

# A Note on Bipartite and Balanced Fuzzy Graph Structures

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**ABSTRACT:** In this paper, we introduce  $\rho_i$ -bipartite fuzzy graph structure, completely  $\rho_i$ -bipartite fuzzy graph structure and some characteristics. Also we introduce  $\rho_i$ -balanced fuzzy graph structure, completely  $\rho_i$ -balanced fuzzy graph structure and study some properties. Also generalize all these concepts.

**KEY WORDS:**  $\rho_i$ -bipartite fuzzy graph structure, completely  $\rho_i$ -bipartite fuzzy graph structure,  $\rho_i$ -balanced fuzzy graph structure, completely  $\rho_i$ -balanced fuzzy graph structure.

## I. INTRODUCTION

The notion of fuzzy set theory was introduced by L.A.Zadeh in 1965 [13]. Using the concept of fuzzy subsets the concept of fuzzy graph was introduced by A.Rosenfeld in 1975 [11]. After that several authors including Moderson and Nair [9], Bhattacharya and suraweera [1], Bhutani and Rosenfeld [1-3], etc. have took initiative in growing fuzzy graph theory. E. Sampathkumar [12] introduced the concept of graph structure, which in particular is a generalization of the notions like graphs, signed graphs and edge-coloured graphs with the colorings. The study of graph structure is relevant because it helps to study the various relations and the corresponding edges simultaneously. Dinesh.T and T.V Ramakrishnan [5,6,7] introduced the concept of Fuzzy graph structures using the concept of graph structure in [12]. Fuzzy graph structure concept is developed by grouping the edges with same membership function.

In this paper, we introduce  $\rho_i$ -bipartite fuzzy graph structure, completely  $\rho_i$ -bipartite fuzzy graph structure and some characteristics. Also we introduce  $\rho_i$ -balanced fuzzy graph structure, completely  $\rho_i$ -balanced fuzzy graph structure and study some properties. Also generalize all these concepts.

## II. PRELIMINARIES

**Definition 2.1[13]** A **fuzzy set** of a base set  $V = \{v_1, v_2, \dots, v_n\}$  (non empty set) is specified by its membership function  $\sigma$ ; where  $\sigma : V \rightarrow [0,1]$  assigning to each  $v_i \in V$ , the degree or grade to which  $V \in \sigma$ .

**Definition 2.2[10]** A **fuzzy graph**  $G=(\sigma, \mu)$  is a pair of functions  $\sigma:V \rightarrow [0,1]$  and  $\mu:V \times V \rightarrow [0,1]$ , where for all  $v_i, v_j \in V$ , we have  $\mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$  for each  $(v_i, v_j) \in V \times V$ . Here  $\sigma$  and  $\mu$  are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph  $G=(\sigma, \mu)$ .

**Definition 2.3[12]** A **graph structure**  $G=(V, R_1, R_2, \dots, R_k)$  consists of a non-empty set  $V$  together with relations  $R_1, R_2, \dots, R_k$  on  $V$  which are mutually disjoint such that each  $R_i, 1 \leq i \leq k$ , is symmetric and irreflexive. If  $(u, v) \in R_i$ , for some  $i, 1 \leq i \leq k$ , we call it an  $R_i$ - edge and write it as  $uv$ .

**Example 2.4[12]** Any graph can be regarded as a graph structure  $G=(V, R_1, R_2)$  where  $uv \in R_1$  if and only if  $uv$  is an edge of  $G$  and  $xy \in R_2$  if  $x$  and  $y$  are non adjacent vertices in  $G$ .

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**Definition 2.5[5]** Let  $G=(V,R_1,R_2,\dots,R_k)$  be a graph structure and  $\mu,\rho_1,\rho_2,\dots,\rho_k$  be fuzzy subsets of  $V,R_1,\dots,R_k$  respectively such that  $\rho_i(x,y)\leq\mu(x)\wedge\mu(y)$  for every  $x,y\in V$  and  $i=1,2,\dots,k$ . Then  $\overline{G}=(\mu,\rho_1,\rho_2,\dots,\rho_k)$  is a **fuzzy graph structure** of  $G$ .

