

Analogous Study of Inverse Dominations in Fuzzy Graph and Its Applications

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ABSTRACT: Let $G(\sigma, \mu)$ be a fuzzy graph. Let D be a minimum fuzzy dominating set of G . If $V-D$ contains dominating set say D' of G then D' is called an Inverse dominating set of G with respect to D . The inverse domination number $\gamma_I(G)$ is the minimum cardinality of an inverse dominating set of G . The object of this paper is to generalize the Inverse dominations in fuzzy graphs. In this paper we discuss the concepts of inverse connected edge domination, inverse total edge domination, and inverse domination for intuitionistic fuzzy graphs, inverse split and non-split dominations. In this paper we extended our study to inverse regular connected domination number and derived some results related to the above inverse dominations in fuzzy graphs and prompt some applications on them like as computer communication network, social network theory.

AMS Subject Classification: 05C75

KEYWORDS: : Fuzzy graph, Domination, Inverse connected edge domination, inverse split and non-split domination and inverse regular connected domination.

I. INTRODUCTION

The concept of fuzzy sets and fuzzy relations was introduced by L.A. Zadeh in 1965 [1]. His aim was to develop a mathematical theory to deal with uncertainty and imprecision. The distinction between the set and fuzzy set is that the set divide the universal set into two subsets, namely members and non-members while fuzzy set assigns a sequence of membership values to elements of the universal set varies from 0 to 1. The first definition of fuzzy graph was proposed by Kaufmann [3]. Rosenfeld [4] introduced elaborated definition including fuzzy vertex and fuzzy edge. The concept of domination in graphs found its origin in 1850s with the interest of several chess players. Chess enthusiasts in Europe considered the problem of determining the minimum number of queens that can be placed on a chess board so that all the squares are either attacked by a queen or occupied by a queen. The domination number is introduced by Cockayne and Hedetniemi. The study of domination set in graphs was begun by Ore and Berge [6]. V.R. Kulli wrote on theory of domination in graphs. A. Somasundram and S. Somasundram [7] discussed domination in Fuzzy graphs. Among the numerous applications of the domination theory in graphs, the most often discussed is a communication network. This network consists of communication links between a fixed set of sites. The problem is to select a smallest set of sites at which the transmitters are placed so that every other site in the network is joined by a direct communication link to the site, which has a transmitter. In other words the problem is to find a minimum dominating set in the graph corresponding to this network.

The problem of selecting two disjoint sets of transmitting stations so that one set can provide service in the case of failure of some of the transmitting stations of the other set. This led them to define the inverse domination number. In this aspect, it is worthwhile to concentrate on dominating and inverse dominating sets. The Inverse domination in graphs was introduced by V.R. Kulli and Sigarkanti [9]. Many other inverse domination parameters in domination theory were studied, for example, in [7, 8, 10, 11,12]. In this paper we discuss the inverse connected edge domination

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number of fuzzy graph using fuzzy edge cardinality inverse total edge domination, inverse split and non split edge domination and inverse regular domination in fuzzy graphs , we establish the relationship with other parameter which is also investigated. In this paper we offer a survey of selected recent results on inverse domination in fuzzy graphs.

II. RELATED WORK

Definition: 2.1 A fuzzy graph $G(\sigma, \mu)$ is a set with two functions, $\sigma: V \rightarrow [0,1]$ and $\mu: E \rightarrow [0,1]$ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y) \forall x, y \in V$.

Definition: 2.2 Let $G(\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define $\sigma_1 = \sigma(x)$ for all $x \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(x, y) = \mu(x, y)$ for all $x, y \in V_1$. Then (σ_1, μ_1) is called the fuzzy sub graph of G induced by V_1 and is denoted by $\langle V_1 \rangle$.

Definition: 2.3 The order p and size q of a fuzzy graph $G(\sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{xy \in E} \mu(x, y)$.

