} (T - T_\infty) \tag{3}$$

$$\bar{u} \frac{\partial C}{\partial \bar{x}} + \bar{v} \frac{\partial C}{\partial \bar{y}} = D_B \frac{\partial^2 C}{\partial \bar{y}^2} + \left( \frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial \bar{y}^2} - k_c (C - C_\infty) \tag{4}$$

The appropriate boundary conditions are following Datta and Karniadakis,

$$\bar{u} = \bar{u}_w + \bar{u}_{slip}; \bar{v} = 0, -k \frac{\partial T}{\partial \bar{y}} = h_f (T_f - T), -D_B \frac{\partial C}{\partial \bar{y}} = h_m (C_f - C) \text{ at } \bar{y} = 0, \tag{5}$$

$\bar{u}$  tends to 0, T tends to  $T_\infty$ , C tends to  $C_\infty$  as  $\bar{y}$  tend to  $\infty$

Here  $\alpha = k / (\rho c)_f$  thermal diffusivity of the fluid;  $\tau = (\rho c)_p / (\rho c)_f$ , ratio of heat capacity of the nanoparticle and fluid;  $Kp$ : permeability of the medium;  $(\bar{u}, \bar{v})$ : velocity components along  $\bar{x}$  and  $\bar{y}$  axes,  $\bar{u}_w = U_r (\bar{x}/L)$  velocity of the plate,  $L$ : characteristic length of the plate,  $\bar{u}_{slip} = N_1 v \frac{\partial \bar{u}}{\partial \bar{y}}$ : linear slip velocity,  $N_1$  velocity slip factor with dimension s/m,  $\rho_f$ : density of the base fluid,  $\sigma$ : electric conductivity,  $\mu$ : dynamic viscosity of the base fluid,  $\rho_p$ : density of the nanoparticles,  $(\rho c p)_f$ : effective heat capacity of the fluid,  $(\rho c p)_p$  effective heat capacity of the nanoparticle material,  $Q_0$  volumetric rate of heat generation/absorption,  $\varepsilon$ : porosity,  $D_B$ : Brownian diffusion coefficient,  $D_T$ : thermophoretic diffusion coefficient and  $q_r$ : radiative heat transfer in  $\bar{y}$ -direction. We consider the fluid to be a gray, absorbing-emitting but non scattering medium. We also assume that the boundary layer is optically thick and the Rosseland approximation or diffusion approximation for radiation is valid [29, 30]. Thus, the radioactive heat flux for an optically thick boundary layer (with intensive absorption), as elaborated by Sparrow and Cess [13], is defined as  $q_r = \left( \frac{\sigma_1}{3k_1} \right) (\partial T^4 / \partial \bar{y})$  where  $\sigma_1 (= 5.67 \times 10^{-8} W / m^2 K^4)$  is the Stefan-Boltzmann constant and  $k_1 (m^{-1})$  Rosseland mean absorption coefficient. Purely analytical solutions to the partial differential boundary value problem defined by (2)–(4) are not possible. Even a numerical solution is challenging. Hence we aim to transform the problem to a system of ordinary differential equations. We define the following dimensionless transformation variables:

$$\eta = \frac{\bar{y}}{\sqrt{Kp}}, \psi = U_r (\bar{x}/L) \sqrt{Kp} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \varphi(\eta) = \frac{C - C_\infty}{C_f - C_\infty}, \tag{6}$$

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where  $L$  is the characteristic length. From (4), we have  $T = T_\infty\{1 + (T_r - 1)\theta\}$ , where  $T_r = T_f/T_\infty$  (the wall temperature excess ratio parameter) and hence  $T^4 = T_\infty^4\{1 + (T_r - 1)\theta\}^4$ . Substitution of (6) into (2)-(4) generates the following similarity equations:

$$f''' + Re Da (ff'' - f'^2 - M f') - f' = 0 \tag{7}$$

$$\theta'' + Re Pr Da f \theta' + Pr [Nb \theta' f \varphi' + Nt \theta'^2] + \frac{4}{3R} [1 + (T_r - 1)\theta]^3 \theta' + Q\theta = 0 \tag{8}$$

$$\varphi'' + Re Le Da f \varphi' + \frac{Nt}{Nb} \theta'' - \gamma \varphi = 0 \tag{9}$$

