

Evaluation of Longitudinal Proton Structure Function and Gluon Momentum Distribution by Thermodynamical Bag Model

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ABSTRACT: We evaluate the unpolarized and longitudinal structure function of proton separately in the moderate kinematic region at low order square four momentum transfer and gluon momentum distribution in the large x region at $Q^2 = 3 \text{ GeV}^2$ and 10 GeV^2 by Thermodynamical Bag Model(TBM). The evaluated structure function results are compared with JLab experimental data and gluon momentum distribution evaluation is compared with world data, NNPDF and DSSV fit collaboration data. Theoretical results are in good agreement with the experimental results and obtain the integral value of gluon in different kinematic region at $Q^2 = 10 \text{ GeV}^2$ which is consistent with NNPDF and DSSV fit.

KEYWORDS: DIS, Lonitudinal structure function, gluon momentum distribution, TBM

I. INTRODUCTION

Since few decades, the structure function of the nucleon is extracted from inclusive lepton-nucleon cross section as measured in Deep Inelastic Scattering (DIS). In this measurement, the longitudinal structure function of proton F_L is proportional to the cross section through the interaction of virtual photon and nucleon. In quark parton model, F_2 is the sum of the quark and anti-quark distributions weighted square of the electric charge of quarks in which F_L is zero. In Quantum Chromo Dynamics (QCD) model, F_L has non zero quantity which is due to the contribution of quarks and gluons[1]. From the measurement of longitudinal structure function, gluon distribution can be extracted and provides a sensitive test of the perturbative Quantum Chromo Dynamics (pQCD)[2-3].

In general, the experimental determination of F_L is difficult and it requires a measurement of the inelastic cross section at the same values of Bjorken variable x and square four momentum transfer Q^2 with incoming beam energy has different center of mass energy which was achieved electron-proton collider at HERA by changing the proton energy with fixed lepton energy. HERA[4] collected electron-proton data with H1 and ZESUS detector at a positron beam energy of 27.5 GeV and proton beam energies $920,575$ and 460 GeV which is allowed the measurement of structure function in kinematic region $6.5 \times 10^{-4} \leq x \leq 0.65$ at $0.2 \leq Q^2 \leq 800 \text{ GeV}^2$.

Several experiments are investigated of F_L on low kinematic region for the extraction of gluon density. Monghan[5] extracted lowest moment of F_L at $0.75 \leq Q^2 \leq 45 \text{ GeV}^2$ in the low kinematic region. This result reveals need for whether increased high x gluon distribution is possible given its large uncertainty or possibly non-negligible higher twist effects. Aaron et.al[6] presented the measurement of F_L in low x region at $12 \leq Q^2 \leq 90 \text{ GeV}^2$. This result exhibits gluon distribution is dominated at low x region. Liang et.al[7] presented the measurement of longitudinal and

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transverse structure functions in the resonant region in the squared four momentum transfer $0.2 < Q^2 < 5.5 \text{ GeV}^2$ at high x region. The longitudinal structure function is found to be substantial. Boroun[8] estimated the dynamical behavior of F_L at low x and high Q^2 . This result shows that the contribution of heavy quark in the measurement of F_L is considerable and cannot be vanished. In the present work, we focus large kinematic region and theoretical results are compared with JLab E00-002[9], NNPDF[10] and DSSV fit[11-12].

II. MATERIALS AND METHODS

The Methods used to Evaluate Longitudinal Proton Structure Function is Thermodynamical Bag Model.

III. THERMODYNAMICAL BAG MODEL

Thermodynamical Bag Model (TBM)[13-17] is modified from MIT bag model in which quarks and gluons are treated as fermions and bosons respectively. The invariant mass(W) of the final hadron and the equation of states can be expressed as

$$W^2 = M^2 + 2M\nu - Q^2 \quad \dots \quad (1)$$

$$6(n_u - n_{\bar{u}}) = \frac{2}{V} = \mu_u T^2 + \frac{\mu_u^3}{\pi^2} \quad \dots \quad (2)$$

$$6(n_d - n_{\bar{d}}) = \frac{1}{V} = \mu_d T^2 + \frac{\mu_d^3}{\pi^2} \quad \dots \quad (3)$$

