

A Note on Uniform Fuzzy Graphs and the Relationship between Regularity and Uniformity of a Fuzzy Graph

K.Bhanu Chander¹, Dr.T.V.Pradeep Kumar²

Assistant Professor, Department of Mathematics, Potti Sriramulu Chalavadi Mallikharjuna Rao College of Engineering and Technology, Vijayawada, A.P., India¹

Assistant Professor, Department of Mathematics, Acharya Nagarjuna University, Guntur, A.P, India²

ABSTRACT: Fuzzy graph was introduced by Kaufmann [6] in 1973. Later many authors like Bhattacharya, P [1], Bhutani, K.R [2], NagoorGani, A. and Radha, K [3] studied and develop the concept of Fuzzy graphs and its applications. In this paper, we introduced the concepts of Uniform fuzzy graph with an example, discussed the relationship between uniformity and regularity as well as the relationship between uniformity and total regularity of a fuzzy graph, calculated the order and size of a uniform fuzzy graph and established the facts that the union as well as the join of two k-level uniform fuzzy graphs is also a k-level uniform fuzzy graph..

KEYWORDS: Union of two fuzzy graphs, join of two fuzzy graphs, regular fuzzy graph, totally regular fuzzy graph, uniform fuzzy graph, order and size of a uniform fuzzy graph.

I.INTRODUCTION

The concept of a Graph is introduced for the first time by Leonhard Euler [3] in the year 1736. It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'Fuzzy Graph Model'.

Application of fuzzy relations are widespread and important in the field of clustering analysis, neural networks, computer networks, pattern recognition, decision making and expert systems. In each of these, the basic mathematical structure is that of a fuzzy graph. We know that a graph is a symmetric binary relation on a nonempty set V .

Similarly, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The first definition of a fuzzy graph was by Kaufmann[6] in 1973, based on Zadeh's fuzzy relations [13]. But it was Azriel Rosenfeld [8] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. Later on so many Mathematicians produced various results on Fuzzy Graphs in various aspects.

II. PRELIMINARIES

In this section we have given the definitions and examples related to the concepts that are discussed in the introduction.

2.1 Definition [4]: A graph G is a triple consisting of a vertex set $V(G)$, an edge set $E(G)$, and a relation that associates with each edge, two vertices called its endpoints (not necessarily distinct). Graphically, we represent a graph

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

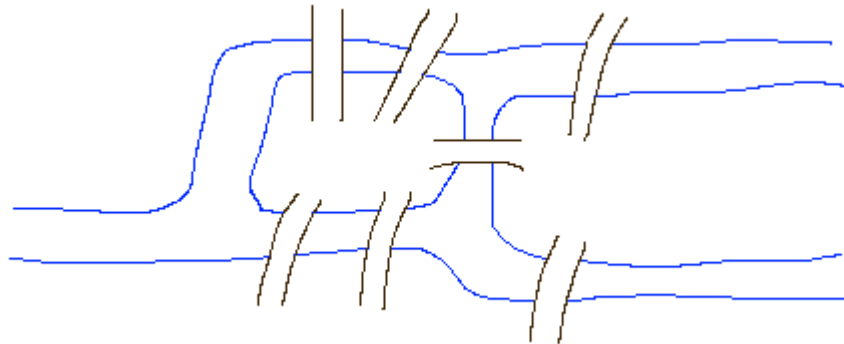
Website: www.ijirset.com

Vol. 6, Issue 2, February 2017

by drawing a point for each vertex and representing each edge by a curve joining its endpoints. (A graph G is said to be an empty graph if its edge set E(G) is empty.)

2.2 Example [3]: Königsberg Bridge’s problem

Königsberg was a city in Prussia situated on the Pregel River in the 16th century. (Today, the city is named Kaliningrad, western Russia.) The river Pregel flowed through the town, creating an island, as in the following picture. Seven bridges spanned the various branches of the river, as shown. A famous problem concerning Königsberg was whether it was possible to take a walk through the town in such a way as to cross over every bridge once, and only once



Euler's approach was to regard the spots of land (there are 4 of them) as vertices, and the bridges as edges between those points. The mathematical essentials of the map of Königsberg can then be reduced to the following diagram, which is an example of what is called a *graph*:

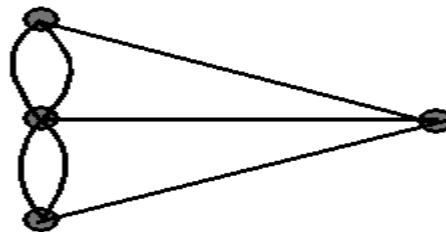


Fig 2.2.1

2.3 Definition [[8],[13]]: Let S be a non - empty set. Let $\mu: S \times S \rightarrow [0,1]$ and $\sigma: S \rightarrow [0, 1]$ be two fuzzy sets. Then μ is said to be a fuzzy relation on σ if $\mu(x, y) \leq \sigma(x) \cap \sigma(y) \forall x, y \in S$.