The relevant boundary conditions are

$$\begin{aligned} f(0) = 0, f'(0) = 1 + a f''(0), f'(\infty) = 0, \\ \theta'(0) = -Nc[1 - \theta(0)], \theta(\infty) = 0 \\ \varphi'(0) = -Nd[1 - \varphi(0)], \varphi(\infty) = 0 \end{aligned} \tag{10}$$

where primes denote differentiation with respect to  $\eta$ . The thermo physical dimensionless parameters arising in (7)–(9) are defined as follows:  $Re = U_r L / \nu$  is the Reynolds number,  $Da = k_p / L^2$  is the Darcy number,  $M = \sigma B_0^2 L / U_r \rho$  is the magnetic field parameter,  $Pr = \nu / \alpha$  is the Prandtl number,  $Nt = \tau D_T (T_f - T_\infty) / \nu T_\infty$  is the thermophoresis parameter,  $Nb = \tau D_B (C_f - C_\infty) / \nu$  is the Brownian motion parameter,  $R = k k_1 / 4 \sigma_1 T_\infty^3$  is the convection-radiation parameter,  $Le = \nu / D_B$  is the Lewis number,  $a = N_1 \nu / \sqrt{K_p}$  is the hydrodynamic (momentum) slip parameter,  $Nd = h_m \sqrt{K_p} / D_B$  is the convection-diffusion parameter,  $Nc = h_f \sqrt{K_p} / k$  is the convection-conduction parameter and the chemical reaction parameter  $\gamma = k_c (C - C_\infty)$ . Quantities of physical interest are the local friction factor,  $C_{f_x}$ , the local Nusselt number,  $Nu_x$  and the local Sherwood number,  $Sh_x$ . Physically,  $C_{f_x}$  represents the wall shear stress,  $Nu_x$  defines the heat transfer rates and  $sh_x$  defines the mass transfer rates:

$$\begin{aligned} C_{f_x} Re_x^{-1} Da_x^{0.5} = 2f''(0), \quad Sh_x Da_x^{0.5} = -\varphi'(0) \\ Nu_x Da_x^{0.5} = -[1 + \frac{4}{3R} \{1 + (T_r - 1)\theta(0)\}^3] \theta'(0) \end{aligned} \tag{11}$$

### III. RESULTS AND DISCUSSION

The numerical simulation of equations (7)-(9) subject to the boundary condition (10) are carried out for various values of the physical parameters such as Reynolds number  $Re$ , magnetic field parameter  $M$ , Prandtl number  $Pr$ , thermophoresis parameter  $Nt$ , Brownian motion parameter  $Nb$ , convection-radiation parameter  $R$ , Lewis number  $Le$ , hydrodynamic (momentum) slip parameter  $a$ , convection-diffusion parameter  $Nd$ , convection-conduction parameter  $Nc$ . In order to get a physical understanding of the problem the results have been performed for the velocity, temperature and concentration profiles. The results are presented graphically in figures 1-14.

Figures 1 to3 illustrate the velocity, temperature and concentration profiles for different values of heat source/sink parameter  $Q$ . From figure 1, it reveal that with the effect of heat source/sink parameter ( $Q < 0$ ), the velocity profiles decreases and the velocity profiles increase with heat source ( $Q > 0$ ). There is a significant variation is observed from figure.2 the temperature profiles decreases with heat source/sink parameter ( $Q < 0$ ) on the contrary, the temperature profiles increases with heat source/sink ( $Q > 0$ ) for both cases with and without convection-conduction parameter  $Nc$ . Figure.3 represents the effect of heat source/sink parameter  $Q$ , for different values of convection diffusion parameter  $Nd$  on concentration profiles. The concentration profiles increase in case of heat source/sink  $Q < 0$ , while the concentration profiles decreases with heat source/sink parameter  $Q > 0$ . It is interested to note that the heat sources effect is higher for higher conduction-diffusion parameter  $Nd$ . Therefore, the thermal and concentration boundary layer thickness are enhanced. Figures 4-5 depict the influence of chemical reaction on the dimensionless temperature and solutal concentration profiles with the fixed values of other parameters. It is obvious that an increase in the chemical reaction parameter results a decreasing in the solutal concentration profile. The distribution of solutal concentration becomes weak in the presence of chemical reaction. So, the solutal concentration boundary layer becomes thin as the