Where V is the volume of bag, B is the bag constant, W is the invariant mass of excited nucleon at T , ν is the energy transfer, Q^2 is square of four momentum transfer, M is the mass of the nucleon at ground state, $6(n_u - n_{\bar{u}})$ is number density of up quark, $6(n_d - n_{\bar{d}})$ is the number density of down quark, μ_u is the chemical potential of up quark and μ_d is the chemical potential of down quark. Solving equations (1),(2) and (3) simultaneously we obtain the up and down quark chemical potentials with fixed Q^2 at any temperature. Here temperature and chemical potentials are evaluated for fixed Q^2 . The total energy density $\epsilon(T)$ of the bag can be written by the sum of energy densities of up quark, down quark and gluon is given by

$$\epsilon_u + \epsilon_{\bar{u}} = \left(\frac{1}{8\pi^2}\right)\mu_u^4 + \left(\frac{1}{4}\right)\mu_u^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad \dots \quad (4)$$

$$\epsilon_d + \epsilon_{\bar{d}} = \left(\frac{1}{8\pi^2}\right)\mu_d^4 + \left(\frac{1}{4}\right)\mu_d^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad \dots \quad (5)$$

$$\epsilon_g = \frac{\pi^2 T^4}{30} \quad \dots \quad (6)$$

$$\epsilon(T) = d_q(\epsilon_{u+\bar{u}}) + d_q(\epsilon_{d+\bar{d}}) + d_g \epsilon_g \quad \dots \quad (7)$$

Where $d_q = 6$ and $d_g = 16$ denotes the degeneracy of quarks and gluon orderly. The invariant mass is obtained by the energy transfer to the nucleon causes the heating up to the quarks and gluon. As the four momentum transfer tends to zero, temperature of the bag decreases and only the valence quarks are dominated. When $T \approx 0 \text{ MeV}$, the invariant mass W is equal to the mass of the nucleon at rest. Increasing Q^2 , the temperature of the bag increases and the invariant mass corresponds to the excited state of nucleon for $T > 0 \text{ MeV}$. Quark and gluon distribution function are derived from Fermi and Bose functions which are transferred to the infinite momentum frame and they are expressed in terms of temperature and chemical potentials as,

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$$q_i(x, Q^2) = \left(\frac{6V}{4\pi^2}\right) M^2 T x \ln \left\{ 1 + \exp \left[\left(\frac{1}{T}\right) \left(\mu_i - \frac{Mx}{2}\right) \right] \right\} \quad \dots \quad (8)$$

$$\bar{q}_i(x, Q^2) = \left(\frac{6V}{4\pi^2}\right) M^2 T x \ln \left\{ 1 + \exp \left[\left(\frac{1}{T}\right) \left(-\mu_i - \frac{Mx}{2}\right) \right] \right\} \quad \dots \quad (9)$$

μ_i is the chemical potential of quark with the flavour 'i'. Here 'i' denotes u or d quark. In order to relate the PDF's

with Λ_{QCD} , which is quark gluon coupling parameter, we introduce the strong quark gluon coupling constant. The experimental fit could be made by considering only with the QCD corrections. The quark and anti-quark distributions are modified by including QCD parameters as,

$$q_i'(x, Q^2) = q_i(x, Q^2) \left(1 - \frac{\alpha_s(Q^2)}{2\pi} \right) \quad \dots \quad (10)$$

$$\bar{q}_i'(x, Q^2) = \bar{q}_i(x, Q^2) \left(1 - \frac{\alpha_s(Q^2)}{2\pi} \right) \quad \dots \quad (11)$$

The strong running coupling constant (α_s) for various Q2 is evaluated using the Next to Leading Order (NLO) solution.

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2 / \Lambda^2)} \left[1 - \frac{\beta_1 (\ln(\ln(Q^2 / \Lambda^2)))}{\beta_0 \ln(Q^2 / \Lambda^2)} \right] \quad \dots \quad (12)$$

Where $\beta_0 = 11 - (2 N_f / 3)$ and $\beta_1 = 102 - (38 N_f / 3)$. In order to account for heavy quark threshold correction and target mass effect together, a substitution of x is made with ξ .

$$\xi = \frac{2x((1+m_s^2)/Q^2)}{1 + [(1 + (4M^2 x^2 / Q^2)(1+m^2)/Q^2)]^{1/2}} \quad \dots \quad (13)$$

m_s is the mass of the strange quark. Here we assume strange quark mass as 100 MeV and $\Lambda_{QCD} = 200$ MeV.

IV. EVALUATION OF STRUCTURE FUNCTIONS

The structure function F_1 and F_2 are related by Callon-Gross equation[18],

$$2xF_1 = F_2 \quad \dots \quad (14)$$

Structure function can be obtained by the quark distribution function and cross section for deep inelastic scattering. Using quark distribution functions of up and down quark and their anti-quarks, the unpolarized structure function of nucleon can be expressed as,

$$F_2^p = x \left[\frac{4}{9}(u' + \bar{u}') + \frac{1}{9}(d' + \bar{d}') \right] \quad \dots \quad (15)$$

$$F_2^n = x \left[\frac{1}{9}(u' + \bar{u}') + \frac{4}{9}(d' + \bar{d}') \right] \quad \dots \quad (16)$$

The unpolarized structure function has a weak dependence of Q2. The structure function of neutron more than the proton structure function at low x region whereas F_2^p increases than F_2^n at high x region. In TBM, the up and down

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quark momentum distribution for proton and neutron being merely equal and there is a greater probability of nucleon spin parallel to the target. The proton structure function as a function of $W^2(\text{GeV}^2)$ at $Q^2 = 0.3, 0.7$ and 1.1 GeV^2 as shown in figure 1.