Note: $\sigma(x) \cap \sigma(y) = \min \{ \sigma(x), \sigma(y) \}$.

2.4 Definition [[8],[13]]:(i) A fuzzy relation is called Symmetric if $\mu(x, y) = \mu(y, x), \forall x, y \in S$.

(ii) A fuzzy relation is called Reflexive if $\mu(x, x) = \sigma(x), \forall x \in S$

2.5 Definition-[[6],[8]]: A fuzzy graph G is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ . The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*(V,E)$ where $E \subseteq V \times V$.

2.6 Example: The following figure gives us an example for a fuzzy graph. Here ‘ σ ’ values are written in the brackets at vertices and ‘ μ ’ values are written in the middle of the edges.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 2, February 2017

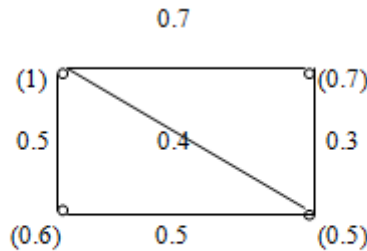


Figure (2.6.1.)

In fig (2.6.1), we observe that $\mu(x, y) \leq \sigma(x) \cap \sigma(y)$ at each edge. So it represents a fuzzy graph.

2.7 Note: $\sigma^* = \{x \in V / \sigma(x) > 0\}$ and $\mu^* = \{(x, y) \in V \times V / \mu(x, y) > 0\}$

2.8 Note: In any fuzzy graph $G: (\sigma, \mu)$ on $G^*(V, E)$ we assume that V is finite and μ is reflexive. Also we need not consider the loops and parallel edges.

2.9 Definition- [9]: A fuzzy graph $G: (\sigma, \mu)$ is said to be a Strong Fuzzy Graph if $\mu(u, v) = \sigma(u) \cap \sigma(v) \forall (u, v) \in \mu^*$.

2.10 Definition- [9]: A fuzzy graph $G: (\sigma, \mu)$ is said to be a Complete Fuzzy Graph if $\mu(u, v) = \sigma(u) \cap \sigma(v) \forall u, v \in \sigma^*$.

2.11 Definition [12]: Let $G: (\sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is defined as $d_G(u) = \sum_{u \neq v} \mu(u, v)$.

2.12 Definition [[5],[10]]: The complement of a fuzzy graph $G: (\sigma, \mu)$ is a fuzzy graph $\bar{G}: (\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} = \sigma$ and $\bar{\mu}(u, v) = \sigma(u) \cap \sigma(v) - \mu(u, v) \forall u, v \in V$.

2.13 Definition [7]: Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If $d_G(u) = k \forall u \in V$ (i.e.) if each vertex has same degree then it is called a Regular Graph of degree k or k -Regular Graph.

2.14 Definition [7]: Let $G: (\sigma, \mu)$ be a fuzzy graph. The Total degree of a vertex u is defined as $td_G(u) = \sum_{u \neq v} \mu(x, y) + \sigma(u)$. i.e. $td_G(u) = d_G(u) + \sigma(u)$.

2.15 Definition [7]: Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If $td_G(u) = k \forall u \in V$. i.e., if each vertex has same total degree then it is called a Totally Regular Graph of degree k or k -Totally Regular Graph.

2.16 Definition [11]: Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs with $G_1^*: (V_1, E_1)$ and $G_2^*: (V_2, E_2)$ with and be union of G_1^* and G_2^* . Then the union of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G = G_1 \cup G_2: (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ defined by

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_2 - V_1 \end{cases} \quad \text{and} \quad (\mu_1 \cup \mu_2)(uv) = \begin{cases} \mu_1(u) & \text{if } uv \in E_1 - E_2 \\ \mu_2(u) & \text{if } uv \in E_2 - E_1 \end{cases}$$

2.17 Definition [11]: Consider the join $G^* = G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E^l)$ of graphs where E^l is the set of all arcs joining the nodes of V_1 and V_2 where we assume that $V_1 \cap V_2 = \emptyset$. Then the join of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G = G_1 + G_2: (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ defined by

$$(\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u), u \in V_1 \cup V_2 \quad \text{and} \quad (\mu_1 + \mu_2)(uv) = \begin{cases} (\mu_1 \cup \mu_2)(uv) & \text{if } uv \in E_1 \cup E_2 \\ \sigma_1(u) \wedge \sigma_2(v) & \text{if } uv \in E^l \end{cases}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