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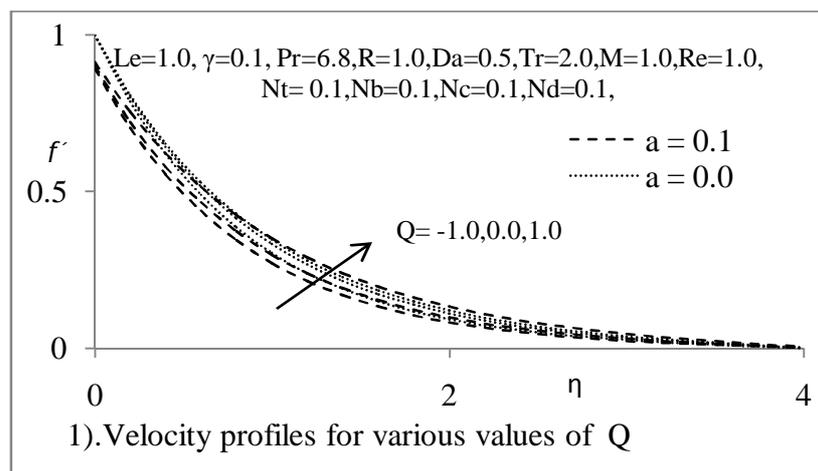
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chemical reaction parameter increases. From Figs4-5, it is observed that the chemical reaction influences the solutal concentration field. However, it has a minor effect on thermal diffusion. This explains the minor influence of chemical reaction on temperature profile. It is worth mentioning here that, the large values of  $\gamma$  shows small changes on temperature field.

The velocity, temperature and concentration display in 6 and 7. It is depicted that the effect of magnetic fluid is to decrease the velocity field for the cases slip and no slip boundary conditions. It is due to Lorentz forces in the saturated porous medium. When the strength of the magnetic field increase the velocity gradient at the interface has been diminishes, the temperature profiles increases with the increase of magnetic field  $M$ . Figure 8 and 9 demonstrates that an increase Chemical reaction parameter  $\gamma$  increases velocity as well as temperature profiles. It can be also noticed that the convection parameter  $Nc$  increases the temperature profiles.

Figure 10, represents the effect of thermophoresis parameter  $Nt$  when conduction parameter  $Nc=0$ (without biot number),  $Nc=0.1$ (with biot number) with influence of thermophoresis parameter  $Nt$ . The concentration profiles increase with increase of thermophoresis parameter  $Nt$ . It is worth mentioning that the thermophoresis parameter  $Nt$  effect is more in the presence of Biot number  $Nc$ . The effect of Brownian motion parameter  $Nb$  is to display on concentration profiles in Figure 11. It can be seen that the Brownian motion parameter  $Nb$  effects lead to decrease the concentration profiles. It is evident from Figure.12, the variation of skin fraction parameter  $f'(0)$  decreases with the increase of radiation parameter  $R$  and increases with the increase of magnetic parameter  $M$ . The Figure 13 is done for the values of  $\theta'(0)$  verses the magnetic field parameter  $M$ . From figure it revealed that the heat transfer coefficient  $\theta'(0)$  is to increase with the increase of  $R$  and reduces the values with increase of values of magnetic parameter  $M$ . Figure 14 is plotted for the Sherwood number  $\phi'(0)$  verses the magnetic parameter  $M$  for different values of radiation parameter  $R$ . The results indicate that an increasing magnetic parameter  $M$  is to decrease the Sherwood parameter coefficients  $\phi'(0)$  values. It is also noticed that the Sherwood parameter coefficients  $\phi'(0)$  values increasing with the increase of radiation parameter  $R$ .