Definition: 2.4 Let $G(\sigma, \mu)$ be a fuzzy graph on V and $S \subseteq V$. Then the fuzzy cardinality of S is defined to be $\sum_{v \in S} \sigma(v)$.

Definition: 2.5 Let $G(\sigma, \mu)$ be a fuzzy graph on E and $D \subseteq E$. Then the fuzzy edge cardinality of D is defined to be $\sum_{e \in D} \mu(e)$.

Definition: 2.6 Let $G(\sigma, \mu)$ be a fuzzy graph. Define the degree of a vertex v to be $d(v) = \sum_{u \neq v} \mu(u, v)$. The minimum degree of G is $\delta(G) = \wedge \{d(v) / v \in V\}$ and maximum degree of G is $\Delta(G) = \vee \{d(v) / v \in V\}$.

Definition: 2.8 Let $G(\sigma, \mu)$ be a fuzzy graph. A subset S of X is said to be an edge domination set in G if for every edge in $X - S$ is adjacent to at least one effective edge in S . The minimum Fuzzy cardinality of an edge domination set G is called the edge Domination number of G and is denoted by $\gamma'(G)$.

Definition: 2.7 Let $G(\sigma, \mu)$ be a fuzzy graph. A subset D of V is said to be a dominating set of G if for every $v \in V - D$ there exists an element $u \in D$ such that $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. A dominating set D of G is called the minimal dominating set of G if every node $v \in D$, $D - \{v\}$ is not a dominating set. The minimum scalar cardinality of D is called the domination number and it is denoted by $\gamma(G)$.

II. INVERSE DOMINATIONS IN FUZZY GRAPHS

Definition: 3.1 Let $G(\sigma, \mu)$ be a fuzzy graph on (V, X) . A subset D of X is a minimal edge dominating set of a fuzzy graph G . If $V - D$ contains an edge dominating set D' is called an inverse edge dominating set of G with respect to D . The minimum fuzzy edge cardinality taken over all inverse Edge domination number of the fuzzy graph G and it is denoted by $\gamma'_1(G)$.

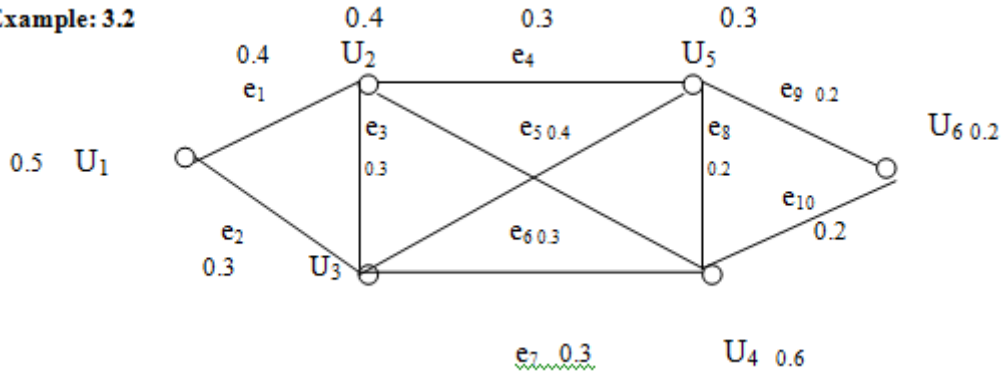
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Example: 3.2



$$\gamma'(G) = 0.5 \quad \text{and} \quad \gamma'_i(G) = 0.6$$

Definition: 3.3 Let $G(\sigma, \mu)$ be a fuzzy graph on (V, X) . A subset D of X is a minimal edge dominating set of a fuzzy graph G . If $V - D$ contains an edge dominating set D' of G and $\langle D' \rangle$ is connected is called an inverse connected edge dominating set of G with respect to D . The minimum fuzzy edge cardinality taken over all inverse connected edge dominating set of G is called the inverse connected edge domination number of the fuzzy graph G and it is denoted by $\gamma'_{ic}(G)$.