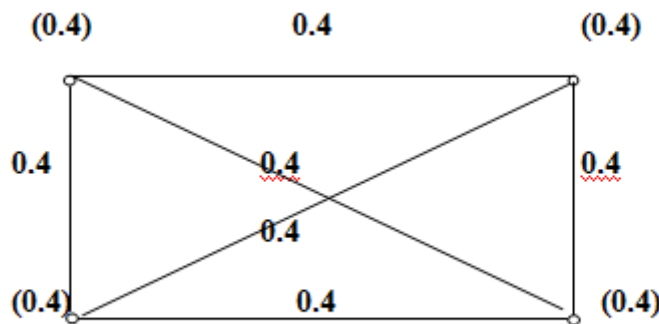
Vol. 6, Issue 2, February 2017

III. MAIN RESULTS

In this section we introduced the concept of Uniform fuzzy graph and obtained some results related to its relationship with regular fuzzy graphs, totally regular Fuzzy graphs. Also derived some basic results on calculation of order, size of uniform fuzzy graphs.

3.1 Definition: Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Let $\sigma^* = \{x \in V / \sigma(x) > 0\}$. Then G is said to be Uniform Fuzzy Graph of level k if $\mu(x, y) = k, \forall (x, y) \in \sigma^* \times \sigma^*$ and $\sigma(x) = k \forall x \in \sigma^*$ where k is some positive real such that $0 < k \leq 1$.

3.2 Example:



(Fig 3.2.1)

In Fig (3.2.1), the graph is an example of uniform fuzzy graph . Here σ values are written in the bracket at vertices and μ values are written in the middle of the edges.

3.3 Note: The Crisp graph of the compliment fuzzy graph of a uniform fuzzy graph always an empty graph.

3.4 Example:



(Fig 3.2.2)

Fig (3.2.2) represents the compliment fuzzy graph G^c for the uniform fuzzy graph G represented by fig (3.2.1). From the above diagram it is clear that the crisp graph of G^c is an empty graph.

3.5 Theorem: If $G: (\sigma, \mu)$ is a uniform fuzzy graph on $G^*(V, E)$ then graph G is a Regular.

Proof: Let V be the finite set of vertices.

Since V is finite, we have

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 2, February 2017

$\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite. Let $|\sigma^*| = n$.

Given that G is a Uniform fuzzy graph. Then

$$\mu(x, y) = k, \forall (x, y) \in \sigma^* \times \sigma^* \quad \dots\dots\dots (1).$$

$$\text{And } \sigma(z) = k \forall z \in \sigma^* \text{ for some real } k \text{ where } 0 < k \leq 1 \dots\dots\dots (2).$$

Let $u \in \sigma^*$.

$$\text{Now } d_G(u) = \sum_{u \neq v} \mu(u, v) = \sum_{v \in \sigma^*} k = (n-1)k$$

$$\therefore d_G(u) = (n-1)k, \forall u \in \sigma^*.$$

Therefore G is regular Fuzzy graph. This completes the proof.

3.6 Theorem: If $G : (\sigma, \mu)$ is a Uniform fuzzy graph on $G^*(V, E)$ then G is a Totally Regular fuzzy graph.

Proof: Let V be the finite set of vertices.

Since V is finite, we have

$\sigma^* = \{x \in V / \sigma(x) > 0\}$ is also finite. Let $|\sigma^*| = n$.

Suppose G is a Uniform fuzzy graph. Then by definition we have

$$\mu(x, y) = k, \forall (x, y) \in \sigma^* \times \sigma^* \quad \dots\dots\dots (1).$$

$$\text{And } \sigma(z) = k, \forall z \in \sigma^* \text{ for some real } k \text{ where } 0 < k \leq 1 \dots\dots\dots (2).$$

Let $u \in \sigma^*$.

$$\begin{aligned} \text{Now } td_G(u) &= d_G(u) + \sigma(u) \\ &= \sum_{u \neq v} \mu(u, v) + \sigma(u) = \left(\sum_{v \in \sigma^*} k \right) + k \\ &= (n-1)k + k = nk \end{aligned}$$

$$\therefore td_G(u) = nk \forall u \in \sigma^*.$$

Therefore G is Totally Regular Fuzzy graph. This completes the proof.

3.7 Theorem-: If $G : (\sigma, \mu)$ is a Uniform fuzzy graph of level k on $G^*(V, E)$ then the order of G $O(G) = nk$, where $|\sigma^*| = n$.