## Graphs:

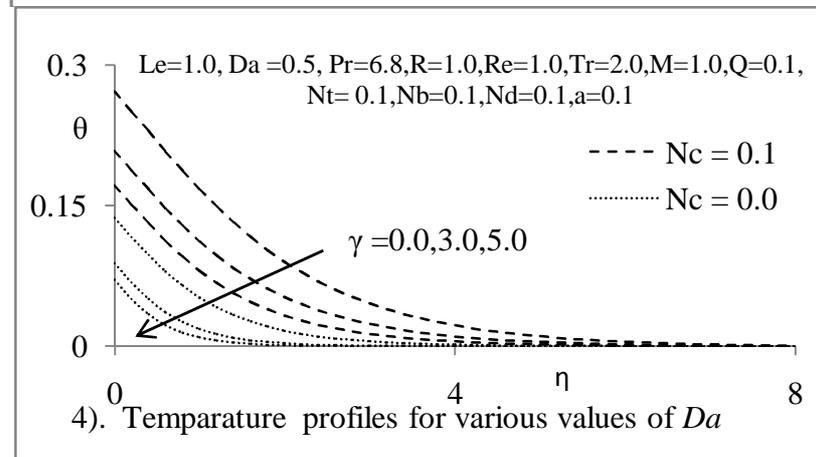
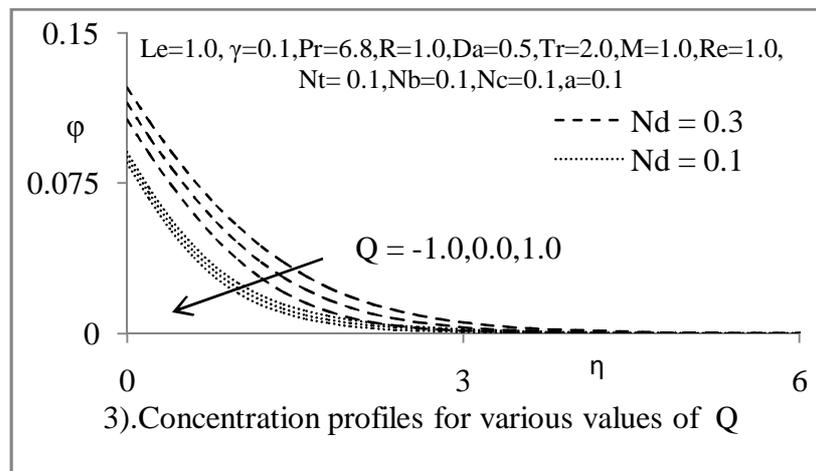
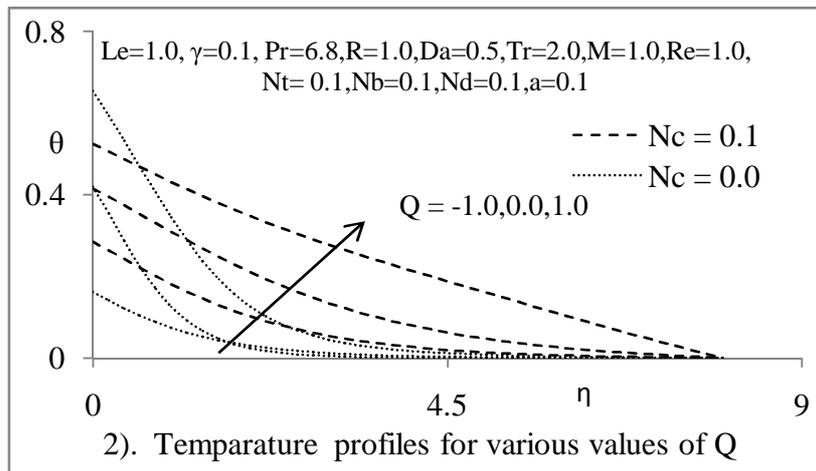