The longitudinal structure function of proton can be written in terms of F_1 and F_2 ,

$$F_L = F_2(x, Q^2) \left(1 + \frac{Q^2}{\nu^2}\right) - 2xF_1 \quad \dots(17)$$

V. EVALUATION OF GLUON DISTRIBUTION

Here we consider a bag contains non-interacting valence quarks and sea quarks in gluons. The gluon distribution increases with increasing the temperature. At low temperature, proton contains only valence quarks. Gluon distribution can be written as,

$$G(x) = -\frac{16V}{4\pi^2} M^2 T x \ln \left[1 + \exp\left(-\frac{Mx}{2T}\right) \right] \quad \dots \quad (18)$$

Gluon carried large amount of spin compared with valence quarks inside a proton. The spin carried by the gluons Δg is obtained using x for the spin dilution of the gluon.

$$\Delta g = xG(x) \quad \dots \quad (19)$$

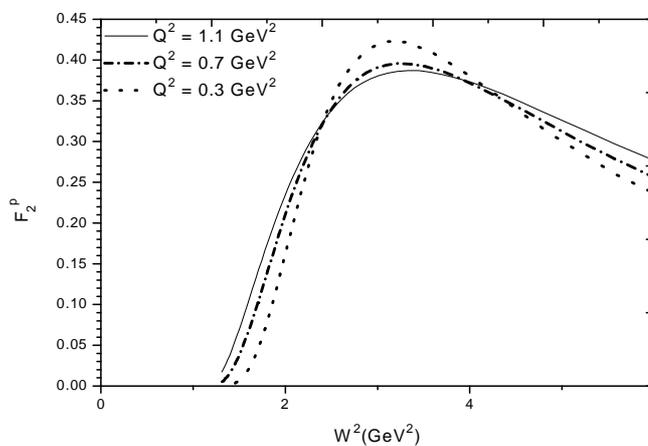


Fig. 1 – Variation of proton structure function with x .

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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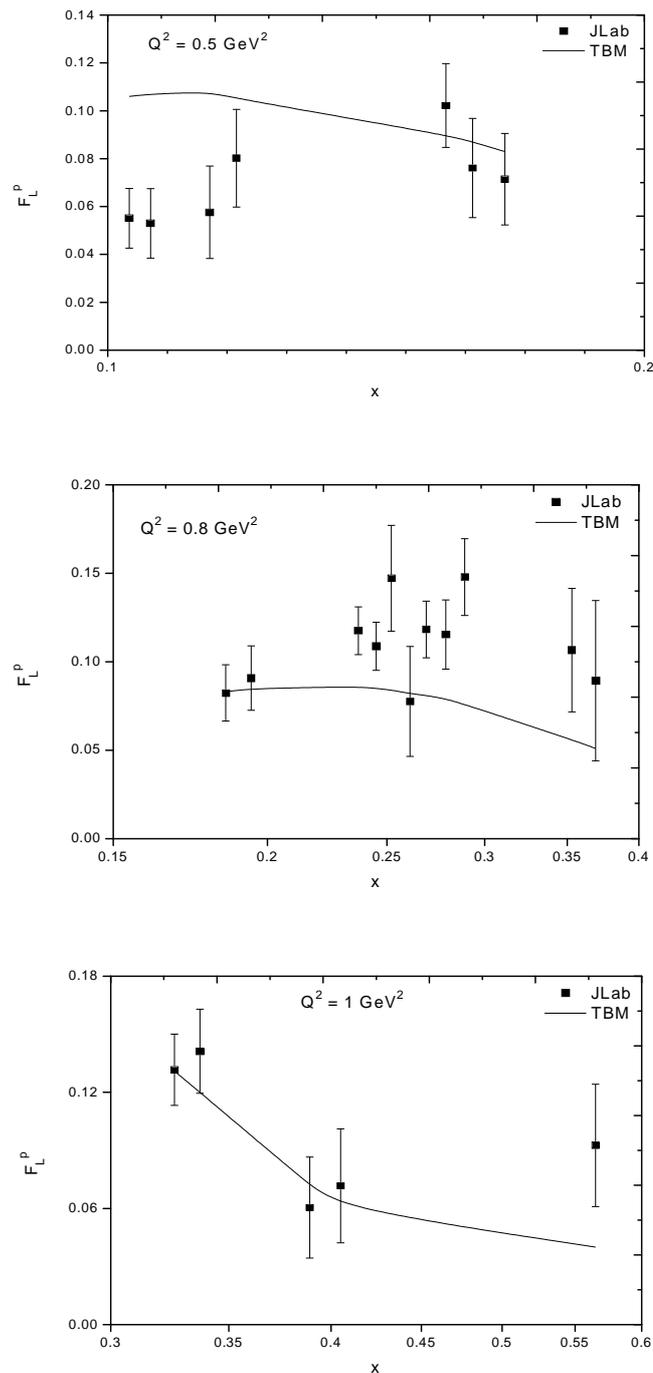


Fig. 2 – Longitudinal proton structure function as function of x and Q^2 . TBM results(solid line) are compared with JLab experimental data(shaded squares).