Proof: Given that G is a Uniform fuzzy graph of level k. Then

$$\mu(x, y) = k, \forall (x, y) \in \sigma^* \times \sigma^* \quad \dots\dots\dots (1).$$

$$\text{And } \sigma(z) = k \forall z \in \sigma^* \text{ for some real } k \text{ where } 0 < k \leq 1 \dots\dots\dots (2).$$

$$\text{Also given } |\sigma^*| = n. \quad \dots\dots\dots (3).$$

$$\begin{aligned} \text{Now } O(G) &= \sum_{u \in V} \sigma(u) \\ &= \sum_{u \in \sigma^*} \sigma(u) \\ &= \sum_{u \in \sigma^*} k \\ O(G) &= nk \end{aligned}$$

$\therefore O(G) = nk$. This completes the proof.

3.8 Theorem-: If $G : (\sigma, \mu)$ is a Uniform fuzzy graph of level k on $G^*(V, E)$ then the size of G $S(G) = n_{c_2} k$, where $|\sigma^*| = n$.

Proof: Given that G is a Uniform fuzzy graph of level k. Then

$$\mu(x, y) = k, \forall (x, y) \in \sigma^* \times \sigma^* \quad \dots\dots\dots (1).$$

$$\text{And } \sigma(z) = k \forall z \in \sigma^* \text{ for some real } k \text{ where } 0 < k \leq 1 \dots\dots\dots (2).$$

$$\text{Also given } |\sigma^*| = n. \quad \dots\dots\dots (3).$$

$$\begin{aligned} \text{Now } S(G) &= \sum_{u, v \in E} \mu(u, v) = \sum_{u, v \in E} k = n_{c_2} k \\ &\quad \text{(Since } E \subseteq \sigma^* \times \sigma^* \text{ and } |E| = \frac{n^2 - n}{2} = n_{c_2}) \end{aligned}$$

3.9 Theorem-: If $G : (\sigma, \mu)$ and $H : (\tau, \vartheta)$ are two Uniform fuzzy graphs of level k on $G^*(V_G, E_G)$ and $H^*(V_H, E_H)$ then the union $G \cup H : (\sigma \cup \tau, \mu \cup \vartheta)$ is also a k-level uniform fuzzy graph.

Proof: Given that G, H are two k-level Uniform fuzzy graphs.

Then $\sigma(x) = k \forall x \in V_G$

$$\mu(x, y) = k, \forall (x, y) \in \sigma^* \times \sigma^*$$

Also $\tau(x) = k \forall x \in V_H$

$$\vartheta(x, y) = k, \forall (x, y) \in \tau^* \times \tau^*$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 2, February 2017

To show that $(\sigma \cup \tau)(x) = k \forall x \in V_G \cup V_H$

Let $x \in V_G \cup V_H$

Case (i): If $x \in V_G - V_H$

Then $(\sigma \cup \tau)(x) = \sigma(x) = k$

Case (ii): If $x \in V_H - V_G$

Then $(\sigma \cup \tau)(x) = \tau(x) = k$

$\therefore (\sigma \cup \tau)(x) = k \forall x \in V_G \cup V_H$

To show that $(\mu \cup \vartheta)(x, y) = k \forall (x, y) \in E_{G \cup H}$

Let $(x, y) \in E_{G \cup H}$

Case (i): If $(x, y) \in E_G - E_H$

Then $(\mu \cup \vartheta)(x, y) = \mu(x, y) = k$

Case (ii): If $(x, y) \in E_H - E_G$

Then $(\mu \cup \vartheta)(x, y) = \vartheta(x, y) = k$

$\therefore (\mu \cup \vartheta)(x, y) = k \forall (x, y) \in E_{G \cup H}$

So $G \cup H$ is a uniform fuzzy graph.

3.10 Theorem-: If $G : (\sigma, \mu)$ and $H : (\tau, \vartheta)$ are two Uniform fuzzy graphs of level k on $G^*(V_G, E_G)$ and $H^*(V_H, E_H)$ then the join $G + H : (\sigma + \tau, \mu + \vartheta)$ is also a k -level uniform fuzzy graph.

Proof: Given that G, H are two k -level Uniform fuzzy graphs.