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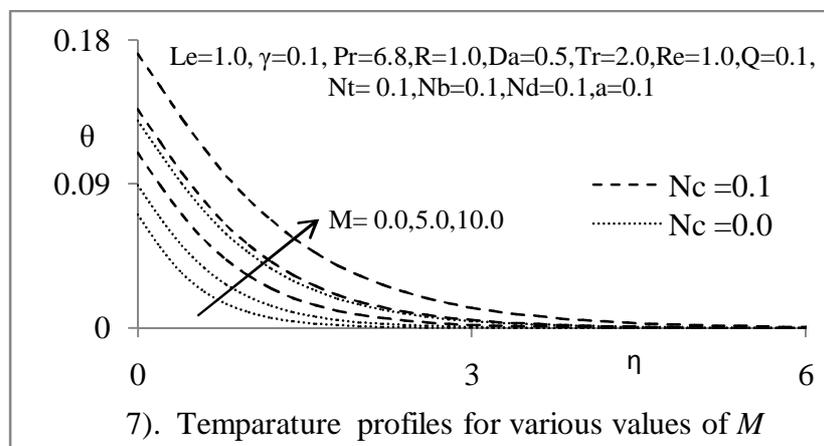
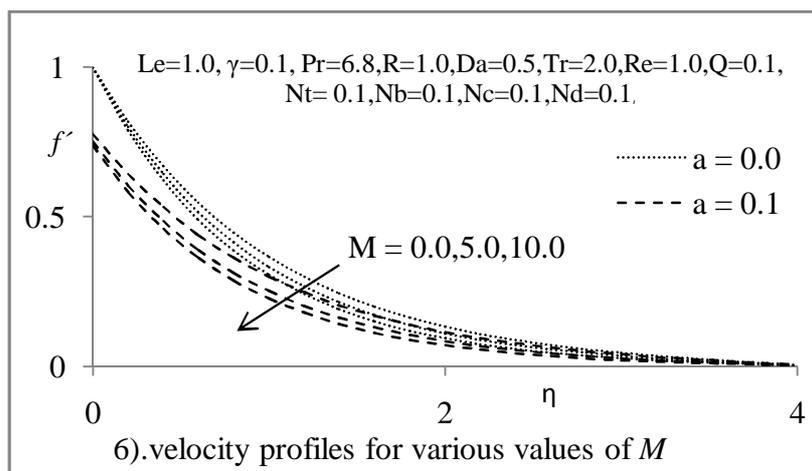
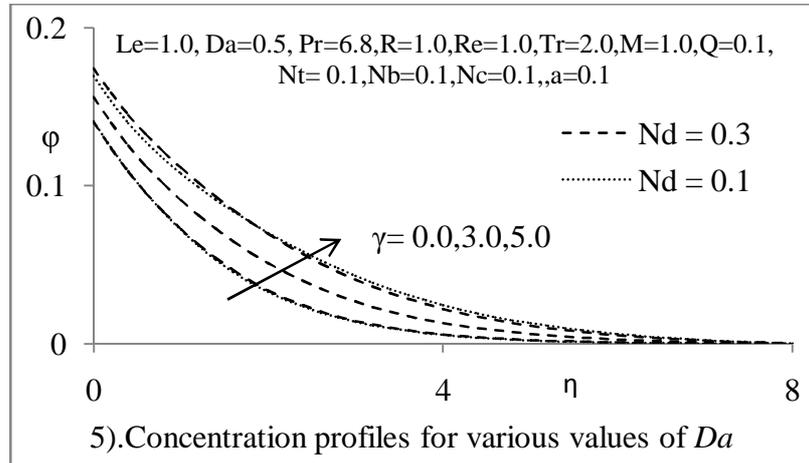