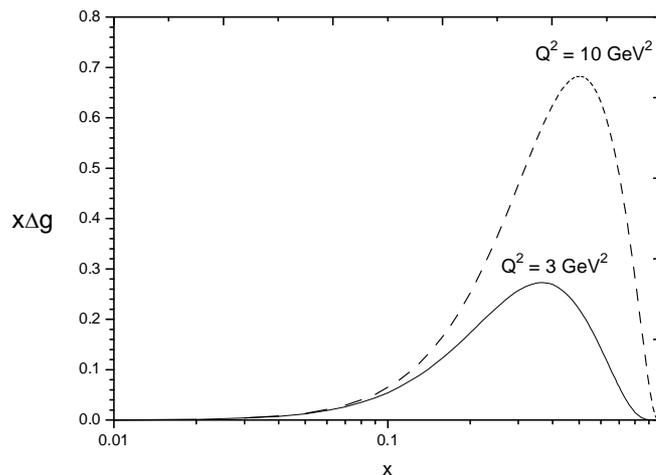


Fig. 3 – Gluon momentum distribution as a function of x at $Q^2 = 3$ and 10 GeV^2 .

VI. RESULTS AND DISCUSSION

In this present work, we evaluate the longitudinal structure function of proton using the quark distribution based on thermodynamical bag model. By use of up and down quark distribution, the structure function of proton is estimated. The invariant mass of the nucleon of the final hadronic system increases when decreasing the Bjorken variable x and it yields to the production of sea quarks and gluons. At low x , energy transfer is greater than the square four momentum transfer. Figure 1 shows that the variation of proton structure function with the square invariant mass of the nucleon W^2 . At low Q^2 , F_2^p increases and then it decreases with W^2 which is deviated with experimental data. This is due to more contribution of sea quarks in the low x region which is the natural consequence of this model. Increasing the value of Q^2 , proton structure function has similar behavior with experimental results when W^2 is more than 4 GeV^2 . The momentum distribution of up quark dominated over down quark in the proton structure function. This is fact that the role of their electric charge. The momentum distribution of \bar{d} is more compared with \bar{u} . Figure 2 shows that the variation of longitudinal structure functions with Bjorken variable x for different Q^2 values. At $Q^2 = 0.5$ and 0.8 GeV^2 , F_L increases with x and then it decreases. At $Q^2 = 1 \text{ GeV}^2$, longitudinal structure function F_L decreases with increasing the value of x . However, F_L is positive in the evaluated kinematic region due the dominating of up quark distribution over that of down quark distribution. The gluon momentum distribution with Bjorken variable x for different four momentum transfer is shown in figure 3.

At $Q^2 = 3 \text{ GeV}^2$, gluon distribution is close agreement with world data and disfavoured with NNPDF data at $Q^2 = 10 \text{ GeV}^2$ which is due to increasing Q^2 value gluon momentum distribution attains the maximum value up to $x = 0.48$ and then it decreases. The shape of the distribution is different between $x = 0.07$ and $x = 0.97$ with NNPDF data. The integral value obtained for $\Delta g = 0.21$ is comparable with NNPDF original fit data 0.06 ± 0.18 in the kinematic region of $0.05 < x < 0.5$. The evaluated gluon distribution is positive in the entire kinematic region. Our evaluated all theoretical results are in good agreement with experimental data.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 1, January 2017

Table 1: The evaluated integral value of Δg at $Q^2 = 10 \text{ GeV}^2$ is compared with experimental data.

	$0 < x < 1$	TBM	$0.001 < x < 1$	TBM	$0.05 < x < 0.2$	TBM
NNPDFpol1.0	-0.95 ± 3.87		-0.06 ± 1.12		0.05 ± 0.15	
NNPDFpol1.1	0.03 ± 3.24	0.54	0.49 ± 0.75	0.536	0.17 ± 0.06	0.20
DSSV08	-		$0.03^{+0.702}_{-0.314}$		$0.005^{+0.120}_{-0.164}$	
DSSV++	-				$0.10^{+0.06}_{-0.07}$	

VII. CONCLUSION

Our TBM evaluation of Longitudinal Proton Structure Function and Gluon Momentum Distribution are good agreement with the available experimental data. The integral value Δg is consistent with NNPDF and DSSV fit.

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