**Example 2.6[5]** Let  $V=\{v_0,v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$ ,  $R_1=\{(v_0,v_1),(v_1,v_2),(v_2,v_3),(v_0,v_3)\}$ ,  $R_2=\{(v_0,v_4), R_3=\{(v_3,v_4),(v_4,v_5),(v_5,v_0),(v_6,v_7)\}$ . Let  $G=(V,R_1,R_2,R_3)$  be a graph structure.

Let  $\overline{G}=(\mu,\rho_1,\rho_2,\rho_3)$  where  $\mu(v_0)=0.8, \mu(v_1)=0.9, \mu(v_2)=0.6, \mu(v_3)=0.5, \mu(v_4)=0.6, \mu(v_5)=0.7, \mu(v_6)=0.6, \mu(v_7)=0.5,$   
 $\rho_1(v_0,v_1)=0.8, \rho_1(v_1,v_2)=0.5, \rho_1(v_2,v_3)=0.5, \rho_1(v_0,v_3)=0.5$   
 $\rho_2(v_0,v_4)=0.6, \rho_3(v_3,v_4)=0.5, \rho_3(v_4,v_5)=0.6, \rho_3(v_5,v_0)=0.4, \rho_3(v_6,v_7)=0.5.$

$\overline{G}=(\mu,\rho_1,\rho_2,\rho_3)$  is a fuzzy graph structure of  $G=(V,R_1,R_2,R_3)$ .

In cases where  $\rho_i$ -values are zeros, do not write them explicitly.

Throughout this paper  $G$  and  $\overline{G}$  represent the graph structure  $G=(V,R_1,R_2,\dots,R_k)$  and its fuzzy graph structure  $\overline{G}=(\mu,\rho_1,\rho_2,\dots,\rho_k)$ , unless otherwise specified.

**Definition 2.7[5]** Let  $G$  be a graph structure and  $\overline{G}$  be a fuzzy graph structure of  $G$ . If  $(x,y)\in\sup p(\rho_i)$ , then  $(x,y)$  is said to be a  $\rho_i$ -edge of  $\overline{G}$ .

**Definition 2.8[5]** An  $R_i$ -cycle of  $\overline{G}$  is an alternating sequence of vertices and edges  $v_0,e_1,v_1,e_2,\dots,v_{n-1},e_n,v_n=v_0$  consisting only of  $R_i$ -edges.

**Definition 2.9[5]**  $\overline{G}$  is a  $\rho_i$ -cycle if and only if  $(\sup p(\mu),\sup p(\rho_1),\sup p(\rho_2),\dots,\sup p(\rho_k))$  is an  $R_i$ -cycle.

**Definition 2.9[5]**  $\overline{G}$  is a fuzzy  $\rho_i$ -cycle if and only if  $(\sup p(\mu),\sup p(\rho_1),\sup p(\rho_2),\dots,\sup p(\rho_k))$  is an  $R_i$ -cycle and there exists no unique  $(x,y)$  in  $\sup p(\rho_i)$  such that  $\rho_i(x,y)=\wedge\{\rho_i(u,v)\in\sup p(\rho_i)\}$ .

**Definition 2.10[5]** An alternating sequence of vertices and  $R_i$ -edges for some  $i\in\{i_1,i_2,\dots,i_r\}, 1\leq r\leq k$ , of a graph structure  $G=(V,R_1,R_2,\dots,R_k)$  is an  $R_{i_1i_2\dots i_r}$ -path where  $R_{i_1},R_{i_2},\dots,R_{i_r}$  are some among  $R_1,R_2,\dots,R_k$  which are represented in it.