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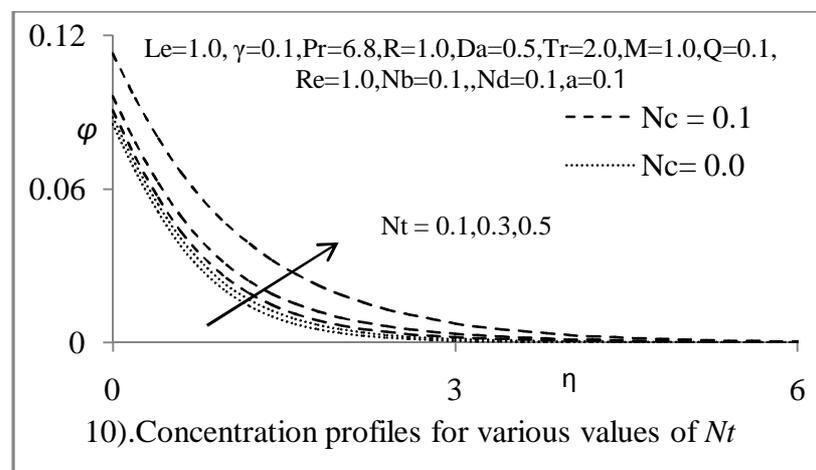
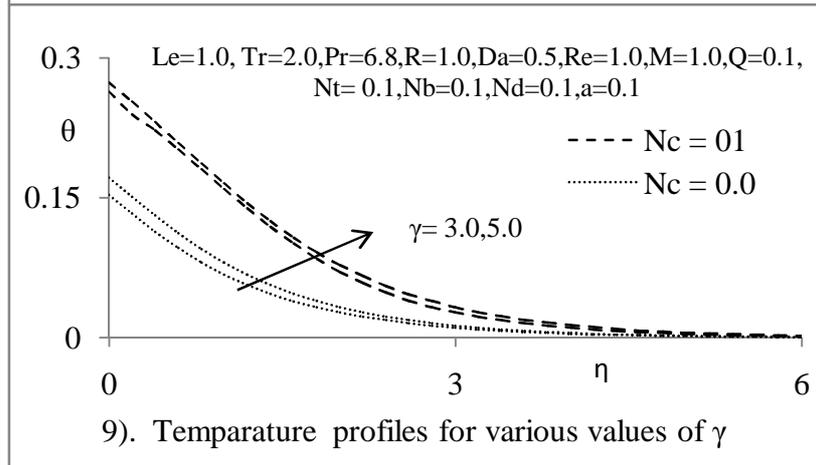
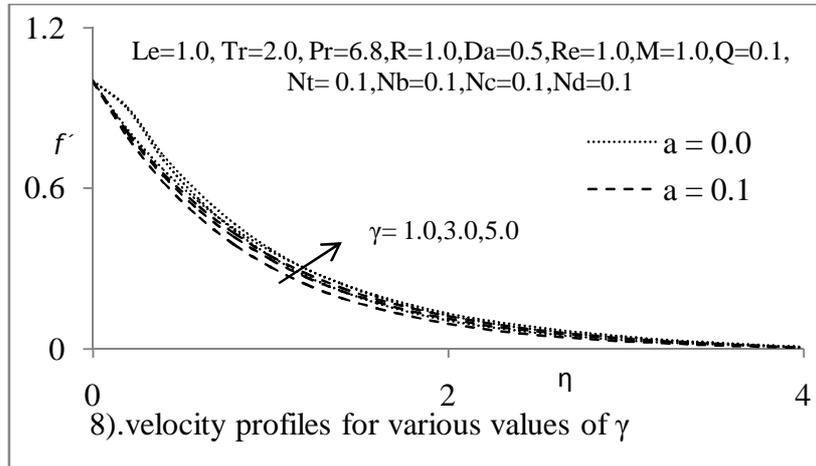