Definition: 3.4 An inverse connected edge dominating set D of a fuzzy graph G is said to be minimal inverse connected edge dominating set if no proper subset D is an inverse connected edge dominating set of G .

Definition: 3.6 Let D' be the minimum inverse dominating set of G with respect to D . Then D' is called an inverse split(non split) dominating set of G if the induced subgraph $\langle V - D' \rangle$ is disconnected (connected). The inverse split (non-split) domination number is denoted by $\gamma'_{is}(G)$ ($\gamma'_{ins}(G)$) and is the minimum cardinality taken over all the minimal inverse split (non- split) dominating sets of G .

Theorem: 3.5

If G is a fuzzy graph without isolated edges then

$$\frac{q}{\Delta'(G)+1} \geq \gamma'_{ic}(G).$$

Proof: Let D be an inverse connected edge dominating set of G

$$\text{Since, } |D| \Delta'(G) \leq \sum_{e \in D} d_E(e) = \sum_{e \in D} |N(e)|$$

$$\leq \sum_{e \in D} |N(e)|$$

$$\leq |E - D|$$

$$\leq q - |D|$$

$$\therefore |D| \Delta'(G) + |D| \leq q$$

$$\text{Thus } \gamma'_{ic}(G) \leq \frac{q}{\Delta'(G)+1}$$

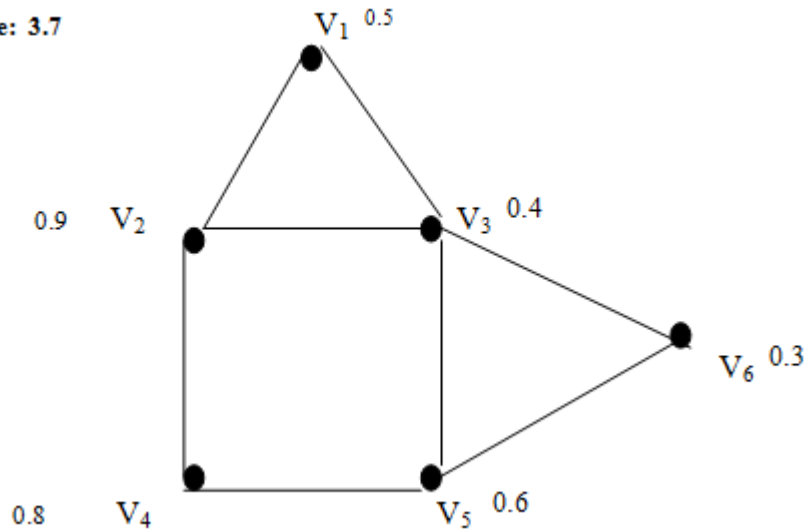
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Example: 3.7



$\sigma(V_1) = 0.5$, $\sigma(V_2) = 0.9$, $\sigma(V_3) = 0.4$
 $\sigma(V_4) = 0.8$, $\sigma(V_5) = 0.6$
 $D = \{V_2, V_5\}$
 $D' = \{V_3, V_4\}$
 $\langle V - D' \rangle$ is disconnected.

Corollary: 3.8 If G is a fuzzy graph without isolated edges then $\frac{q}{\Delta(G)+1} \geq \gamma'_{ins}(G)$.

Proof: We can prove this as we proved above for inverse connected edge dominating set. In the similar way we can prove that for inverse non split dominating set of G .

III. INVERS REGULAR DOMINATION IN FUZZY GRAPHS

Definition: 4.1 Let $G(\sigma, \mu)$ be a regular fuzzy graph on $G^* = (V, E)$.

If $d_G(v) = k$ for all $v \in V$, i.e. if each vertex has same degree k , then G is said to be a regular fuzzy graph of degree k or k -regular fuzzy graph.

Definition: 4.2 The domination set D of the fuzzy graph $G(\sigma, \mu)$ is said to be regular dominating set if the following conditions are satisfied. (i). Every vertex in D is of same degree. The fuzzy regular domination number $r(G)$ is the minimum fuzzy cardinality taken over all minimal regular dominating sets of G .