**Definition 2.11[5]** Let  $v_0,v_1,\dots,v_n$  be a sequence of vertices of  $\overline{G}$ . Let  $\rho_{i_p}(v_{j-1},v_j)>0$  for every  $j=1,2,\dots,n$  for some  $p\in\{1,2,\dots,r\}$  where  $\rho_{i_1},\rho_{i_2},\dots,\rho_{i_r}$  are some among  $\rho_1,\rho_2,\dots,\rho_k$ . Then  $v_0v_1\dots v_n$  is a  $\rho_{i_1i_2\dots i_r}$ -path.

**Definition 2.12[5]**  $\overline{G}$  is a  $\rho_{i_1i_2\dots i_r}$ -cycle if and only if  $(\sup p(\mu),\sup p(\rho_1),\sup p(\rho_2),\dots,\sup p(\rho_k))$  is an  $R_{i_1i_2\dots i_r}$ -cycle ( $\rho_{i_1},\rho_{i_2},\dots,\rho_{i_r}$  are some among  $\rho_1,\rho_2,\dots,\rho_k$  which correspond to  $R_{i_1},R_{i_2},\dots,R_{i_r}$ ).

**Definition 2.13[5]**  $\overline{G}$  is a fuzzy  $\rho_{i_1i_2\dots i_r}$ -cycle if and only if  $(\sup p(\mu),\sup p(\rho_1),\sup p(\rho_2),\dots,\sup p(\rho_k))$  is an  $R_{i_1i_2\dots i_r}$ -cycle and there exists no unique  $(x,y)$  in  $\bigcup_{q=1}^r\sup p(\rho_{i_q})$  such that  $\bigvee_{q=1}^r\rho_{i_q}(x,y)=\wedge\{\bigvee_{q=1}^r\rho_{i_q}(u,v)\in\bigcup_{q=1}^r\sup p(\rho_{i_q})\}$ .

**Definition 2.14[12]** A set  $\{R_1,R_2,\dots,R_k\}$  of relations on a set  $V$  is mutually independent if for any two relations  $ab$  and  $R_j, i\neq j$ , the following holds: if  $ab$  is an  $R_i$ -edge, then  $\{a,b\}$  is not a subset of  $V(R_j)$ . In other words, if  $ab$  is an  $R_i$ -edge then  $|\{a,b\}\cap V(R_j)|\leq 1$ .

**Definition 2.15[8]** A signed graph is balanced if no cycle in  $G$  contains an odd no. of negative edges.

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**Proposition 2.16[8]** A signed graph is balanced if and only if its vertex set can be partitioned into two sets  $V_1$  and  $V_2$  such that each positive edge joins vertices of the same set and each negative edge joins vertices of different sets.

### III. BIPARTITE AND BALANCED FUZZY GRAPH STRUCTURES

**Definition 3.1**  $\bar{G}$  is said to be  $\rho_i$ -bipartite fuzzy graph structure for some  $i, 1 \leq i \leq k$  if the vertex set  $V$  can be partitioned into two sets  $V_1, V_2$  such that every  $\rho_i$ -edge joins a vertex in  $V_1$  and a vertex in  $V_2$ .

**Definition 3.2**  $\bar{G}$  is said to be completely  $\rho_i$ -bipartite fuzzy graph structure if for each  $i, 1 \leq i \leq k$ ,  $\bar{G}$  is  $\rho_i$ -bipartite.

**Definition 3.3**  $\bar{G}$  is said to be  $\rho_i \rho_{i+1} \dots \rho_r$ -bipartite fuzzy graph structure,  $1 \leq i \leq r \leq k$  if the vertex set  $V$  can be partitioned into two  $\rho_i \rho_{i+1} \dots \rho_r$  independent sets  $V_1, V_2$  such that there is no  $\rho_j$ -edge  $1 \leq j \leq i$  between  $V_1$  and  $V_2$ .