Then $\sigma(x) = k \forall x \in V_G$

$\mu(x, y) = k, \forall (x, y) \in \sigma^* \times \sigma^*$

Also $\tau(x) = k \forall x \in V_H$

$\vartheta(x, y) = k, \forall (x, y) \in \tau^* \times \tau^*$

To show that $(\sigma + \tau)(x) = k \forall x \in V_G \cup V_H$

Let $x \in V_G \cup V_H$

$(\sigma + \tau)(x) = (\sigma \cup \tau)(x) = k$

(Since by Theorem 3.9)

$\therefore (\sigma + \tau)(x) = k \forall x \in V_G \cup V_H$

To show that $(\mu + \vartheta)(x, y) = k \forall (x, y) \in E_{G+H}$

Let $(x, y) \in E_{G+H}$

Case (i): If $(x, y) \in E_G \cup E_H$

$(\mu + \vartheta)(x, y) = (\mu \cup \vartheta)(x, y) = k$

(Since by Theorem 3.9)

Case (ii): If $(x, y) \in E^1$

Then $(\mu + \vartheta)(x, y) = \sigma(x) \wedge \tau(y)$

$$= k \wedge k = k$$

$\therefore (\mu + \vartheta)(x, y) = k \forall (x, y) \in E_{G+H}$

This completes the proof.

IV. CONCLUSION

In this paper we introduced the concepts of uniform fuzzy graph with an illustrated example. Also we produced various results related to uniform fuzzy graph. These results can be extended to of various products of two fuzzy graphs.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 2, February 2017



ACKNOWLEDGMENT

My Sincere Thanks to My Research supervisor **Dr. T. V. PRADEEP KUMAR** for all his immense support and valuable guidance and encouragement.

REFERENCES

1. Bhattacharya, P., Some Remarks on Fuzzy Graphs, Pattern Recognition Lett. 6(1987) 297- 302.
2. Bhutani, K.R., On Automorphism of fuzzy Graphs, Pattern Recognition Letters 12:413-420, 1991.
3. Euler, L., "Solutio problematis ad geometriam situs pertinentis", Comment. Academiae Sci. I. Petropolitanae 8 (1736), 128-140.
4. Frank Harary, Graph Theory, Narosa / Addison Wesley, Indian Student Edition, 1988.
5. John N. Mordeson and Premchand S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica-verlag, Heidelberg 2000.
6. Kauffman A., Introduction a la Theorie des Sous-ensembles Flous, Vol.I, Masson et Cie, (1973).
7. Nagoor Gani, A., and Radha, K., On Regular Fuzzy Graphs, Journal of Physical Sciences, Vol. 12, 2008, 33-40.
8. Rosenfeld, A., Fuzzy Graphs, In: L.A. Zadeh, K.S. Fu, M. Shimura, Eds., Fuzzy Sets and Their Applications, Academic Press (1975), 77-95
9. Sunitha, M.S., Studies on Fuzzy Graphs, Thesis submitted to the Cochin University Science and Technology For the award of the degree of *DOCTOR OF PHILOSOPHY* Under the Faculty of Science, Department of Mathematics, Cochin University Of Science And Technology, Cochin, April 2001.
10. Sunitha, M.S., Complement of a Fuzzy Graph, Indian J. pure appl. Math., 33(9) : 1451-1464, September 2002.
11. Venugopalam, D., Naga Maruthi Kumari and Vijay Kumar, M., operations on fuzzy graphs, south asian journal of mathematics, 2013, vol.3 (5) : 333-338
12. Yeh R. T. and Bang S. Y., Fuzzy Relations, Fuzzy Graphs and their Applications to Clustering Analysis, In Fuzzy Sets and their Applications to Cognitive and Decision Processes, Zadeh, L.A., Fu, K.S., Shimura, M. Eds; Academic Press, New York (1975) pp 125-149
13. Zadeh, L.A., Similarity Relations and Fuzzy Ordering, Inform. Sci. 3 (1971), 177-200.

BIOGRAPHY

	<p>K. Bhanu Chander is an Assistant professor in the Department of Mathematics Potti Sriramulu Chalavadi Mallikharjuna Rao College of Engineering and Technology, Vijayawada-520001, Krishna (DT), Andhra Pradesh, India. He received his Master's degree in mathematics in the year 2006 from P. B. Siddhartha college of arts & sciences, Vijayawada, A.P. He is been imparting mathematics into both the graduate students for the past eight years. His main research field is fuzzy graphs and their applications.</p>
	<p>Dr. Tumurukota Venkata Pradeep Kumar is presently working as Assistant Professor in Mathematics in University Engineering College, Acharya Nagarjuna University, Nagarjuna Nagar, Guntur (dt), A.P., India. He obtained his M. Sc and Ph.D., from Acharya Nagarjuna University under the guidance of Prof. Dr. Bhavanari Satyanarayana. He had brilliant academic career. His field of work is Algebra. He has nearly two decades of teaching experience in various capacities. He has published more than 40 research papers in various National and International Journals. He also published 10 books for U.G / P.G students. He attended more than 20 conferences/workshops / seminars and presented papers.</p>