Definition: 4.3 The dominating set D'_{rc} is an inverse regular connected dominating set if D'_{rc} is subset of $V - D_{rc}$ is a regular connected dominating set with respect to D_{rc} . The fuzzy inverse regular connected domination number $\gamma'_{rc}(G)$ is the minimum fuzzy cardinality taken over all minimal inverse regular connected dominating sets of G .

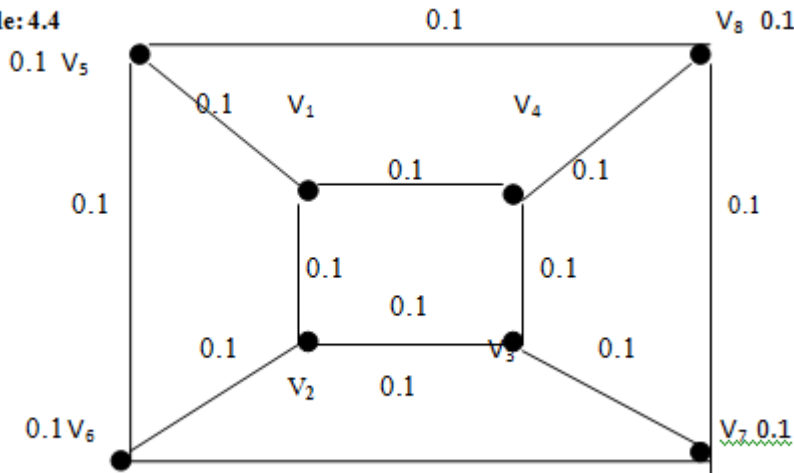
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Example: 4.4



$D_{Irc} = \{V_1, V_2, V_3, V_4\}$ $\langle D_{Irc} \rangle$ is connected
 $D'_{Irc}(G) \subseteq V - D_{Irc} = \{V_5, V_6, V_7, V_8\}$
 $\langle D'_{Irc} \rangle$ is connected.

Remark: 4.5 $\gamma_{Irc}(G) + \gamma'_{Irc}(G) = p$

Proposition: 4.6 Subset of regular fuzzy dominating set need not be regular fuzzy dominating set.

Example: 4.7 Consider $G(\sigma, \mu)$ be fuzzy graph. Define $\sigma(u) = 0.3$, $\sigma(v) = 0.6$, $\sigma(w) = 0.7$, $\sigma(x) = 0.5$, $\sigma(y) = 0.2$ and $\mu(u, v) = 0.3$, $\mu(v, w) = 0.4$, $\mu(y, u) = 0.1$. Here $S = \{v, x\}$ is regular fuzzy dominating set but the subset is not even a fuzzy dominating set.

Definition 3.7: A set S subset of V is called minimal regular fuzzy dominating set if

- (i) Any subset of S is not a fuzzy dominating set.
- (ii) All the vertices of S have same degree.

IV. APPLICATIONS

The applications of domination in fuzzy graphs lies in various fields in solving real life problems. It includes social network theory, land surveying, radio station computer communication networks, school bus routing sets of representatives, interconnection networks etc. The online social network has been developed significantly in the recent years as a medium of communication, sharing the information and spreading the influence. The dominating set plays a vital role in analyzing the effect on a real online social network data set through simulation. The dominating set concept can be applied to the social network data set through simulation. The dominating set concept can be applied to the social network graph to determine the amount of positive influence that is possessed by an individual as well as its impact to their related neighbor.

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Dominating Set in Mobile Ad-hoc Network

A Mobile Ad-hoc Network is a self configurable infrastructure less network connecting the mobile devices in wireless mode. The dominating set has been commonly used for routing and broadcasting the information to the mobile devices in mobile ad-hoc networks [4]. The connected dominating set is widely used as a virtual backbone for Mobile ad-hoc network to provide different communication primitive such as routing, broadcasting etc.

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BIOGRAPHY



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