**Definition 3.4**  $\bar{G}$  is said to be  $\rho_1 \rho_2 \dots \rho_i$ -bipartite fuzzy graph structure  $1 \leq i \leq k$ , if the vertex set  $V$  can be partitioned into two sets  $V_1, V_2$  such that every  $\rho_r$ -edge,  $1 \leq r \leq i$  joins a vertex in  $V_1$  and a vertex in  $V_2$ .

**Proposition 3.5** A fuzzy graph structure  $\bar{G}$  is  $\rho_i$ -bipartite fuzzy graph structure if and only if it does not have an odd fuzzy  $\rho_i$ -cycle,  $1 \leq i \leq k$ .

**Proposition 3.6** A fuzzy graph structure  $\bar{G}$  is  $\rho_i \rho_{i+1} \dots \rho_r$ -bipartite fuzzy graph structure if and only if every fuzzy  $\rho_{i_1 i_2 \dots i_r}$ -cycle from the set  $\{\rho_i, \rho_{i+1}, \dots, \rho_r\}$  which corresponds to  $\{R_i, R_{i+1}, \dots, R_r\}$ ,  $1 \leq i \leq r \leq k$  contains even number of edges.

**Proposition 3.7** A fuzzy graph structure  $\bar{G}$  is  $\rho_1 \rho_2 \dots \rho_i$ -bipartite fuzzy graph structure if and only if it does not have an odd fuzzy  $\rho_{i_1 i_2 \dots i_r}$ -cycle from the set  $\{\rho_1, \rho_2, \dots, \rho_i\}$  which corresponds to  $\{R_1, R_2, \dots, R_i\}$ ,  $1 \leq i \leq k$ .

**Definition 3.8** A fuzzy graph structure  $\bar{G}$  is said to be fuzzy  $\rho_i$ -balanced if  $V$  can be partitioned into two  $\rho_i$ -independent sets  $V_1, V_2$  such that the only edges joining  $V_1$  and  $V_2$  are  $\rho_i$ -edges.

**Definition 3.9** A fuzzy graph structure  $\bar{G}$  is said to be completely  $\rho_i$ -balanced fuzzy graph structure if  $\bar{G}$  is  $\rho_i$ -balanced for each  $i$ .

**Definition 3.10** A fuzzy graph structure  $\bar{G}$  is said to be fuzzy  $\rho_i \rho_{i+1} \dots \rho_r$ -balanced if  $V$  can be partitioned into two  $\rho_i \rho_{i+1} \dots \rho_r$ -independent sets  $V_1, V_2$  such that the edges joining  $V_1$  and  $V_2$  are from  $\rho_i, \rho_{i+1}, \dots, \rho_r$ -edges,  $1 \leq i \leq r \leq k$ .

**Proposition 3.11** A fuzzy graph structure  $\bar{G}$  is  $\rho_i$ -balanced,  $1 \leq i \leq k$  if and only if every  $\rho_i$ -cycle in  $\bar{G}$  contains even number of  $\rho_i$ -edges.

**Remark 3.13** If  $\bar{G}$  is completely  $\rho_i$ -balanced for each  $i, 1 \leq i \leq k$ , then number of  $\rho_i$ -edges is  $2r$  for some integer  $r \geq 0$ .

**Proposition 3.14** A fuzzy graph structure  $\bar{G}$  is  $\rho_i$ -balanced, then  $\bar{G}$  is  $\rho_i$ -bipartite,  $1 \leq i \leq k$ . Converse need not be true.

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**Proposition 3.15** A fuzzy graph structure  $\overline{G}$  is completely  $\rho_i$ -balanced, then  $\overline{G}$  is completely  $\rho_i$ -bipartite fuzzy graph structure.

## IV. CONCLUSION

In this paper we extend the concepts of Bipartite and Balanced Graph structure to Fuzzy Graph Structures. The concept of fuzzy graph structures is very useful in solving many real life problems.

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