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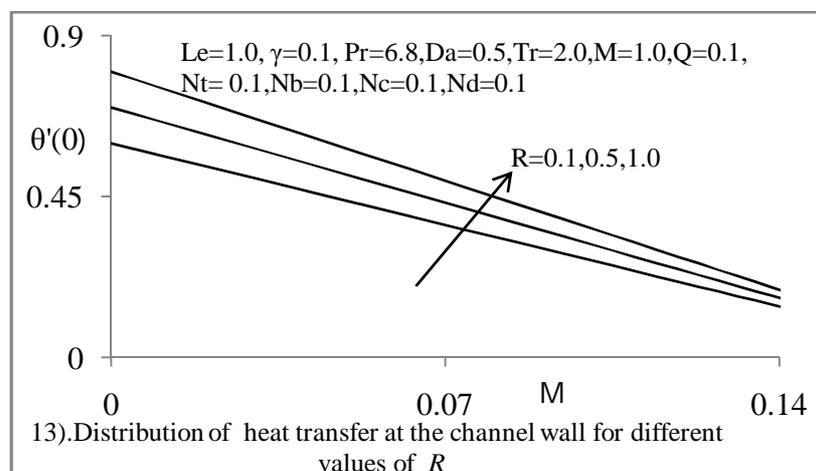
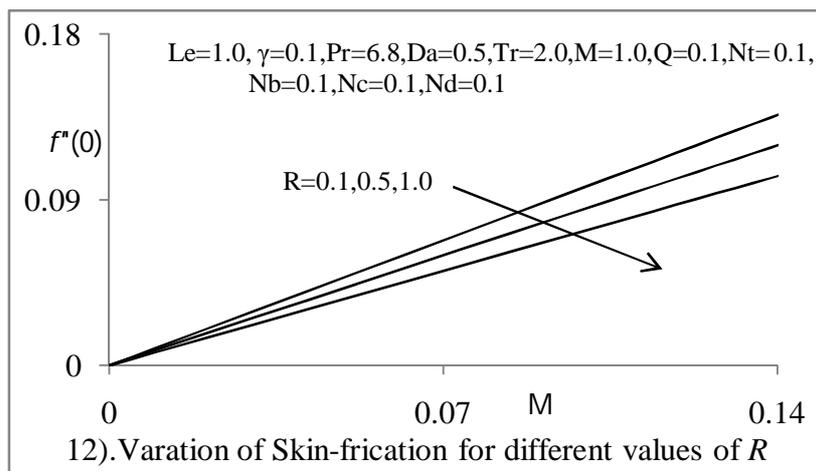
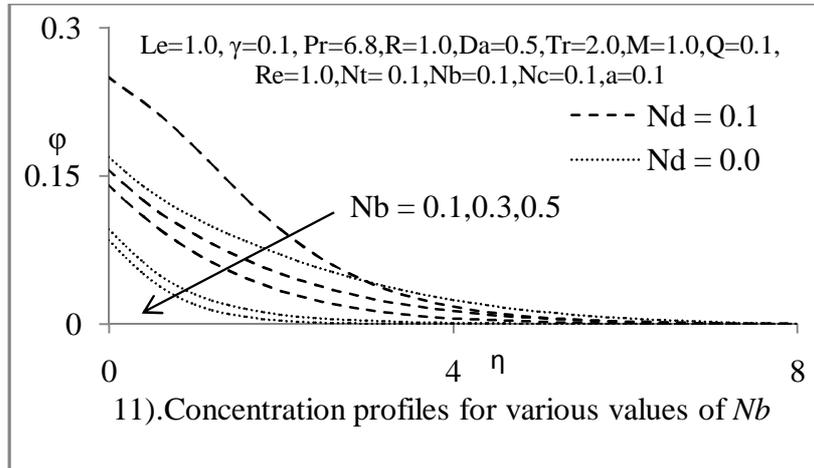


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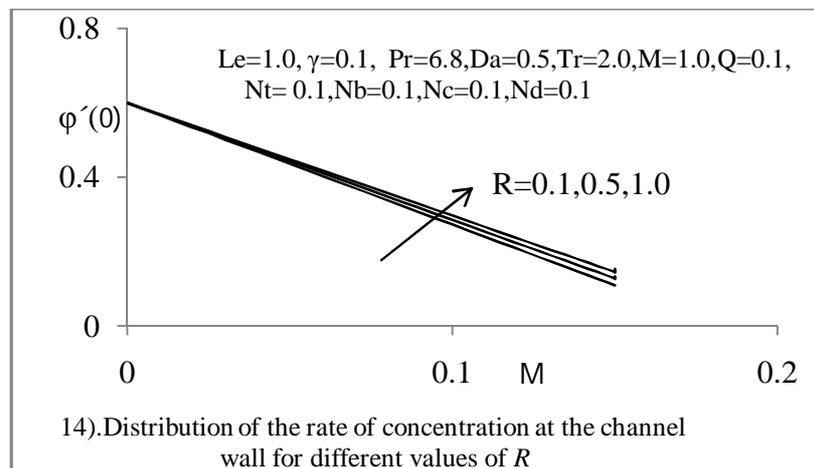


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## IV. CONCLUSION

In the present investigation, an analysis in order to study the chemical reaction effects of magnetohydrodynamics convection slip flow of a thermosolutal nanofluid in a saturated porous media over a radiating stretching sheet with heat source/sink. In this study the density is dependent on velocity, temperature and concentration profiles. The governing equations are transformed to the high non-linear ordinary differential equations by the use of similarity transformation and are solved analytically using finite difference method along with the Gauss-seidel method. The physical parameters such as Reynolds number, chemical reaction parameter, magnetic field parameter, Prandtl number, thermophoresis parameter, Brownian motion parameter, convection-radiation parameter, Lewis number, hydrodynamic (momentum) slip parameter, convection-diffusion parameter, convection-conduction parameter on velocity, temperature and concentration profiles are depicted and discussed in this paper.

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